DYNAMICAL MODELING OF IMPACT CRUSHER'S ROTOR SYSTEM

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Abstract. In this paper is performing a dynamical analysis of 3D forced vibration on a heavy rotor of impact crusher. Dynamical model with three mass and eight degree of freedom has been created. The applied loads are transient impulse load. Differential equations of motion are obtained with account of mass, elastic, damping and geometrical properties. The numerical solution of differential equations of motion is obtained with MATLAB program. Numerical experiments show influences of dynamical characteristics of crusher are performed.

Key words: nonlinear vibrations, vibration of rotary shaft, impulse load, impact crusher

1. Introduction

The purpose of this work is to create of dynamical model of impact crusher's rotor system described forced vibration excited from impact load. Modeling elastically coupled rigid bodies is an important problem in multibody dynamics. Many sources discuss the problem for spatial vibration of rigid body with elastic support for example [2] and [7]. The problem for determine of the impulse forces is described in [3]. The sources [4] and [5] discuss the problem of measurement of impact forces and related deformation with strainmeter instruments. Other scientists established stationary character of working process with some noise.

2. Theoretical base

The study of spatial vibration of impact crusher's rotor imposes some simplifications. We consider that continues structure of the rotor and the shaft is discrete. We assume that rotor's mass is higher and shaft's mass is a small. This assumption allows the entire mass of the system is concentrated in the rotor's center of mass. The spring's stiffness is equivalent of the shaft's stiffness. Points of the elastic suspension are accepted with rotor's width. Electromotor's rotor and impact crusher's rotor are elastic coupled by one degree of freedom (free rotation).

The nature of the impact crusher's working process is a stationary with suddenly applied impulse forces.

A part of the energy is consumed for breakage and transformed in surface energy; other part of the rotor's kinetic energy is accelerated comminuted material. The impact is semi elastic and impulse load is determined with equation:

$$S_p = k_p \cdot m_r \cdot v_p \cdot (1+k) \quad [\mathrm{N \cdot s}] \tag{1}$$

where: m_r= 0.0353 [kg] is a mass of a rubble, v_p = 31 [m/s] – peripheral velocity of a rotor's hammer, k – coefficient of elastic restitution (according [3] k = 0.45 for materials with middle hardness). Coefficient k_p is related with penetration

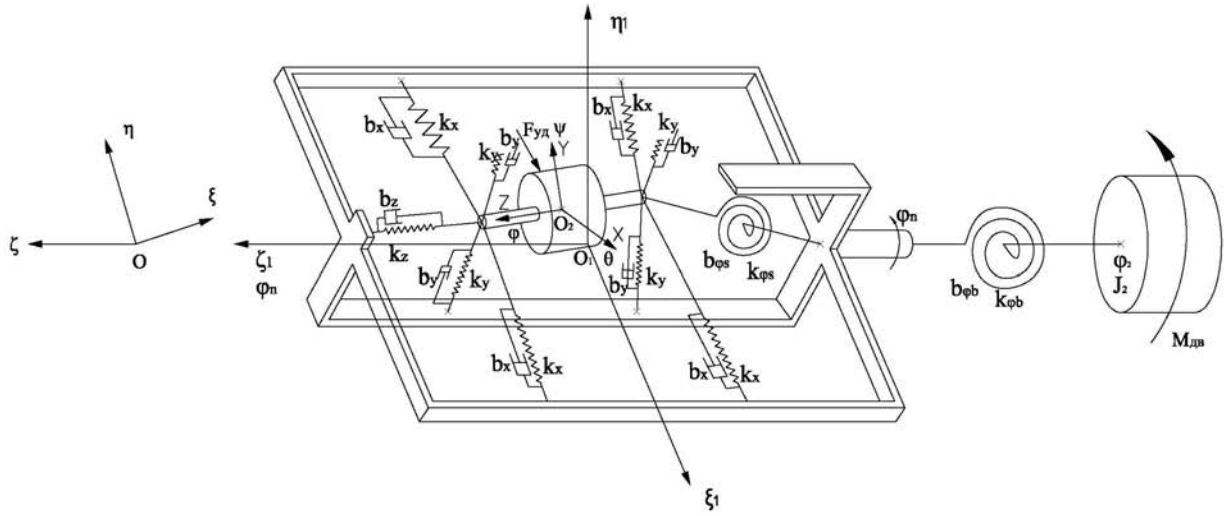

Fig. 1. Dynamical model of impact crusher's rotor system

of the rubble in working space, defined with equation:

$$k_p = 0.75 \cdot k_d^2 \cdot (1 - \frac{k_d}{3}) = 0.91 \quad (2)$$

where: k_d = $(2 \cdot h_h)/D$ = 2×0.035/0.03 = 2.33, h_h = 0.035 [m] – the high of impact hammer, D = 0.03 [m] – average diameter of rubble of the material.

The applied impact force on the rotor is determining from impulse value ($dS_p = F_{im} \cdot dt_{im}$). Assuming that form of impulse is triangle. For example determining of t_{im} is:

$$S_p = \frac{F_{im} \cdot t_{im}}{2} \Rightarrow F_{im} = \frac{2 \cdot S_p}{t_{im}}, \quad (3)$$

$$t_{im} = \frac{k_{im}}{v_p^n} = \frac{0.005}{\sqrt{36}} = 0.0008333s, \quad (4)$$

where: k_{im} and n are variables depended from material's properties: k_{im} = 0.005, and n = 0.5.

$$S_p = 0.91 \cdot 0.0353 \cdot 36 \cdot (1 + 0.45) = 1.6768 [\mathrm{N \cdot s}]$$

$$F_{im} = 400 \cdot S_p \cdot \sqrt{v_p} = 400 \cdot 1.677 \cdot \sqrt{36} = 4024.34 [\mathrm{N}]$$

The computed values of the impact force, impact impulse and time of impact are given in table 1. Accounts for four levels of the peripheral velocity are done. Dynamical model of impact crusher's rotor must be solved for all four levels of the peripheral velocity.

