STATIC MODEL AND SIMULATION OF A PNEUMATIC ARTIFICIAL MUSCLE

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Abstract. A Pneumatic artificial muscle (PAM) is a pneumatic device developed in the 1950's as an orthotic appliance for polio patients by J.L. McKibben. It consists of a rubber bladder encompassed by a tubular braided mesh. When the bladder is inflated, the actuator expands radial and undergoes a lengthwise contraction.

Present paper starts the research regarding PAM (pneumatic artificial muscle) by determining the static model of a PAM and testing this model by some simulations. The parameters resulted from the static model will be the input data for the dynamic model and, for the theoretical simulations, respectively.

Keywords: pneumatic artificial muscle, static model, valve

1. Introduction

In recent years, robotics engineers have begun to rediscover these fascinating devices, and use them as actuators for their robots. These actuators exhibit non-linear force-length properties similar to skeletal muscle, and have a very high strength-to weight ratio.

The static model of a PAM can be done based, as many researchers done it before [1], based on the virtual mechanic work. This will provide a relationship between actuator force, pressure, and length. After that, the obtained equations will be used to derive further relationships between force, pressure, length, and stiffness.

2. Simplified static model

The PAM can be modelled as a cylinder. The non-cylindrical end effects are ignored, and the wall thickness is assumed to be zero. The dimensions of this cylinder are the length, *L*, and diameter, *D*. Neither of these dimensions remains constant. Assuming inextensibility of the mesh material, the geometric constants of the system are the thread length, *b*, and *n*, the number of turns for a single thread. The final dimension used for this formulation is the interweave angle, θ. *Theta* is the angle between the thread and the long axis of the cylinder. The interweave angle changes as the length of the actuator changes.

The relationship between these parameters is presented in figure 1.

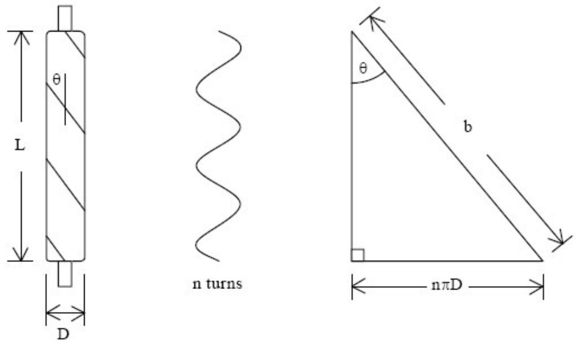

Figure 1. Constructive design of a BPA [1]

During operation *L, D* and θ are variable (θ changes as length *L* changes), while, based on the above hypotheses, the *b* and *n* parameters are constant.

As it is shown in figure 1, the relationships between these parameters are:

$$L = b\cos\theta\,; \tag{1}$$

$$D = \frac{b\sin\theta}{n\pi}\,. \tag{2}$$

The volume of any cylinder is equal to its length times the cross sectional area.

$$V = \frac{\pi D^2}{4}L\,. \tag{3}$$

Substituting equations (1), (2) into (3) results:

$$V = \frac{b^3}{4\pi\cdot n^2}\sin^2\theta\cos\theta\,. \tag{4}$$

The maximum contracted length (minimum length) occurs when the actuator volume is at its greatest. This results in equilibrium of the system.

To be able to simulate the static model it is chosen a PAM that has the maximum interweave angle $\theta = 54.7°$ and the corresponding length and diameter L = 40 mm, D = 10.865 mm.

These value together the hypothesis that b and n parameters are constant during operation, are used to determine this functional parameters. Thus, from equation (1) results:

$$b = \frac{L}{\cos\theta} = \frac{40}{\cos(54.7°)} = 69.221 \text{ mm}. \tag{5}$$

Using equations (2), (5) it will be determined the number of turns for a single threat. Thus:

$$n = \frac{b\sin\theta}{D\pi} = \frac{69.221 \cdot \sin(54.7°)}{10.865 \cdot \pi} = 2.248. \tag{6}$$

Thus it is obtained that a single threat has 2 complete turns and 25% from the third.

