FRACTURE MECHANICS COMPUTER MODEL FOR THE CALCULATION OF THE CONTACT FATIGUE LIFE OF SPUR GEARS

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Abstract. The Fracture Mechanics approach is a relatively new approach used for the calculation of mechanical components fatigue life. In the paper presented a computer model, based on the fracture mechanics approach and on finite element analysis for determination of gear teeth fatigue life, is demonstrated. The model allows to follow the growth of a net of micro-cracks and their mutual interaction. It is considered that cracks initiate on the surface and finish on the surface. It is assumed, that the gear tooth contacting surface fails due to pitting when the size of pits increases 40 μm.

Keywords: gears, gears tooth contact fatigue, fracture mechanics, crack propagation, finite element analysis

1. Introduction

The Fracture Mechanics Approach is recently introduced powerful approach for the determination of fatigue life of mechanical components. The focus of this approach is on the phases on crack growth until it becomes critical, and the particle is removed from the surface. The analysis tool is “Linear Elastic Fracture Mechanics”.

The contact fatigue or pitting is the most common failure for the gears operating in oil [1, 2]. Figure 1 shows a gear tooth with intensive pitting which causes loosing of its working ability.

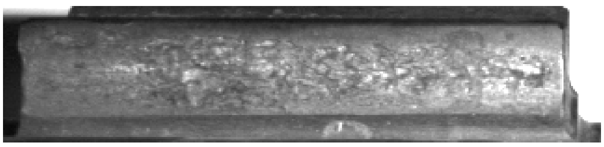

Figure 1. A spur gear tooth with intensive pitting

There are many models describing pitting phenomena [3, 4, 5, 6]. All these models deal with one single crack and its growth. Actually contact fatigue failure begins not from one crack only. During the operation of gears, a net of micro-cracks generates over the all gear contacting surface. With the increase of the number of loading cycles these micro-cracks mutually interact. Considering such mechanism of pitting phenomena, we can conclude that the number of cycles till developing of pits is smaller than in models with the following of one crack growth [7, 8].

The aim of this work is to suggest a model for adequate description of pitting phenomena for gears operating with elastohydrodynamic lubrication. This model should include growth of a net of cracks.

2. Prerequisites

2.1. Location of zones with pitting over gear tooth flange

The development of the model starts with the determination of the zone where pitting is the most likely to occur. From figure 2 it can be seen that the pitting friendly zone is the zone where speed of rotation and speed of slip during the gear teeth interaction is in different directions (case II). This is the zone below the pitch circle, where, additionally, the loading conditions are heavier because only one pair of teeth is in contact.

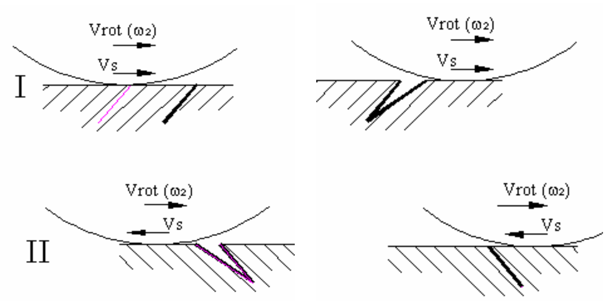

Figure 2. Unfavourable (I) and favourable (II) circumstances on gear tooth contacting surface for micro-cracks to grow

The flank zone, as a favourable zone on gear tooth contacting surface for initiation of pitting is proposed by many authors [2, 9]. In this model, the opposite direction of rolling and sliding linear velocities is considered.

2.2. Contact surface loading

The gear tooth contact surface is subjected to normal force F_{bn} (transmitted force) in the mesh and to frictional force F_{fr}. The resultant surface pressure $p(x)$ and frictional pressure $q(x)$, as a function of normal pressure, are shown in figure 3.

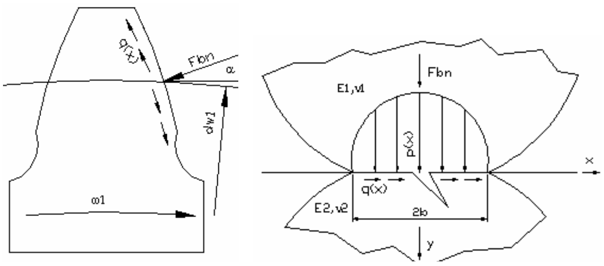

Figure 3. Gear teeth contacting surfaces loading

We consider that in a gear mesh, the contacting surfaces are separated with a thin fluid film, which transmits contact stresses, by its pressure. This kind of lubrication – elastohydrodynamic (EHD) is favorable for reduction of friction but the presence of a pressure peak in the outlet region produces large shear stress that is localized very close to the surface. Figure 4 shows Hertz and EHD lubrication pressure in a gear mesh.