Table 1. Values of impact loads

v_p [m/s]	t_{im} [s]	F_{im} [N]	S_p [N·s]
36	0.000833	4024.34	1.6768
31	0.000898	3216.04	1.444
24.2	0.001	2254.4	1.1272
10.5	0.0016	611.375	0.4891

The dynamical model of the rotor is shown of figure 1. The dynamical model's parameters are: M = 47.87 kg; C = 0.35635 kg·m²; A = B = = 0.28675 kg·m²; L = 0.11 m; k = 2.91545´10^8 N/m; $k_x = k/4$; k_Z = 3´10^9 N/m; $k_{\varphi s}$ = 2.3311´10^5 N/rad; $k_{\varphi b}$ = 1.6´10^5 N·m/rad.

The vector of the degree of freedom in dynamical model is described as:

$$q = [\xi \quad \eta \quad \zeta \quad \theta \quad \psi \quad \varphi \quad \varphi_n \quad \varphi_2]$$

The common case of vibration of rigid body is separated absolute motion of two parts: relative motion and frame motion. Absolute frame is Oξηζ, translated frame is $O_1\xi'_1\eta'_1\zeta'_1$, doing frame motion with degree of freedom ξ_n, η_n, ζ_n translation on axis Oξ, Oη, Oζ. Frame $O_1\xi_1\eta_1\zeta_1$ is rotating about center O_1, i.e. doing frame motion (translation ξ_n, η_n, ζ_n about center O_1 in relation to center O) and rotation about center O_1 with degree of freedom θ_n, ψ_n, φ_n, where θ_n, ψ_n, φ_n are Euler angles.

The relative motion is deriving like on the frame motion. The frame O_2xyz is defined as fixed with rotor's body. Axis O_{2x}, O_{2y}, O_{2z} are principal axis of inertia and center O_2 is a center of mass. Motion of frame O_2xyz is expressed in frame $O_1\xi_1\eta_1\zeta_1$ by six degree of freedom ξ, η, ζ determine translation of O_2 along axis $O_1\xi_1$, $O_1\eta_1$ and $O_1\zeta_1$ and θ, ψ, φ Euler angles determined rotating of the body about the center of mass. Defined in this way frame $O_1\xi_1\eta_1\zeta_1$ described frame motion and frame O_2xyz, fixed to body, defined relative motion of body in relation to frame $O_1\xi_1\eta_1\zeta_1$. The degree of freedom ξ_n, η_n, ζ_n, θ_n, ψ_n, φ_n, ξ, η, ζ, θ, ψ, φ fully determined position of the body in frame Oξηζ. For the reviewed dynamical model degree of freedom reduced because few constrains acting over system. This constrains are: $\xi_n = 0$, $\eta_n = 0$, $\zeta_n = 0$, $\theta_n = 0$, $\psi_n = 0$.

The law of motion of the center of mass is:

$$M \cdot \varpi_C = H + (-M \cdot W) \quad (5)$$

where v_C is acceleration of the center of mass about frame $O_1\xi_1\eta_1\zeta_1$, M is a mass of the body, W – frame acceleration and H – the main vector of the external forces.

After projection of equation (5) over axis $O_1\xi_1$, $O_1\eta_1$ and $O_1\zeta_1$ is obtained system differential equations that expressed translations of the body:

$$\begin{cases} M\left(\frac{\partial v_{c\xi}}{\partial t} + q_1 \cdot v_{c\zeta} - r_1 \cdot v_{c\eta}\right) = H_\xi \\ M\left(\frac{\partial v_{c\eta}}{\partial t} + r_1 \cdot v_{c\xi} - p_1 \cdot v_{c\zeta}\right) = H_\eta \\ M\left(\frac{\partial v_{c\zeta}}{\partial t} + p_1 \cdot v_{c\eta} - q_1 \cdot v_{c\xi}\right) = H_\zeta \end{cases} \quad (6)$$

where v_{cx}, v_{ch}, and v_{cz} are projections of velocity vector v_c about axis $O_1\xi_1$, $O_1\eta_1$ and $O_1\zeta_1$, and p_1, q_1 and r_1 are projection of the frame angular velocity over same axis. They are obtained from Euler kinematics equations:

$$\begin{cases} p_1 = \dot\psi_n \sin\varphi_n \cos\theta_n + \dot\theta_n \cos\varphi_n \\ q_1 = \dot\psi_n \cos\varphi_n \cos\theta_n - \dot\theta_n \sin\varphi_n \\ r_1 = \dot\varphi_n - \dot\psi_n \sin\theta_n \end{cases} \quad (7)$$

where: $\omega_1 = [p_1 \quad q_1 \quad r_1]^T$; $p_1 = 0$; $q_1 = 0$; $r_1 = \varphi_n$; $v_{cx} = \xi$; $v_{ch} = \eta$; $v_{cz} = \zeta$.

After simplification, equations described translations of the body are:

$$\begin{cases} M\left(\ddot{\xi}-\dot{\eta}\cdot\dot{\varphi}_n\right)=H_\xi \\ M\left(\ddot{\eta}+\dot{\xi}\cdot\dot{\varphi}_n\right)=H_\eta \\ M\cdot\ddot{\zeta}=H_\zeta \end{cases} \quad (8)$$

The equations that expressed angular momentum of the body about center O_1, are:

$$\frac{dK_{O_1}}{dt}=M^{(e)}+r_c\times(-M\cdot W) \quad (9)$$

Appling the rule for absolute derivative of vector and obtain:

$$\frac{\partial K_{O_1}}{\partial t}+\omega_1\times K_{O_1}=M^{(e)} \quad (10)$$

When projected equation (10) over axis $O_1\xi_1$, $O_1\eta_1$ and $O_1\zeta_1$ is obtained system differential equations that expressed rotation of the body about center O_1:

$$\begin{cases} \frac{\partial K_\xi}{\partial t}+q_1\cdot K_\zeta-r_1\cdot K_\eta=M_\xi \\ \frac{\partial K_\eta}{\partial t}+r_1\cdot K_\xi-p_1\cdot K_\zeta=M_\eta \\ \frac{\partial K_\zeta}{\partial t}+p_1\cdot K_\eta-q_1\cdot K_\xi=M_\zeta \end{cases} \quad (11)$$

where K_ξ, K_h and K_ζ are projections of K_{O1} over axis $O_1\xi_1$, $O_1\eta_1$ and $O_1\zeta_1$, and M_ξ, M_h and M_ζ are projections of main vector of moment of external forces $M^{(e)}$ about center O_1 over same axis. K_ξ, K_h and K_ζ are determined from next equations:

$$\begin{cases} K_\xi=[r_c\times M\,v_c]_\xi+K_{c\xi} \\ K_\eta=[r_c\times M\,v_c]_\eta+K_{c\eta} \\ K_\zeta=[r_c\times M\,v_c]_\zeta+K_{c\zeta} \end{cases} \quad (12)$$