After the geometry is determined it can be developed a relationship between the force as function of pressure, and the length of PAM. To obtain this relationship it will be done a simple energy analysis. The assumption is conservation of mechanical work, W_i = W_e. The losses will be neglected in this step. Work is input to the actuator when the air pressure moves the inner bladder surface. Thus, the pressure variations determine a variation of input mechanical work:

$$\begin{aligned} dW_i &= \int_{S_i} (P_{abs} - P_{atm}) dl_i \cdot ds_i = \\ &= (P_{abs} - P_{atm}) \int_{S_i} dl_i \cdot ds_i = P_g dV, \end{aligned} \tag{7}$$

where: P_{abs} is absolute internal gas pressure; P_{atm} – atmospheric pressure; P_g – gage pressure; S_i – total inner surface; ds_i – area vector; dl_i – inner surface displacement; dV – volume change.

The output work occurs when the actuator shortens due to the change in volume.

$$dW_e = -F \cdot dL. \tag{8}$$

Considering the ideal system can be applied the mechanical work conservation law:

$$dW_i = dW_e \tag{9}$$

Substituting equations (7) and (8) into (9) results:

$$P_m dV = -F dL; \tag{10}$$

$$F = -P_g \frac{dV}{dL}. \tag{11}$$

Using the geometry that was established above (equations (1) ÷ (3)), can be developed an equation for force as a function of pressure and interweave angle.

$$\begin{aligned} F &= -P_g \frac{dV}{dL} = -P_g \frac{dV/d\theta}{dL/d\theta} \\ &= \frac{P_g b^2 \left(2\cos^2\theta - \sin^2\theta\right)}{4\pi \cdot n^2} \end{aligned} \tag{12}$$

Thus, results an equation of force as function of P_g and θ.

$$F = \frac{P_g b^2 \left(3\cos^2\theta - 1\right)}{4\pi \cdot n^2}. \tag{13}$$

Note that at the maximum interweave angle, 54.7°, the force output of the actuator is zero. The geometric variables used above provide a straightforward formulation, but to use the resulting equations in practice, they first need to be modified. The first step is to develop a method to accurately measure the braid length, b, and count the non-integer number of thread wraps [1]. If the cylindrical mesh is opened and laid flat, the trapezoidal geometry is easily observed. The shape of the trapezoid is governed by the interweave angle, θ, and the length of the trapezoid side, ℓ.

$$L = 2A\ell\cos\theta \tag{14}$$

$$C = 2B\ell\sin\theta \tag{15}$$

where A is the number of lengthwise trapezoids and B - number of circumferential trapezoids (around actuator)

The diameter is proportional to the circumference and so:

$$D = \frac{C}{\pi} = \frac{2B\ell\sin\theta}{n\pi}. \tag{16}$$

Setting the length (equations (1) and (14)) and diameter (equations (2) and (16)) equations equal results:

$$b = 2A\ell \qquad n = \frac{A}{B} \tag{17}$$

Thus, to characterize such and actuator it is enough to know the trapezoid size and number of circumferential trapezoids in both directions.

The results obtained above used the maximum value of the interweave angle. To be able to apply a control to this system it is necessary to eliminate this angle. This is justified by the fact that it is difficult to sense the interweave angle during operation of the actuator. It is much easier to measure the length. Thus, in the following these equations will be rewritten in terms of force, pressure, and length, because these variables can be measured most easily.