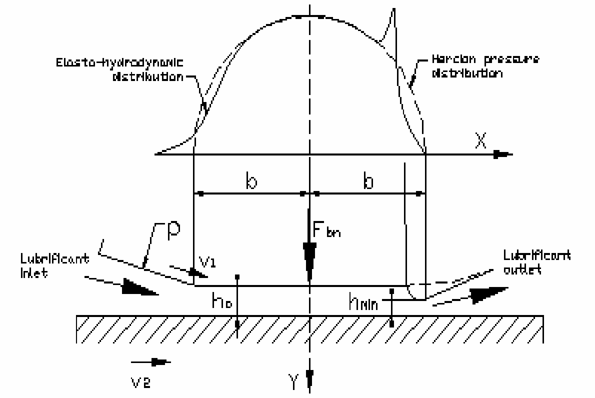

Figure 4. Graphical interpretation of EHD pressure and Hertzian contact pressure

It can be seen from figure 4 that EHD lubrication as a whole coincides with the contact pressure calculated by Hertz [10] for the case of two cylinders in contact. The peaks on the right side on the figure in the model are determined according to suggested by Hamrock [11] experimental relations:

$$Y = 0.267W^{-0,375}U^{0,174}G^{0,219} \quad (1)$$

$$X = 1 - 2.469W^{-0,941}U^{0,206}G^{-0,848} \quad (2)$$

In equation (1) Y is the peak amplitude and X is its coordinate in equation (2). W, U, and G are dimensionless coefficients which could be found as follows [11]:

$$W = \frac{F_{NB}}{E_\Sigma \rho_\Sigma} \text{ - } \textit{loading coefficient} \quad (3)$$

In equation (3) $E_\Sigma = 2E_1E_2/(E_1 + E_2)$, where E_1 and E_2 are modules of elasticity; $\rho_\Sigma = r_1r_2/(r_1 + r_2)$, where r_1 and r_2 are radii of curvature in the point of contact.

$$U = \frac{\eta\nu(\nu_1 + \nu_2)}{2E_\Sigma \rho_\Sigma} \text{ - } \textit{speed coefficient} \quad (4)$$

In equation (4) η is fluid density; ν is kinematics viscosity; v_1 and v_2 are contacting surfaces linear velocities.

$$G = \alpha_p E_\Sigma \text{ - } \textit{material coefficient} \quad (5)$$

In equation (5) α_p is a coefficient taking into account the surface pressure as a function of viscosity.

The coefficient of friction is assumed to be constant all over the contact and is assumed 0.04 [12]. Then the quantity of sliding friction is calculates as:

$$q(x) = 0.04p(x) \quad (6)$$

3. Description of the model

The model is based on the use of *Mechanical Desktop* and *Matlab* computer packages. The result of computer simulations is the number of cycles till the first appearance of pits on the gear tooth contact surface.

3.1. Assumptions

In the process of the model development the following assumptions are made:

1. The materials used for gears are isotropic.
2. There is a net of microcracks available on the gear tooth contacting surface (the so called “micro pitting”). It is assumed that the difference between “micro pitting” and “pitting” is the cracks length. Pitting is associated with cracks longer than 15 μm [13].
3. “Critical” pieces are considered formations of pits size above 40 μm [2, 13].
4. Initial micro-cracks are inclined to the surface in angle of $\theta_1 = 22^0$ [15].
5. The only one gear teeth pair is in contact.
6. The load is constant.
7. The contact between two gear teeth is approximated with two cylinders with radii equal to the radii of curvature in the contacting point.

3.2. Method of model operation

Figure 5 shows the initial circumstances concerning the arrangement of finite elements.

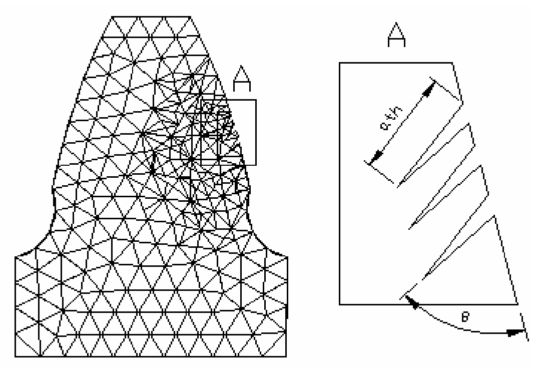

Figure 5. Graphical interpretation on initial circumstances

In the model theory of short cracks growth is used. It is assumed that the plastic zone in the tip of a crack is comparatively very small and the linear fracture mechanics could be applied and the Paris [16] equation could be used:

$$\frac{dA}{dN} = C \cdot [\Delta K(a)]^m \tag{7}$$

In equation (7) C and m are experimentally obtained coefficients, $\Delta K(a)$ is effective stress intensity factor. For short cracks, we can write:

$$\Delta K = F_{IS} K_t \Delta\sigma\sqrt{\pi a} \tag{8}$$

In equation (8) F_{IS} is a coefficient accounting short cracks interaction effect; K_t is stress concentration factor; $\Delta\sigma = \sigma_{max} - \sigma_{min}$ - applied stress range; a is crack length, α is a coefficient depending on the shape of surface and type of loading. In this case $\alpha = 1.12$.