After projecting of the vector production over principal axis are obtained:

$$\begin{cases} K_\xi=M\cdot(\eta\cdot\dot{\zeta}-\zeta\cdot\dot{\eta})+K_{c\xi} \\ K_\eta=M\cdot(\zeta\cdot\dot{\xi}-\xi\cdot\dot{\zeta})+K_{c\eta} \\ K_\zeta=M\cdot(\xi\cdot\dot{\eta}-\eta\cdot\dot{\xi})+K_{c\zeta} \end{cases} \quad (13)$$

where $K_{c\xi}$, K_{ch} and $K_{c\zeta}$ are projections of angular momentum over axis $O_2\xi''_1$, $O_2h''_1$ and $O_2z''_1$, which are parallel of axis $O_2\xi_1$, O_2h_1 and O_2z_1 and pass through center O_2. Determining of $K_{c\xi}$, K_{ch} and $K_{c\zeta}$ are made with rotational matrix that contained directional cosines $K_c = a\cdot K^0$, from where:

$$\begin{cases} K_{c\xi}=a_{11}\cdot K_X^0+a_{12}\cdot K_Y^0+a_{13}\cdot K_Z^0 \\ K_{c\eta}=a_{21}\cdot K_X^0+a_{22}\cdot K_Y^0+a_{23}\cdot K_Z^0 \\ K_{c\zeta}=a_{31}\cdot K_X^0+a_{32}\cdot K_Y^0+a_{33}\cdot K_Z^0 \end{cases} \quad (14)$$

where K^0_X, K^0_Y, K^0_Z are projections of the angular momentum K^0 over principal axis of inertia O_2x, O_2y O_2z determined from the next expressions:

$$K_X^0=A\cdot\omega_{0X}\,;\; K_Y^0=B\cdot\omega_{0Y}\,;\; K_Z^0=C\cdot\omega_{0Z} \quad (15)$$

A, *B* and *C* are principal moments of inertia.

Assuming that range of θ, ψ and φ are smaller then 5°. For small θ, ψ and φ rotational matrix *a* contained directional cosines my take linear form and obtained next description:

$$a=\begin{vmatrix} 1 & -\varphi & \psi \\ \varphi & 1 & -\theta \\ -\psi & \theta & 1 \end{vmatrix}$$

From where are obtained:

$$\begin{cases} K_{c\xi}=K_X^0-\varphi\cdot K_Y^0+\psi\cdot K_Z^0 \\ K_{c\eta}=\varphi\cdot K_X^0+K_Y^0-\theta\cdot K_Z^0 \\ K_{c\zeta}=-\psi\cdot K_X^0+\theta\cdot K_Y^0+K_Z^0 \end{cases} \quad (16)$$

Projections of the absolute angular velocity over principal axis of inertia are expressed with next equations:

$$\begin{cases} \omega_{0X}=p+a_{11}\cdot p_1+a_{21}\cdot q_1+a_{31}\cdot r_1 \\ \omega_{0Y}=q+a_{12}\cdot p_1+a_{22}\cdot q_1+a_{32}\cdot r_1 \\ \omega_{0Z}=r+a_{13}\cdot p_1+a_{23}\cdot q_1+a_{33}\cdot r_1 \end{cases} \quad (17)$$

Projections of the relative angular velocity over principal axis of inertia are:

$$\begin{cases} p=\dot{\psi}\cdot\sin\varphi\cdot\cos\theta+\dot{\theta}\cdot\cos\varphi \\ q=\dot{\psi}\cdot\cos\varphi\cdot\cos\theta-\dot{\theta}\cdot\sin\varphi \\ r=\dot{\varphi}-\dot{\psi}\cdot\sin\theta \end{cases} \quad (18)$$

For small vibrations:

$$\begin{cases} p=\dot{\psi}\cdot\varphi+\dot{\theta} \\ q=\dot{\psi}-\dot{\theta}\cdot\varphi \\ r=\dot{\varphi}-\dot{\psi}\cdot\theta \end{cases} \quad (19)$$

Projections of the absolute angular velocity over principal axis of inertia are obtained next description:

$$\begin{cases} \omega_{0X} = \dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n \\ \omega_{0Y} = \dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n \\ \omega_{0Z} = \dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n \end{cases} \quad (20)$$

Projections of angular momentum over principal axis of inertia are given bellow:

$$\begin{cases} K_X^0 = A\cdot\left(\dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n\right) \\ K_Y^0 = B\cdot\left(\dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n\right) \\ K_Z^0 = C\cdot\left(\dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n\right) \end{cases} \quad (21)$$

Substitute the projections of angular momentum in equation (14) and is obtained system (22):

$$\begin{cases} K_{c\xi} = A\cdot\left(\dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n\right) - \varphi\cdot\left(B\cdot\left(\dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n\right)\right) + \\ \qquad + \psi\cdot\left(C\cdot\left(\dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n\right)\right) \\ K_{c\eta} = \varphi\cdot\left(A\cdot\left(\dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n\right)\right) + B\cdot\left(\dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n\right) - \\ \qquad - \theta\cdot\left(C\cdot\left(\dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n\right)\right) \\ K_{c\zeta} = -\psi\cdot\left(A\cdot\left(\dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n\right)\right) + \theta\cdot\left(B\cdot\left(\dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n\right)\right) + \\ \qquad + C\cdot\left(\dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n\right) \end{cases} \quad (22)$$

