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FORWARD-LOOKING MATHEMATICAL MODEL DEVELOPED TO FORECAST THE TECHNICAL-ECONOMIC ENVIRONMENT IN MANUFACTURING HEAT-INSULATION CARPENTRY. APPLICATION CONDITIONS

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Abstract. The paper presents the application conditions of a model developed to forecast the technical-economic environment in manufacturing heat-insulation carpentry; the model consists of product definition, initial data and previously calculated values.

The following are defined: products, production volumes, related probabilities, production percentage, forecast probability, assumed risk, uncertain environment, decision under uncertainty conditions, decision robustness; a calculation procedure of forecast probabilities is also forwarded.

The paper approaches the application conditions of the model under conditions of flexible manufacturing of heat-insulation materials on sets of products.

The model is probalistic or uncertain, by taking into account the assumed risk.

Keywords: mathematical model, technical-economic environment

1. Object of the paper

The authors state the application conditions of the model, founding it on the basis of papers [1, 4, 6].

2. Application conditions

The industry of metallic, PVC and other composite materials carpentry focus on the manufacturing of some standard products P_k, i.e.: fixed window – FF, turn window– FR, turn and tilt window– FRB, PVC door – UPVC, aluminum door – UAl, mixed panels – C. All these types, being alike morpho-geometrically and morpho-technologically, are affined products and, consequently, they form a set of products expressed through the relations: FPk={FF, FR, FRB, UPVC, UAl, C}$=\bigcup_k P_k$, k= FF, FR, FRB, UPVC, UAl, C, where $\bigcup_k P_k \neq \Phi$, $\bigcap_k P_k \neq \Phi$, which expresses the fact that they are affined products, make up a set of products, products have common constituents [2]. This allows their manufacturing on unique flexible manufacturing system (SP).

P_k in particular, and FP_k in general, are seasonal products, of low technological complexity (the labor expressed in the total cost does not weigh much); increased consumption of imported materials (the material expressed in the total cost weighs a lot); small-batch manufacturing, frequent change of batch, varied composition of batches, all call for frequent adjustments of SFF to requirements.

According to the company's size, the SP may be: industrial or manufacturing; completely manual, less mechanized or automated; with rigid or flexible manufacturing; with manufacturing flows organized on basis of pushing or pulling systems.

3. Initial data

Using indices: t for „past", as previous season S , p – „present" and v – „future", (S_t, S_p, S_v) from historical statistical data („past") and from current statistical data („present") of the company, through extrapolation, future items of information (parameters, efficiency indices, etc) may be forecasted, under probalistic, determinist and uncertainty conditions (with assumed risk). The extrapolation may be done by using any distribution, frequently using the short-time linear distribution, on time intervals ranging from a few months to 2 or 3 years, which requires additional information.

The main initial data required for model development are: production volume, in physical Q_{fk} [buc./τ,k] and value Q_{vk} [uv/τ,k] units per product P_k and total per company Q_{fT} [buc./τ], Q_{vT} [uv/τ], unitary costs per product C_{uk} [uv/buc.] and total costs per company C_T [uv/ τ], being added the seasonal indices to which t, p, v are related.

Another very important initial datum is the probability forecasted for the future (p), with indices k, T, v, by the marketing department.

A very important part in developing the model is played by the additional information, which is taken into account through the I_a automated coefficient; the additional information is comprised in the interval $0<I_a\leq 1$. For $I_a=1$ the additional information is complete, which leads to a determinist and probalistic model; when $I_a\rightarrow 0$ the additional information does not occur, which leads to an uncertain model, with maximum risk noted with r_{max}. For any I_a value ranging from 0 to 1, there results an uncertain probalistic model, with assumed risk r_a, through the company's policy, defined as $r_a=(1/ I_a)$. When $I_a\rightarrow 0$ there results $r_a\rightarrow \infty= r_{max}$, and when $I_a\rightarrow 1$ there results $r_a\rightarrow 1$ and, therefore, $1\leq r_a<\infty$. The company's management determines the percentage value of the risk.

4. Calculated values

The time interval τ to which the production is related may be: day – z, week – spt, month – L, season – S, year – A. The time interval τ may be sampled in subintervals Δτ=Ti, with i=z, spt, L, S, A so that τ can be expressed as sum:

$$\tau=\sum_{i} T_i \quad \text{[ut/z, spt, L, S, A, 3L or 6L]} \tag{1}$$

In case i=S, it must be stated the season and the time interval T_i it covers, as the time interval is different for different seasons $S_t\neq S_p\neq S_v$.