Considering the triangle from figure 1 there can be determined the sin and cos function of θ interweave angle. Thus,

$$\cos\theta = \frac{L}{b}; \qquad \sin\theta = \frac{\sqrt{b^2 - L^2}}{b}. \tag{18}$$

Substituting these equations into (4) and (13) equations will result the expression of the volume and force as function of geometric parameters.

$$V = \frac{L\left(b^2 - L^2\right)}{4\pi \cdot n^2} \tag{19}$$

$$F = \frac{P_g b^2}{4\pi \cdot n^2}\left(\frac{3L^2}{b^2} - 1\right) \tag{20}$$

Relation (20) sustains the affirmation that the PAM exhibit properties of a variable stiffness spring. This approximation is an advantage relative to control but the biggest disadvantage is the stiffness that is not easy to be measured. Therefore this parameter should be mathematically determined by the force derivate with respect to length:

$$k = \frac{dF}{dL} \tag{21}$$

The derivate was done by neglecting the pressure P_g change relative to L, thus $dP_g / dL \approx 0$. Thus, after derivation, the stiffness is

$$k = \frac{3LP_g}{2\pi \cdot n^2} \tag{22}$$

Extracting P_g from equation (19) and substituting into (22) results,

$$P_g = \frac{4\pi \cdot n^2}{3L^2 - b^2} F. \tag{23}$$

Substituting equation (23) into (22) results the expression of stiffness

$$k = \frac{6L}{3L^2 - b^2} F. \tag{24}$$

Based on equations (20) and (22) there were drawn the change of stiffness k (figure 2) and actuator force F (figure 3) as functions of PAM's length. It was considered that the length L is between 40 and 50 mm, the actuator diameter start at 10mm and the feed pressure is between 3 and 6 atm.

Figures 2 and 3 shows that force is a nonlinear function of length, tending to zero at maximum contraction, and stiffness is a linearly increasing function of length and pressure. In figure 4 this nonlinearity is hard to be seen because the input interval is a small one. Thus, if the muscle contracts between 50 and 10 mm, the force, F, variations relative to length L, will be that shown in figure 4, the nonlinearity dependence and tendency to zero being observable.

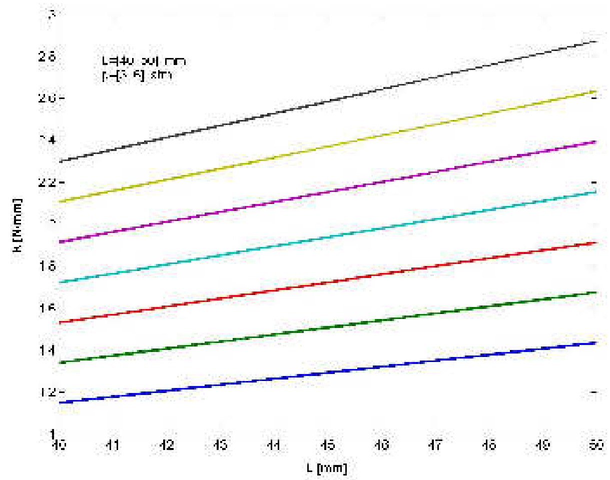

Figure 2. Stiffness k changes as function of length L, for a given pressure variation

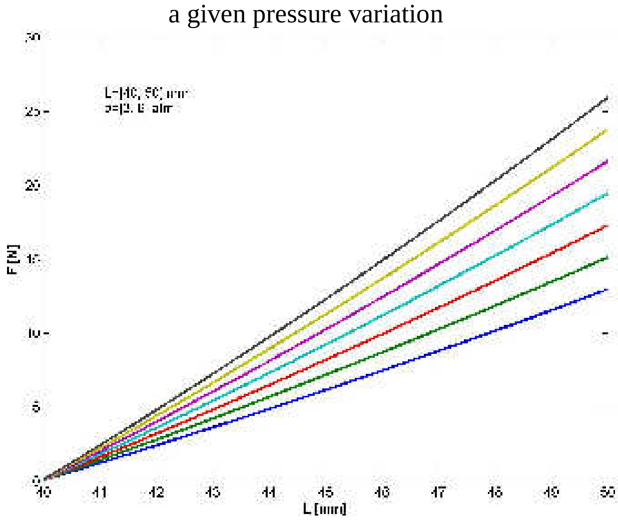

Figure 3. Force F changes as function of length L, for a given pressure variation