From where with substitute in (13) and is derived the system (23):

$$\begin{cases} K_\xi = M\cdot(\eta\cdot\dot{\zeta} - \zeta\cdot\dot{\eta}) + A\cdot\left(\dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n\right) - \\ \qquad - \varphi\cdot\left(B\cdot\left(\dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n\right)\right) + \psi\cdot\left(C\cdot\left(\dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n\right)\right) \\ K_\eta = M\cdot(\zeta\cdot\dot{\xi} - \xi\cdot\dot{\zeta}) + \varphi\cdot\left(A\cdot\left(\dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n\right)\right) + \\ \qquad + B\cdot\left(\dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n\right) - \theta\cdot\left(C\cdot\left(\dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n\right)\right) \\ K_\zeta = M\cdot(\xi\cdot\dot{\eta} - \eta\cdot\dot{\xi}) - \psi\cdot\left(A\cdot\left(\dot{\psi}\cdot\varphi + \dot{\theta} - \psi\cdot\dot{\varphi}_n\right)\right) + \\ \qquad + \theta\cdot\left(B\cdot\left(\dot{\psi} - \dot{\theta}\cdot\varphi + \theta\cdot\dot{\varphi}_n\right)\right) + C\cdot\left(\dot{\varphi} - \dot{\psi}\cdot\theta + \dot{\varphi}_n\right) \end{cases} \quad (23)$$

The system of differential equations of motion (24) that described dynamical model of impact crusher's rotor has description:

$$\begin{cases} M\cdot\ddot{\xi} - M\cdot\dot{\eta}\cdot\dot{\varphi}_n = H_\xi \\ M\cdot\ddot{\eta} + M\cdot\dot{\xi}\cdot\dot{\varphi}_n = H_\eta \\ M\cdot\ddot{\zeta} = H_\zeta \\ (C-B)\cdot\theta\cdot\dot{\varphi}_n^2 - A\cdot\varphi\cdot\psi\cdot\dot{\varphi}_n^2 + M\cdot\dot{\varphi}_n\cdot\left(\xi\cdot\dot{\zeta} - \zeta\cdot\dot{\xi}\right) + \\ \quad + A\cdot\varphi\cdot\dot{\varphi}_n\cdot\dot{\theta} + (C-B)\cdot\theta\cdot\dot{\varphi}_n\cdot\dot{\varphi} + \\ \quad + \left(C\cdot(1-\theta^2) - B - A\cdot(1-\varphi^2)\right)\cdot\dot{\varphi}_n\cdot\dot{\psi} - \\ \quad - C\cdot\psi\cdot\dot{\theta}\cdot\dot{\psi} + (A-B+C)\cdot\dot{\varphi}\cdot\dot{\psi} - C\cdot\theta\cdot\dot{\psi}^2 + \\ \quad + M\cdot\left(\eta\cdot\ddot{\zeta} - \zeta\cdot\ddot{\eta}\right) + \left(A + B\cdot\varphi^2\right)\cdot\ddot{\theta} + C\cdot\psi\cdot\ddot{\varphi} + \\ \quad + \left((A-B)\cdot\varphi - C\cdot\theta\cdot\psi\right)\cdot\ddot{\psi} + 2\cdot B\cdot\varphi\cdot\dot{\theta}\cdot\dot{\varphi} + \\ \quad + \left((C-A)\cdot\psi - B\cdot\theta\cdot\varphi\right)\cdot\ddot{\varphi}_n = M_\xi \\ \left((C-A)\cdot\psi - B\cdot\theta\cdot\varphi\right)\cdot\dot{\varphi}_n^2 + \\ \quad \left(A + B\cdot(1+\varphi^2) - C\right)\cdot\dot{\theta}\cdot\dot{\varphi}_n + (A+C)\cdot\psi\cdot\dot{\varphi}_n\cdot\dot{\varphi} - \\ \quad - (A+B+C)\cdot\dot{\theta}\cdot\dot{\varphi} + (2\cdot A - B)\cdot\varphi\cdot\dot{\varphi}_n\cdot\dot{\psi} - \\ \quad - C\cdot\theta\cdot\psi\cdot\dot{\varphi}_n\cdot\dot{\psi} + 2\cdot C\cdot\theta\cdot\dot{\theta}\cdot\dot{\psi} - 2\cdot A\cdot\varphi\cdot\dot{\varphi}\cdot\dot{\psi} + \\ \quad + \left((B-C)\cdot\theta + A\cdot\varphi\cdot\psi\right)\cdot\ddot{\varphi}_n + M\cdot\left(\zeta\cdot\ddot{\xi} - \xi\cdot\ddot{\zeta}\right) - \\ \quad - (A+B)\cdot\varphi\cdot\ddot{\theta} - C\cdot\theta\cdot\ddot{\varphi} + \left(C\cdot\theta^2 + B - A\cdot\varphi^2\right)\cdot\ddot{\psi} + \\ \quad + M\cdot\dot{\varphi}_n\cdot\left(\eta\cdot\dot{\zeta} - \zeta\cdot\dot{\eta}\right) = M_\eta \\ 2\cdot B\cdot\theta\cdot\dot{\varphi}_n\cdot\dot{\theta} - B\cdot\varphi\cdot\dot{\theta}^2 - B\cdot\theta\cdot\dot{\theta}\cdot\dot{\varphi} + 2\cdot A\cdot\psi\cdot\dot{\varphi}_n\cdot\dot{\psi} + \\ \quad + (B-A-C)\cdot\dot{\theta}\cdot\dot{\psi} - A\cdot\psi\cdot\dot{\psi}\cdot\dot{\varphi} + \\ \quad + \left(C + B\cdot\theta^2 + A\cdot\psi^2\right)\cdot\ddot{\varphi}_n + M\cdot\left(\xi\cdot\ddot{\eta} - \eta\cdot\ddot{\xi}\right) - \\ \quad - A\cdot\varphi\cdot\dot{\psi}^2 - (A\cdot\psi + B\cdot\theta\cdot\varphi)\cdot\ddot{\theta} + C\cdot\ddot{\varphi} + \\ \quad + (B\cdot\theta - A\cdot\varphi\cdot\psi - C\cdot\theta)\cdot\ddot{\psi} = M_\zeta \\ J_p\cdot\ddot{\varphi}_n = M_{\varphi n} \\ J_2\cdot\ddot{\varphi}_2 = M_{\varphi 2} \end{cases} \quad (24)$$