The production volume per product Q_k depends on the measurement unit – physical units Q_{fk} [buc.] or value units Q_{vk} [uv]; the time interval T_i is related to, resulting $Q_{f\tau k}$ [buc./T_i,k], and $Q_{v\tau k}$ [uv/ T_i,k], is expressed as follows:

$$Q_{f\tau k}=\sum_{i=z}^{spt,L,S,A} Q_{fki} \quad \text{[buc./T}_i\text{,k]}$$
$$Q_{v\tau k}=C_{uk}*\sum_{i=z}^{spt,L,S,A} Q_{fki} \quad \text{[uv/T}_i\text{,k]} \tag{2}$$

The total production Q_T in physical and value units, dependant on the time interval, is expressed thorough the relation:

$$Q_{f\tau T}=\sum_{k=FF}^{FR,FRB,UPVC,UAl,C} Q_{f\tau k} \quad \text{[buc./T}_i\text{]}$$
$$Q_{v\tau T}=\sum_{k=FF}^{FR,FRB,UPVC,UAl,C} (Q_{f\tau k}*C_{uk}) \quad \text{[uv/ T}_i\text{]} \tag{3}$$

The share of the product in total production p_τ in physical and value units, dependant on the time interval, is expressed through the relation:

$$p_{f\tau k}=\frac{Q_{f\tau k}}{Q_{f\tau T}} \quad [\%]$$
$$p_{v\tau k}=\frac{Q_{v\tau k}}{Q_{v\tau T}} \quad [\%] \tag{4}$$

The share of the product in total production is related exclusively to „past – t" and „present – p" time so that τ=t or p.

The forecasted probability p depends on the parameter it is related to, i.e.: product p_k or total production per company p_T; in physical units p_{fk}, p_{fT} or in value units p_{vk}, p_{vT}; considering the time interval it is related to, the forecasted probability is related exclusively by „future – v" forecasting- p_{fkv}, p_{fTv}, p_{vkv}, p_{vTv}.

The forecasted probabilities may be determined through different methods, among which the following are the most important:

- The experimental-oriented method consisting of establishing the probabilities values through market testing conducted by the marketing department. This method provides the most trustful values for probabilities but it is prohibitive, being the most expensive and taking a lot of time to conclude.
- The indirect calculation – oriented method, which uses historical statistical data from the company, consists of indirect calculation of probabilities under conditions of uncertainty through the „strategic games with nature" method.

The uncertainty [5] defines that decisional environment, with well-defined states of nature, in which one or more decisional alternatives have different results in a given set, results whose probability of occurrence is unknown or unlikely to be objectively considered. There are two ways of making a decision under uncertainty conditions:

- the decision maker may use the best available information, the own experience and estimation in order to:

1) identify and associate subjective probabilities, allocated to possible states of nature;
2) to estimate the consequences of each available strategy, in each state of nature;

- provided the level of uncertainty is so high that the decision maker prefer not to forecast the probabilities of different results in different states of nature, the decision maker can consider them equal (equiprobable)

The decision under conditions of uncertainty is characterized in that the probabilities to reach the states of nature are not known, more possibilities of choosing alternatives being available. Under conditions of uncertainty managers can make decisions considering one of the rules: prudence rule (Wald's pessimist criterion), optimism rule (Hurwicz's optimist and balanced optimist criterion), equilibrium rule (Bayes-Laplace's equiprobabilities criterion or maximum indetermination), the rule of regrets (Leonard-Savage's criterion of minimum regret).

The decisional matrix can be operated directly by using forward-looking probabilities values or intermediary parameters upon which the calculus of probabilities depends, such as times, space units, costs, payments, losses, benefits, etc.

The manager is in charge of the choice of the decision criterion and the analyst is bound to respect it.

Under conditions of uncertainty the most appropriate decisional criteria are those based on equiprobabilities – Laplace and balanced optimism – Hurwicz, whose expressions allow us to explain the probabilities dependant on intermediary calculation parameters from the decisional matrix [3].

Once calculated, the forward-looking probabilities must be submitted to a validation process. The validation in real environment is prohibitive provided costs and consumed time are considered. A theoretical mean of validation is given by Taguchi method [4], [1], through the concept of „robustness", according to which „ the lower the dispersion of the calculated parameter is the more robust the calculation will be". Consequently, the forecasted probabilities that were calculated must meet the condition of „decision robustness".

The decision matrix is considered [3],

$$[A] = [a_{ij}] \tag{5}$$

where all elements of matrix a_{ij} denote the parameter's forecasted values, studied indicator, corresponding to line – variant V_i, $1 \le i \le m$, and column – state S_j, $1 \le j \le n$, as different results obtained in different technical-economic environments. a_{ij} parameters are theoretically forecasted with a fixed mean value by the manager, different for i=var, j=var. In real manufacturing environment, due to the influence of disturbance upon the process, the a_{ij} elements are variable even for i=ct şi j=ct, resulting for $\forall (V_i \cap S_j)$, $1 \le i \le m$, $1 \le j \le n$, an a_{ijmax} and an a_{ijmin}, $a_{ij} \in [\ a_{ijmin},\ a_{ijmax}]$ and, therefore, an $a_{ijmed} = \overline{a_{ij}}$ whose value is inscribed at the given cross and is forecasted by the management.