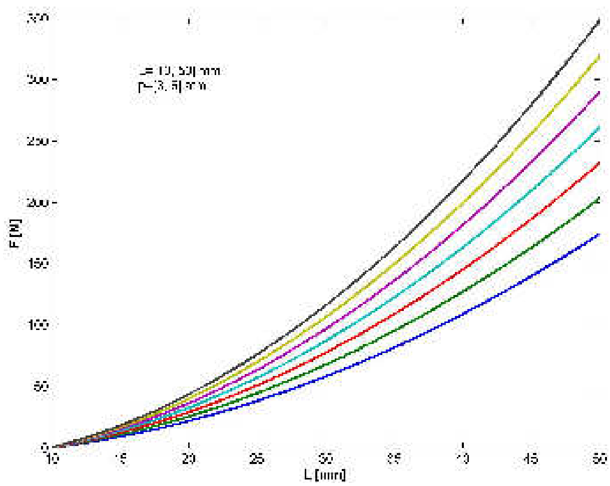

Figure 4. Force F changes as function of length L, for a given pressure variation

The simulations were done considering neglected the effectiveness of the system and the effects of open ends of the PAM. The researchers [1, 2] establish that in theoretical case the output force is higher that the measured one.

This difference forces the introduction of effectiveness, which represents the percentage ratio between the measured force and the theoretical determined force. Considering the experiments done until now it can be considered that the effectiveness is a function of gage pressure. Experiments done by Colbrunn [1] show the dependence between effectiveness and nominal pressure is not a linear one (for a pressure of 60 psi the effectiveness is around 92%).

3. Open ends effects

To obtain a correct static model must be considered the open ends effects. The ends of the PAM are non-cylindrical. The end effects determine changes of output force at the length limits of the actuator. The long end effect occurs when the actuator is at its maximum length. At this point, any increase in length would cause the braids to stretch. Since the stiffness of the braiding material is very high compared to the stiffness of the actuator, the long end effect can be modeled as a spring of very high stiffness when the actuator reaches a maximum length. In [1] Colbrunn shows that an increase of the pressure determines a high increase of gradient of the force-length diagram, showing the effects of the open ends of PAM.

The actuator, like a muscle, can only pull. The model (20) predicts that if the length is less than the maximum contracted length, then the force output will be negative (pushing). Thus, a short end effect is added to the model. It simply states that if the length is less that the minimum length, the force output becomes zero.

Incorporating the effectiveness and the end effects into (20) results (for $L > L_{\min}$ & $L < L_{\min}$):

$$F = \begin{cases} \dfrac{P_g b^2}{4\pi \cdot n^2}\left(\dfrac{3L^2}{b^2} - 1\right) \cdot Ef(P_g) + F_{\max} \\ 0 \end{cases} \quad (25)$$

where ($L > L_{\max}$ & $L < L_{\max}$)

$$F_{\max} = \begin{cases} K_{braid}\left(L - L_{\max}\right) \\ 0 \end{cases} \quad (26)$$

where K_{braid} – braid material stiffness.

Equation (26) is the static model for a PAM. This model ignores bladder thickness, bladder thickness variation, friction-induced hysteresis, and non-linear elastic energy storage of the bladder.

Klute and Hannaford [1] have developed a more accurate model using these factors, but the cost of the more accurate model is much longer equations and therefore more computational time. This model is adequate and yet simple enough that it is suitable for dynamic simulations and control.

4. Conclusions

The first step of PAM research is to mathematical model it both from static and dynamic point of view. The static analysis, done in this paper, shows that this PAM can be approximated with a high stiffness spring. All the results obtained will be used in the dynamic model and in control. Also, the theoretical results will be compared with the measured one.

5. Acknowledgement

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