The main vector of the external forces and main moment of the external forces consist next members:

$$\begin{aligned} H &= H^{el} + H^{damp} + H(t); \\ M &= M^{el} + M^{damp} + M(t). \end{aligned} \quad (25)$$

They are determined as:

$$H_\xi^{el} = \frac{\partial U}{\partial \xi}; \quad H_\eta^{el} = \frac{\partial U}{\partial \eta}; \quad H_\zeta^{el} = \frac{\partial U}{\partial \zeta}; \quad (26)$$

and:

$$\begin{aligned} M_\xi^{el} &= a_{11}\cdot M_X^{el} + a_{12}\cdot M_Y^{el} + a_{13}\cdot M_Z^{el} \\ M_\eta^{el} &= a_{21}\cdot M_X^{el} + a_{22}\cdot M_Y^{el} + a_{23}\cdot M_Z^{el} \\ M_\zeta^{el} &= a_{31}\cdot M_X^{el} + a_{32}\cdot M_Y^{el} + a_{33}\cdot M_Z^{el} \end{aligned} \quad (27)$$

where:

$$\begin{aligned} M_X^{el} &= \frac{\partial U}{\partial \psi}\cdot\sin\varphi\cdot\cos\theta + \frac{\partial U}{\partial \theta}\cdot\cos\varphi \\ M_Y^{el} &= \frac{\partial U}{\partial \psi}\cdot\cos\varphi\cdot\cos\theta - \frac{\partial U}{\partial \theta}\cdot\sin\varphi \\ M_Z^{el} &= \frac{\partial U}{\partial \varphi} - \frac{\partial U}{\partial \psi}\cdot\sin\theta \end{aligned} \quad (28)$$

The work of elastic forces U is expressed with equation (29), where Π is a potential energy of the system:

$$\begin{cases} -U=\Pi=\dfrac{1}{2}\left(\begin{array}{l}\sum_{i=1}^{2}\left(2\cdot k_X^i\cdot u_i^2+2\cdot k_Y^i\cdot v_i^2\right)+k_Z^2\cdot w_2^2\\ \quad +k_{\varphi S}\cdot(\varphi_1-\varphi_n)^2+k_{\varphi b}\cdot(\varphi_n-\varphi_2)^2\end{array}\right)\\ \varphi_1=\varphi+\varphi_n \end{cases} \quad (29)$$

The displacements of the support points, for small vibrations, are:

$$\begin{aligned} u_i &= \xi-\varphi\cdot y_i+\psi\cdot z_i\\ v_i &= \eta-\theta\cdot z_i+\varphi\cdot x_i\\ w_i &= \zeta-\psi\cdot x_i+\theta\cdot y_i \end{aligned} \quad (30)$$

The specific coordinates, for the dynamical model, are:

$$x_i=y_i=0;\quad z_1=-\frac{L}{2};\quad z_2=\frac{L}{2}.$$

From where displacements of the support points yield next description:

$$\begin{aligned} u_i &= \xi+\psi\cdot z_i\\ v_i &= \eta-\theta\cdot z_i\\ w_i &= \zeta \end{aligned} \quad (31)$$

The stiffness is: $k^i_X = k^i_Y = k/4$.

The potential energy's expression yield next description (32):

$$\Pi=\frac{1}{2}\cdot\left[\begin{array}{l} k\cdot\xi^2+k\cdot\psi^2\cdot\left(\frac{L}{2}\right)^2+k\cdot\eta^2+k\cdot\theta^2\cdot\left(\frac{L}{2}\right)^2+\\ \quad +k_Z\cdot\zeta^2+k_{\varphi S}\cdot(\varphi_1-\varphi_n)^2+k_{\varphi b}\cdot(\varphi_n-\varphi_2)^2 \end{array}\right] \quad (32)$$

from where for the elastic forces follow:

$$\begin{aligned} H_\xi^{el} &= \frac{\partial U}{\partial \xi}=-k\cdot\xi\\ H_\eta^{el} &= \frac{\partial U}{\partial \eta}=-k\cdot\eta\\ H_\zeta^{el} &= \frac{\partial U}{\partial \zeta}=-k_Z\cdot\zeta \end{aligned} \quad (33)$$

$$\begin{aligned} \frac{\partial U}{\partial \psi} &= -k\cdot\left(\frac{L}{2}\right)^2\cdot\psi\\ \frac{\partial U}{\partial \theta} &= -k\cdot\left(\frac{L}{2}\right)^2\cdot\theta\\ \frac{\partial U}{\partial \varphi} &= \frac{\partial U}{\partial \varphi_1}\cdot\frac{\partial \varphi_1}{\partial \varphi}=-k_{\varphi S}\cdot(\varphi_1-\varphi_n)\cdot(+1)=\\ &= -k_{\varphi S}\cdot(\varphi+\varphi_n-\varphi_n)=-k_{\varphi S}\cdot\varphi \end{aligned} \quad (34)$$

For small vibrations the substitution of equations (34) in equation (28) are derived equations (35) for the moment of the elastic forces:

$$\begin{aligned} M_X^{el} &= -k\cdot\left(\frac{L}{2}\right)^2\cdot\psi\cdot\varphi-k\cdot\left(\frac{L}{2}\right)^2\cdot\theta=-k\cdot\left(\frac{L}{2}\right)^2\cdot(\psi\cdot\varphi+\theta)\\ M_Y^{el} &= -k\cdot\left(\frac{L}{2}\right)^2\cdot\psi+k\cdot\left(\frac{L}{2}\right)^2\cdot\theta\cdot\varphi=-k\cdot\left(\frac{L}{2}\right)^2\cdot(\psi-\theta\cdot\varphi)\\ M_Z^{el} &= -k_{\varphi S}\cdot\varphi+k\cdot\left(\frac{L}{2}\right)^2\cdot\psi\cdot\theta \end{aligned} \quad (35)$$