According to [4] the function of quality loss is defined as being given by the expression:

$$L(a_{ij}) = k(a_{ij} - a_{ijN})^2 \tag{6}$$

expressing the value of the unitary loss, expressed monetary units, where a_{ij} denotes the forecasted current value due to the parameter in the decisional matrix placed at $V_i \cap S_j$, a_{ijN} – nominal, real, target, standardized and necessary value of the same parameter, k – a constant that adjust the loss to the concrete case (for instance the total cost, in monetary units, of either a wrong decision or the assumed risk).

From the above expression there results that the loss is directly proportional to the squared deviation of the parameter as related to the nominal, target value. As a result, the reduction of financial losses caused by the loss of the decision quality is achieved by the one single way:

$$\min L(a_{ij}) \Rightarrow \min(a_{ij} - a_{ijN})^2 \tag{7}$$

i.e. by minimizing the squared deviation of the parameter.

Provided the parameters of the considered factor's variant are taken into account, by the intermediary of its dispersion around a central value, the following statistical indicators arise:

- ☆ parameter's arithmetic mean:

$$\overline{a_{ij}} = \frac{\sum_{\alpha=1}^{n} (a_{ij})_\alpha}{n} \tag{8}$$

- ☆ squared mean deviation:

$$\sigma = \sqrt{\frac{\sum_{\alpha=1}^{n} (a_{ij} - \overline{a_{ij}})_\alpha^2}{n}} \tag{9}$$

which give the dispersion measurement .

The parameter in the decision matrix has to be either minimized or maximized. Consequently, Taguchi [4] shows that for each case, the function of the quality loss is expresses as follows:

- in case of minimizing a_{ij} parameter, there results (min a_{ij}):

$$L(a_{ij}) = k(\sigma^2 + \overline{a_{ij}}^2) \quad (10)$$

expressing the mean unitary loss and being applied with the lowest possible $\overline{a_{ij}}^2$ and the highest possible σ^2 (low mean unitary loss and high dispersion);

- in case of maximizing the a_{ij} parameter, there results (max a_{ij}):

$$L(a_{ij}) = k\frac{1}{\overline{a_{ij}}^2}[1 + 3(\frac{\sigma}{\overline{a_{ij}}^2})^2] \quad (11)$$

expressing the mean unitary loss and being applied with the highest possible $\overline{a_{ij}}$ and the lowest possible σ (high mean unitary loss and low dispersion).

In the above relations the function of quality loss does not depend on the target nominal value of the considered parameter. Taguchi [4] proposes the relations:

- ✓ in case of minimizing the a_{ij} parameter, (min a_{ij}):

$$L(a_{ij}) = k[\sigma^2 + (\overline{a_{ij}} - a_{ijN})^2] \quad (12)$$

- ✓ in case of maximizing the a_{ij} parameter, (max a_{ij}):

$$L(a_{ij}) = \frac{k}{\overline{a_{ij}}^2}[1 + 3(\frac{\sigma}{\overline{a_{ij}}})^2] \quad (13)$$

The above written relations show that in order to improve the decision quality and to reduce its cost (the increase of decision robustness) the target nominal value a_{ijN} must be correctly established and the dispersion must be reduced as much as possible (min σ). The maximization of any performance indicator is obtained by minimizing the quality loss, that is:

$$\max a_{ij} \Rightarrow \min L(a_{ij}) \quad (14)$$

By meeting the last condition there results:

- min $a_{ij} \Rightarrow a_{ijN\,\min} \le \overline{a_{ij}} - \sigma^2$
- max $a_{ij} \Rightarrow a_{ijN\,\max} \ge \overline{a_{ij}} + (1{,}7...2)\sigma$

The influence of disturbances occurring outside the process leads to real variation of the considered parameter in the interval $a_{ij} \in [a_{ijN\,\min}, a_{ijN\,\max}]$. Their values depend on the mean of values forecasted by the manager $\overline{a_{ij}}$ in $\forall(V_i \bigcap S_j)$ and admitted σ dispersion.

As for the forecasted probabilities $a_{ij} = p_{ij}$ and $\sigma = \sigma_p$, if p_{ij} may be determined through one of the methods previously presented, the squared mean deviation σ_p must meet the following conditions:

- ✰ since $0 < p_{ij} < 1$, σ_p must express a small variation as compared to p_{ij}; therefore, form min $a_{ij} \Rightarrow \sigma_p=(0.,10...0.20)\ p_{ij}$, and for max $a_{ij} \Rightarrow \sigma_p=(0{,}001...0{,}050)\ p_{ij}$;
- ✰ if p_{ijN} values resulted from calculation do not comply with Laplace or Hurwicz criteria, they must comply with one of Wald or Savage criteria.

The automated coefficient I_a or the assumed risk r_a are determined through a foundation identical to the forecasted probability, aiming at the max I_a with min r_a.

5. Conclusions

Using the knowledge from the specialist literature there were established the application conditions of a forward-looking model developed to forecast the technical-economic environment in manufacturing heat-insulation carpentry. The initial data of the model are defined so that the latter one should be forecasted on a probalistic basis. The values that must be calculated before model elaboration are defined; likewise, the min max conditions of the quality loss (robustness) function, target value and decision dispersion are stated.

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