ON THE EFFICIENCY COEFFICIENT OF A WORM GEARING

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Abstract. The efficiency coefficient of a mechanism is an important parameter, which quantifies the rate of energy loss. Compared with other gearing mechanisms, the worm gearings are characterized with lower values of the efficiency coefficient, which makes the ways of studying and measures for its increase particularly relevant. This work offers an analytical expression for the efficiency coefficient as a function of characteristic geometrical and kinematical parameters of the gearing (module, gear teeth numbers and angular velocity of the gearing, speed ratio). A numerical study was carried out to determine the direction and the degree of influence of these factors.

Key words: worm gearing, efficiency coefficient, loss coefficient, friction forces

1. Introduction

The efficiency coefficient is an important characteristic of a gearing. It is a trend in modern engineering design to pay special attention to this parameter because it is an indication of the energy effectiveness and the reliability of the mechanisms and machinery. The problem is of particular interest in the case of the worm gearings, where the losses resulting from the sliding action of the worm thread geared with that of the wheel are high. The efficiency coefficient is influenced by the choice of the materials for the gear elements, as well as by the nature of the friction between them in the mesh. The friction regime can be defined by a set of geometrical (construction) and kinematical (exploitation) parameters, such as the gear teeth number and the rotation speed of the worm, the wheel teeth number, the module, etc. [1, 2]. A further analysis of the degrees of their relative influence could facilitate the constructor's work towards design of energy-efficient worm gearings.

2. Purpose

The purpose of this work is to:

- determine analytically the influence of specific geometrical and kinematical parameters of a worm gearing upon friction losses in the mesh;
- study numerically and visualize the obtained analytic expression(s) in graphical form.

3. Analytical calculations

List of the used parameters

η – Efficiency coefficient;
i – Speed ratio;
m – Module;
N_f – Friction force coefficient;
α – Profile angle ;
ρ' – Reduced friction angle;
z_1 – Worm lead number;
z_2 – Gear teeth number;
F_n – Normal component of the force in the mesh;
g – Coefficient of the worm diameter;
n_1 – Frequency of worm's rotation;
μ' – Reduced friction coefficient;
V_s – Sliding velocity in the mesh;
V_1 – Surface velocity of the worm along the separating diameter;
V_2 – Peripheral velocity of the wheel along the separating diameter;
γ – Elevation angle of the worm's helix line.

A worm gearing is based on the mechanism of helix-tooth meshing, which leads to losses characteristic of both the tooth-surface threaded and bolt-nut gearings [3].

It has become conventional to determine the efficiency coefficient from the well known relation for a bolt-nut pair:

$$\eta = \frac{tg\gamma}{tg(\gamma + \rho')} \tag{1}$$

Using the relations:

$$tg\gamma = \frac{z_1}{g} = \frac{z_2}{g \cdot i}; \quad tg\rho' = \mu' = \frac{\mu}{\cos\alpha} \tag{2}$$

and after some trigonometric and algebraic transformations Eq. (1) has the form:

$$\eta = \frac{1 - \frac{\mu' \cdot z_2}{g \cdot i}}{1 + \frac{\mu' \cdot g \cdot i}{z_2}} \tag{3}$$

By taking into account the resistance / tenacity / insusceptibility condition for the worm

[3, 4] $q_{min}/z_2 = 0.25 = k$ is obtain for the efficiency coefficient

$$\eta = \frac{1 - \frac{\mu'}{k \cdot i}}{1 + \mu' \cdot k \cdot i} \quad (4)$$

The influence of the parameters μ’, *i*, *k* is shown in Figure 1, where the dependence $\eta = f(\mu' i)$ is represented in graphical form for $k = 0.25$ and $k = 0.5$.

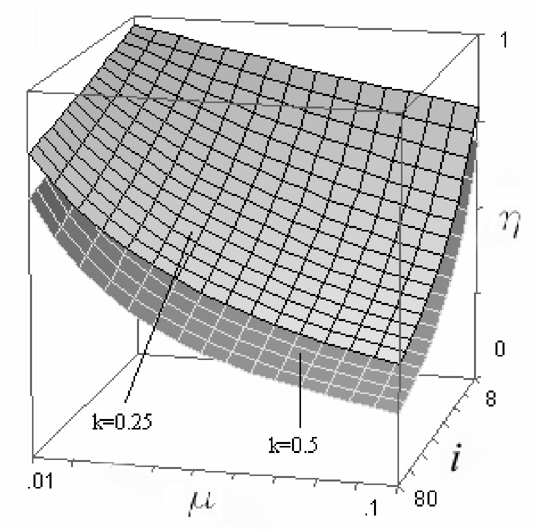

Fig. 1. Dependence of the efficiency coefficient on the reduced friction coefficient and the speed ratio