With substitute and simplification we obtained (36):

$$\begin{aligned} M_\xi^{el} &= k\cdot\left(\frac{L}{2}\right)^2\cdot\left(\psi^2-1-\varphi^2\right)\cdot\theta-k_{\varphi S}\cdot\varphi\cdot\psi\\ M_\eta^{el} &= -k\cdot\left(\frac{L}{2}\right)^2\cdot\left(1+\varphi^2+\theta^2\right)\cdot\psi+k_{\varphi S}\cdot\varphi\cdot\theta\\ M_\zeta^{el} &= k\cdot\left(\frac{L}{2}\right)^2\cdot\left(\varphi\cdot\psi^2+\varphi\cdot\theta^2+\psi\cdot\theta\right)-k_{\varphi S}\cdot\varphi\\ M_{\varphi n} &= \frac{\partial U}{\partial \varphi_n}=-k_{\varphi S}\cdot(\varphi_1-\varphi_n)\cdot(-1)-k_{\varphi b}\cdot(\varphi_n-\varphi_2)=\\ &= k_{\varphi S}\cdot(\varphi+\varphi_n-\varphi_n)-k_{\varphi b}\cdot(\varphi_n-\varphi_2)=\\ &= k_{\varphi S}\cdot\varphi-k_{\varphi b}\cdot(\varphi_n-\varphi_2)\\ M_{\varphi 2} &= \frac{\partial U}{\partial \varphi_2}=-k_{\varphi b}\cdot(\varphi_n-\varphi_2)\cdot(-1)=k_{\varphi b}\cdot(\varphi_n-\varphi_2) \end{aligned} \quad (36)$$

The determining of the damped forces and the damped moment is such as determining of the elastic forces and the elastic moment, equation (37):

$$\begin{aligned} & H_\xi^{damp}=\frac{\partial \Phi}{\partial \xi}=-b\cdot\dot{\xi};\quad H_\eta^{damp}=\frac{\partial \Phi}{\partial \eta}=-b\cdot\dot{\eta};\\ & H_\zeta^{damp}=\frac{\partial \Phi}{\partial \zeta}=-b_Z\cdot\dot{\zeta}\\ & M_\xi^{damp}=b\cdot\left(\frac{L}{2}\right)^2\cdot\left(\dot{\psi}^2-1-\dot{\varphi}^2\right)\cdot\dot{\theta}-b_{\varphi S}\cdot\dot{\varphi}\cdot\dot{\psi}\\ & M_\eta^{damp}=-b\cdot\left(\frac{L}{2}\right)^2\cdot\left(1+\dot{\varphi}^2+\dot{\theta}^2\right)\cdot\dot{\psi}+b_{\varphi S}\cdot\dot{\varphi}\cdot\dot{\theta}\\ & M_\zeta^{damp}=b\cdot\left(\frac{L}{2}\right)^2\cdot\left(\dot{\varphi}\cdot\dot{\psi}^2+\dot{\varphi}\cdot\dot{\theta}^2+\dot{\psi}\cdot\dot{\theta}\right)-b_{\varphi S}\cdot\dot{\varphi} \end{aligned} \quad (37)$$

From where the system differential equations of motion are obtained next ended description (38):

$$\begin{cases} M\cdot\ddot{\xi}=M\cdot\dot{\eta}\cdot\dot{\varphi}_n-k\cdot\xi-b\cdot\dot{\xi}+H_\xi(t)\\ M\cdot\ddot{\eta}=-M\cdot\dot{\xi}\cdot\dot{\varphi}_n-k\cdot\eta-b\cdot\dot{\eta}+H_\eta(t)\\ M\cdot\ddot{\zeta}=-k_Z\cdot\zeta-b_Z\cdot\dot{\zeta}+H_\zeta(t) \end{cases}$$