In order to determine the losses of a worm gearing, the classical definition of the loss coefficient is used [5]:

$$\psi = \frac{N_f}{N_o}, \quad (5)$$

where the input power of the gearing is

$$N_o = T_1 \cdot \omega_1 = F_{t_1} \cdot r_1 \cdot \omega_1 = F_{t_1} \cdot v_1 \quad (6)$$

and the meshing friction power:

$$N_f = \mu \cdot F_n \cdot v_s, \quad (7)$$

where

$$F_n = F_t \cdot \frac{1}{\cos\alpha \cdot \sin\gamma + \mu \cdot \cos\gamma}. \quad (8)$$

Having in mind the expressions (6), (7), and (8), is obtain for the loss coefficient:

$$\psi = \mu \cdot \frac{v_s}{v_1} \cdot \frac{1}{\cos\alpha \cdot \sin\gamma + \mu \cdot \cos\gamma}. \quad (9)$$

From the velocity diagram for the worm gearing, figure 2, is can deduce v_1 and v_2 in the form

$$v_2 = v_1 \cdot \mathrm{tg}\,\gamma; \quad (10)$$

$$v_s = \sqrt{v_1^2 + v_2^2} = \sqrt{v_1^2 \cdot \left(1 + \mathrm{tg}^2\gamma\right)} = = v_1 \cdot \sqrt{1 + \left(\frac{z_2}{g \cdot i}\right)^2} = v_1 \cdot \sqrt{1 + \left(\frac{1}{k \cdot i}\right)^2} \quad (11)$$

or

$$\frac{v_1}{v_s} = \frac{k \cdot i}{\sqrt{(k \cdot i)^2 + 1}} = \cos\gamma, \quad (12)$$

respectively

$$\sin\gamma = \frac{1}{\sqrt{(k \cdot i)^2 + 1}}. \quad (13)$$

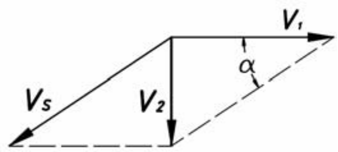

Fig. 2. Speed diagram

After substituting the Equations (12) and (2) into (8) and further mathematical transformations, the relation for ψ takes the form:

$$\psi = \frac{k \cdot i + \frac{1}{k \cdot i}}{k \cdot i + \frac{1}{\mu'}}. \quad (14)$$

Obviously, the losses would depend strongly on the friction coefficient, the value of which is mainly a function of the friction regime. If the lubrication substances are properly chosen, the friction regime is determined by the sliding velocity. The literature offers sets of similar experimental data for the influence of the sliding velocity v_3 on the friction coefficient [2, 6]. The dependence of $\mu' = f(v_s)$ is plotted in Figure 3 from the data provided in [6], while the mathematical expression for the function is mathematically derived by computer calculations, namely

$$\mu' = 0.0417 \cdot v_s^{-0.33}, \quad (15)$$

where v_S has the physical dimension of m/s.

The correlation coefficient of the approximate function is 0.9724.

The sliding velocity v_S can be expressed from (12), with v_1 written as:

$$v_1 = \omega_1 \cdot r_1 = \frac{\pi \cdot n_1 \cdot m \cdot g}{60} = \frac{\pi \cdot n_1 \cdot m \cdot z_1}{60} \cdot k \cdot i \quad (16)$$

and

$$v_s = \frac{\pi \cdot m \cdot z_1 \cdot n_1}{60} \cdot \sqrt{(k \cdot i)^2 + 1}, \; [m/s]. \qquad (17)$$

By taking into account relations (17) and (15), equation (14) takes the form:

$$\psi = \frac{k \cdot i + \frac{1}{k \cdot i}}{ki + 9 \cdot \sqrt[3]{n_1 \cdot m \cdot z_1} \cdot \sqrt[6]{k^2 \cdot i^2 + 1}}. \qquad (18)$$

The analytical obtained relation (18) reflects the effects of some of the gearing parameters on the friction losses in the mesh.