$$
\begin{cases}
-M\cdot\zeta\cdot\ddot{\eta}+M\cdot\eta\cdot\ddot{\zeta}+\left(A+B\cdot\varphi^2\right)\cdot\ddot{\theta}+\\
\quad+\left((A-B)\cdot\varphi-C\cdot\theta\cdot\psi\right)\cdot\ddot{\psi}+C\cdot\psi\cdot\ddot{\varphi}+\\
\quad+\left((C-A)\cdot\psi-B\cdot\theta\cdot\varphi\right)\cdot\ddot{\varphi}_n=M_{\xi}(t)+\\
\quad+A\cdot\varphi\cdot\psi\cdot\dot{\varphi}_n^2-(C-B)\cdot\theta\cdot\dot{\varphi}_n^2-\\
\quad-M\cdot\dot{\varphi}_n\cdot\left(\xi\cdot\dot{\zeta}-\zeta\cdot\dot{\xi}\right)-A\cdot\varphi\cdot\dot{\varphi}_n\cdot\dot{\theta}-\\
\quad-(C-B)\cdot\theta\cdot\dot{\varphi}_n\cdot\dot{\varphi}-\\
\quad-\left(C\cdot(1-\theta^2)-B-A\cdot(1-\varphi^2)\right)\cdot\dot{\varphi}_n\cdot\dot{\psi}+\\
\quad+C\cdot\psi\cdot\dot{\theta}\cdot\dot{\psi}-2\cdot B\cdot\varphi\cdot\dot{\theta}\cdot\dot{\varphi}-\\
\quad-(A-B+C)\cdot\dot{\varphi}\cdot\dot{\psi}+C\cdot\theta\cdot\dot{\psi}^2+\\
\quad k\cdot\left(\frac{L}{2}\right)^2\cdot\left(\psi^2-1-\varphi^2\right)\cdot\theta-k_{\varphi S}\cdot\varphi\cdot\psi+\\
\quad+b\cdot\left(\frac{L}{2}\right)^2\cdot\left(\dot{\psi}^2-1-\dot{\varphi}^2\right)\cdot\dot{\theta}-b_{\varphi S}\cdot\dot{\varphi}\cdot\dot{\psi}\\
M\cdot\zeta\cdot\ddot{\xi}-M\cdot\xi\cdot\ddot{\zeta}-(A+B)\cdot\varphi\cdot\ddot{\theta}+\\
\quad+\left(C\cdot\theta^2+B-A\cdot\varphi^2\right)\cdot\ddot{\psi}-C\cdot\theta\cdot\ddot{\varphi}+\\
\quad+\left((B-C)\cdot\theta+A\cdot\varphi\cdot\psi\right)\cdot\ddot{\varphi}_n=M_{\eta}(t)-\\
\quad-\left((C-A)\cdot\psi-B\cdot\theta\cdot\varphi\right)\cdot\dot{\varphi}_n^2-\\
\quad-M\cdot\dot{\varphi}_n\cdot\left(\eta\cdot\dot{\zeta}-\zeta\cdot\dot{\eta}\right)-\\
\quad-\left(A+B\cdot(1+\varphi^2)-C\right)\cdot\dot{\theta}\cdot\dot{\varphi}_n-\\
\quad-(A+C)\cdot\psi\cdot\dot{\varphi}_n\cdot\dot{\varphi}+(A+B+C)\cdot\dot{\theta}\cdot\dot{\varphi}-\\
\quad-(2\cdot A-B)\cdot\varphi\cdot\dot{\varphi}_n\cdot\dot{\psi}+C\cdot\theta\cdot\psi\cdot\dot{\varphi}_n\cdot\dot{\psi}-\\
\quad-2\cdot C\cdot\theta\cdot\dot{\theta}\cdot\dot{\psi}+2\cdot A\cdot\varphi\cdot\dot{\varphi}\cdot\dot{\psi}-\\
\quad k\cdot\left(\frac{L}{2}\right)^2\cdot\left(1+\varphi^2+\theta^2\right)\cdot\psi+k_{\varphi S}\cdot\varphi\cdot\theta-\\
\quad-b\cdot\left(\frac{L}{2}\right)^2\cdot\left(1+\dot{\varphi}^2+\dot{\theta}^2\right)\cdot\dot{\psi}+b_{\varphi S}\cdot\dot{\varphi}\cdot\dot{\theta}\\
-M\cdot\eta\cdot\ddot{\xi}+M\cdot\xi\cdot\ddot{\eta}-(A\cdot\psi+B\cdot\theta\cdot\varphi)\cdot\ddot{\theta}+\\
\quad+(B\cdot\theta-A\cdot\varphi\cdot\psi-C\cdot\theta)\cdot\ddot{\psi}+\\
\quad+C\cdot\ddot{\varphi}+\left(C+B\cdot\theta^2+A\cdot\psi^2\right)\cdot\ddot{\varphi}_n=\\
\quad=M_{\zeta}(t)-2\cdot B\cdot\theta\cdot\dot{\varphi}_n\cdot\dot{\theta}+B\cdot\varphi\cdot\dot{\theta}^2+B\cdot\theta\cdot\dot{\theta}\cdot\dot{\varphi}-\\
\quad 2\cdot A\cdot\psi\cdot\dot{\varphi}_n\cdot\dot{\psi}-(B-A-C)\cdot\dot{\theta}\cdot\dot{\psi}+A\cdot\psi\cdot\dot{\psi}\cdot\dot{\varphi}+\\
\quad+A\cdot\varphi\cdot\dot{\psi}^2+k\cdot\left(\frac{L}{2}\right)^2\cdot\left(\varphi\cdot\psi^2+\varphi\cdot\theta^2+\psi\cdot\theta\right)-\\
\quad-k_{\varphi S}\cdot\varphi+b\cdot\left(\frac{L}{2}\right)^2\cdot\left(\dot{\varphi}\cdot\dot{\psi}^2+\dot{\varphi}\cdot\dot{\theta}^2+\dot{\psi}\cdot\dot{\theta}\right)-b_{\varphi S}\cdot\dot{\varphi}\\
J_p\cdot\ddot{\varphi}_n=k_{\varphi S}\cdot\varphi-k_{\varphi b}\cdot(\varphi_n-\varphi_2)+b_{\varphi S}\cdot\dot{\varphi}-b_{\varphi b}\cdot(\dot{\varphi}_n-\dot{\varphi}_2)\\
J_2\cdot\ddot{\varphi}_2=k_{\varphi b}\cdot(\varphi_n-\varphi_2)+b_{\varphi b}\cdot(\dot{\varphi}_n-\dot{\varphi}_2)
\end{cases}
$$

For the purpose of solution on the system differential equations of motion is created a computer program in the MATLAB environment. The obtained results are given in graphical form. Obtaining of the value for the damping constant (b) is done with the damping ratio (β).

The relationship is:

$$n=\frac{b}{2m};\quad \omega=\sqrt{\frac{k}{m}}$$

$$\beta=\frac{n}{\omega}=\frac{b}{2m\sqrt{\frac{k}{m}}}=\frac{b}{2\sqrt{k\,m}}=\frac{b}{b_{kp}}\quad\Rightarrow\quad b=2\beta\sqrt{k\,m}$$

For the system with nature damping the value of the damping ratio is in the range: $\beta = 0.2\% \div 20\%$. In this range, with respect sources [6] and [9] is accepted $\beta = 0.5\%$ and damping constants are: $4b_x = 4b_y = b = 1180$ [N·s/m]; $b_z = 3800$ [N·s/m].

The different equations for the performance curve on the electromotor for the different ratios are given on table 2. Reduced moment of inertia for the different ratios also given.

Table 2. Equations for the performance curve of the electromotor

ω_s	$J_2 = J_{elm}\cdot i^2$	M_{elm}
251.33	0.21250	$M_{elm} = 30.875 - 7.74\cdot e^{-9}\cdot\omega$^4
216.66	0.28594	$M_{elm} = 35.815 - 1.63\cdot e^{-8}\cdot\omega$^4
168.90	0.47051	$M_{elm} = 45.942 - 5.65\cdot e^{-8}\cdot\omega$^4
73.06	2.51464	$M_{elm} = 106.21 - 3.73\cdot e^{-6}\cdot\omega$^4

3. Results

1. The solution of the system differential equations is provided in graphical form by frequency domain characteristics. The frequency domain graphics expressed amplitude spectrums are obtained with Fast Fourier Transformation / FFT/ method.
2. On figure 2 is presented solution of the system differential equations for the variation of the angular velocity of the impact crusher's rotor and electro motor's rotor.
3. On figures from 3 to 8 are presented amplitude spectrums for generalized coordinates ξ, ψ and φ for the different peripheral velocity of rotor's hammers.

4. Conclusion

4.1 Three mass dynamical model with eight degree of freedom is created. The forced influence over rotor system is impulse.