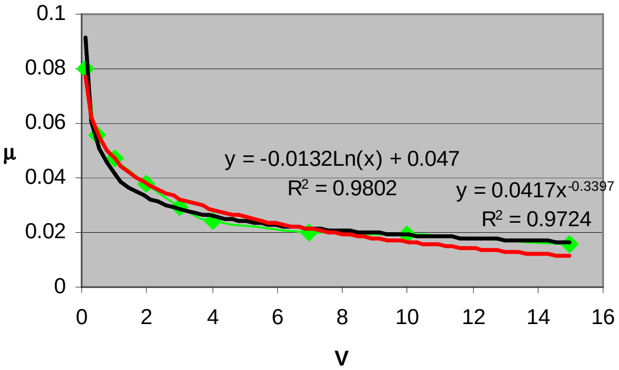

Fig. 3. Dependence of the reduced friction coefficient on the sliding speed

4. Numerical analysis and graphical visualization

The study was been carried out using the software package Derive 6, which offers the possibilities of calculating and displaying functions in graphical form, where two variables can be varied.

In studying the losses, we accepted the following finite set of variables: the gear lead numbers (i = 1, 2, 4) and the worm revolution (n_1 = 750; 1000; 1500; 3000 per minute). The revolutions chosen correspond to those of the standard asynchronous motors. From robustness arguments we accept $k = 0.25$. Consequently within a loop with a certain increment the values of the parameters are assigned as follows $i = 1 \div 8$ and $m = 2 \div 20$.

Figure 4 illustrate the relations $\psi = f(m, i, n_1)$ for z = 1; 2; 4, respectively.

From the graphical form of the relations we can conclude that the influence of all of the factors is nonlinear, and the losses increase with the increase of the speed ratio, and decrease with the increase of the revolutions (n_1), the module (m), and the gear teeth number (z_1). The influence of the factors (m), (z_1) (i) is substantial, while it is negligible for (n_1).

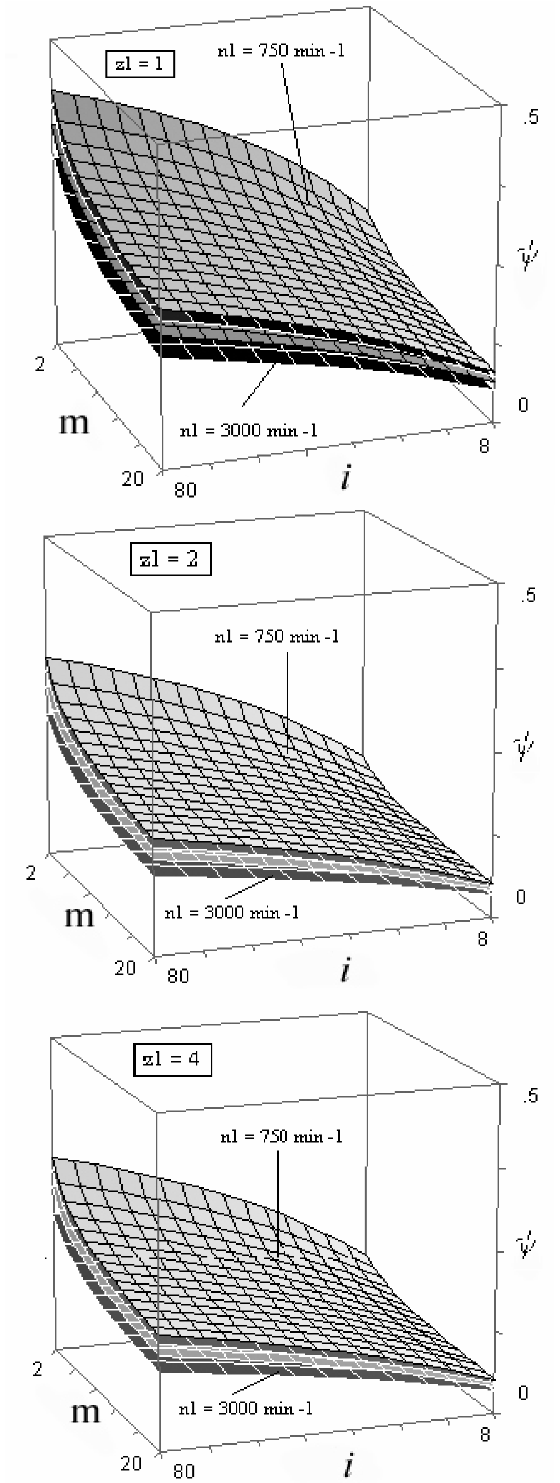

Fig. 4. Dependence of the loss coefficient on the module, the speed ratio and the turning rate of the worm

5. Conclusions

1. A function representing the influence of the module, gear teeth number, the frequency of the

worm element, and the speed ratio on the losses in the meshing of a worm gearing was found analytically.

2. A numerical study has been carried out with its results presented in graphical form, which indicated that:

- the speed ratio has the strongest influence, leading to an increase in the losses;
- the factors (m) and (z_1) have substantial influence – increases in their values would lead to decreasing losses;
- the influence of n_1 is insignificant.

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