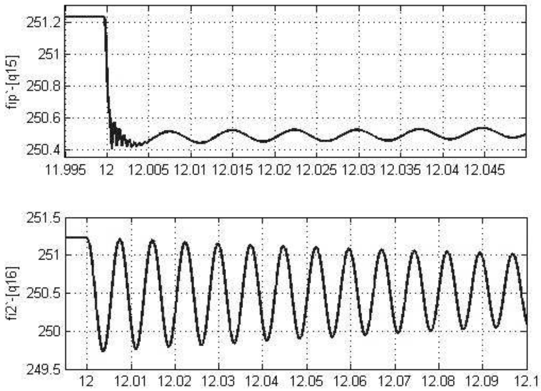

Fig. 2. Variation of the peripheral velocities of both impact crusher's rotor and electro motor's rotor for V_p = 36 [m/s]

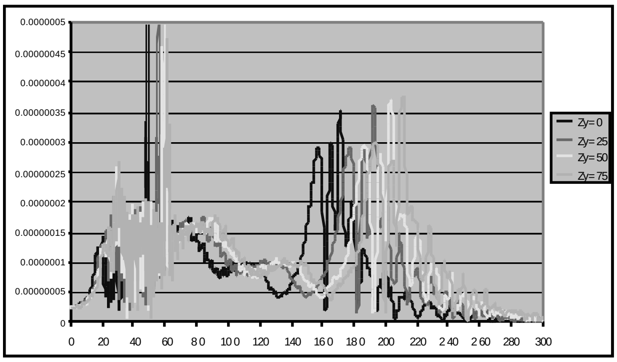

Fig. 3. Amplitude spectrum for the generalized coordinate ξ for V_p = 36 [m/s]

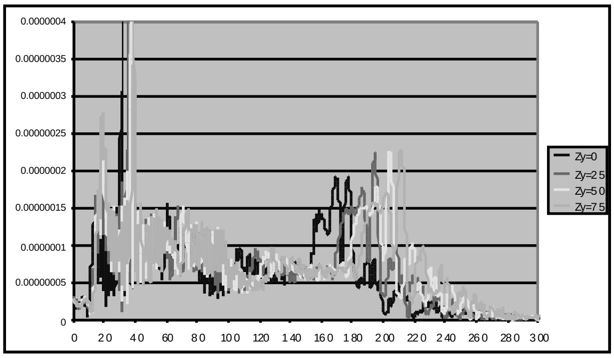

Fig. 6. Amplitude spectrum for the generalized coordinate ξ for V_p = 36 [m/s]

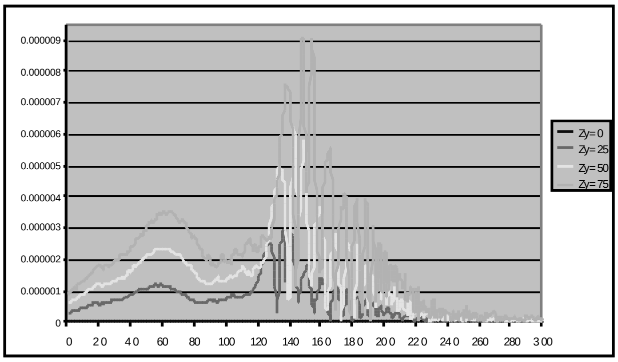

Fig. 4. Amplitude spectrum for the generalized coordinate ψ for V_p = 36 [m/s]

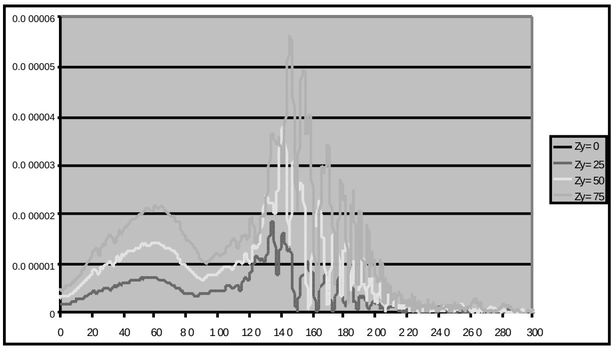

Fig. 7. Amplitude spectrum for the generalized coordinate ψ for V_p = 24 [m/s]

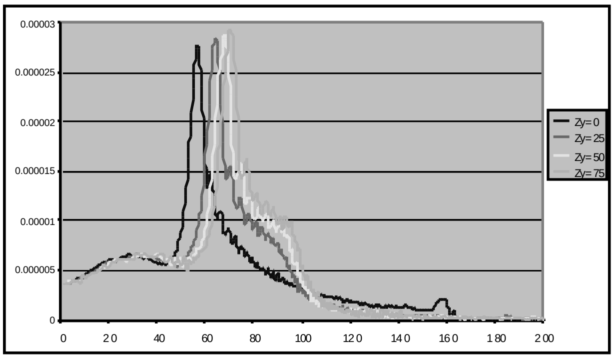

Fig. 5. Amplitude spectrum for the generalized coordinate φ for V_p = 36 [m/s]

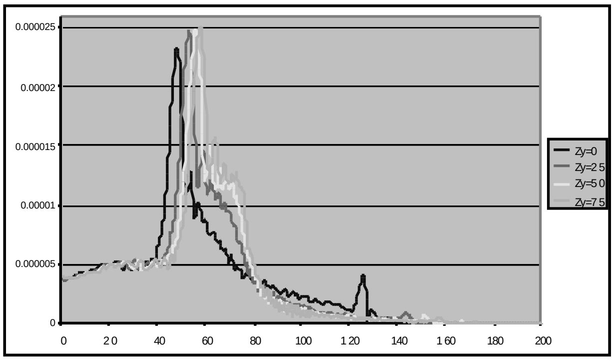

Fig. 8. Amplitude spectrum for the generalized coordinate φ for V_p = 24 [m/s]

4.2 Numerical solutions for the system differential equations of motion are obtained with MATLAB. For the yielding of amplitude spectrums on the vibration process is applied FFT.

4.3 The numerical experiments for the influence of the peripheral velocity on the hammer over generalized coordinates are carry out.

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