When equation (7) is integrated, for the number of cycles we have:

$$N_P = \frac{1}{C}\int_{a_{th}}^{a_c} \frac{da}{[\Delta K(a)]^m} \tag{9}$$

The integral (9) limits are: a_{th} – initial length of the micro-crack; a_c – critical length of the crack.

The level of stress in the pit top is calculated by the use of *Mechanical Desktop* finite element analysis. The number of cycles needed for the crack to grow until pit falls is calculated by the use of program *Matlab*.

The angle of propagation of a crack θ is computed by the use of maximum shear stress criterion. According to this criterion cracks grow from its tips in direction perpendicular to maximum shear stress [16]. Shear stresses in cracks tips are shown in figure 6.

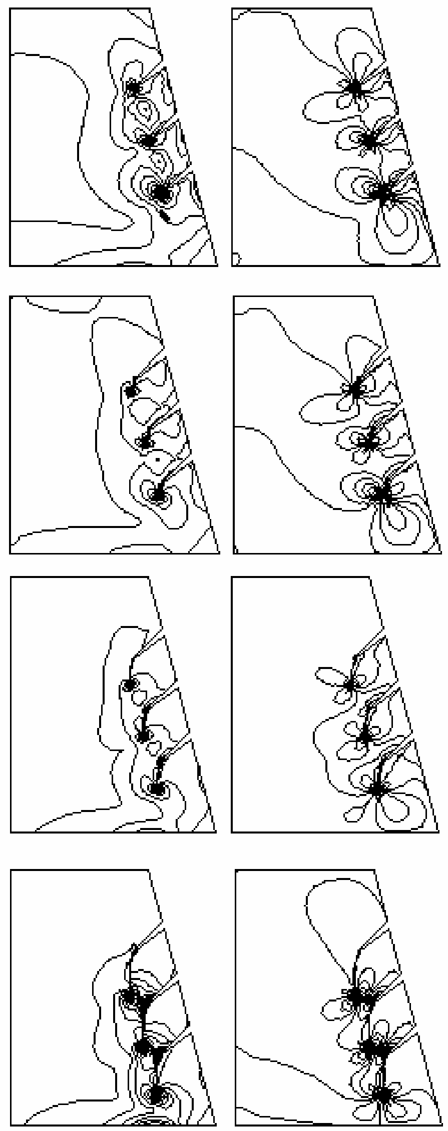

Figure 6. Distribution of stresses in the process of cracks growth

If we accept that gear teeth cracks grow according to *I* and *II* mode [3, 16, 17, 18, 19], then for the angle of growth could be written [14]:

$$\theta_i = 2\tan^{-1}\left(\frac{1}{4}\frac{K_I}{K_{II}} \pm \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right) \tag{10}$$

Stress intensity factors K_I and K_{II} (for *I* and *II* modes) were computed according to fracture mechanics equations [16].

3.3. Roughness stress concentration factor

The influence of concentration of stresses in the tip of roughness is recognized from many years. Researchers suggest different approaches to the role of roughness depth *c* and tip radius of curvature ρ [11, 12, 19]. In this work, for accounting of these parameters we consider the stress concentration factor K_t, which could be determined according to [19].

$$K_t = 1 - 0.719 e^{-0.476 b/d} \left(1 + 2\sqrt{\frac{c}{\rho}}\right) \qquad (11)$$

In equation (11) b is the distance between two adjacent roughness valleys, and d is the width of a valley.

3.4. Short crack interaction effect

In the model, the effect of crack interaction is taken into account. This effect becomes significant when the density of asperities is high and the distance between cracks is relatively small. For short crack case, in the model the following equation for the crack interaction factor is used [19]:

$$F_{IS} = 1.2 - 0.293 \frac{2a}{2a+h}\left[1 - \left(1 - \frac{2a}{2a+h}\right)\right] \qquad (12)$$

In equation (12) a is crack length, and h is distance between adjacent cracks.

4. Results

The functioning of the model was checked with data for steel ANSI 4130 [18] with σ_Y = 900 MPa; K_{th} = 300 N.mm$^{-3/2}$; C_0 = 120.57; m = 3.069, and initial length of cracks of a_{th} =15 μm and θ_1 = 22^0. Tearing of pits (40 μm) was observed when N_p = 2.016.10^7 cycles. These results coincide well with results from [2, 12] and the deviation is about 3.20 %.

Figure 6 illustrates the functioning of the model in four phases. The first column shows stresses according to the Von Mises theory and the second one indicates shear stresses.

Figure 7 shows the crack growth for the same phases. In table 1, the initial data (phase *I*) are considered as 1 and the quantitative evaluation of stress and deformation for other phases is presented.

Table 1. Quantitative evaluation (in times) of stress and deformation growth for the illustrated above four phases

Model	Von Mises	Shear stress	Deformation	
			X	Y
I st stage	1	1	1	1
II nd stage	2.81	1.42	2.52	2.25
III rd stage	4.78	4.23	6.07	4.78
IV th stage	5.92	6.11	21.51	11.8

5. Conclusion

This work illustrates one more application of Fracture Mechanics approach in the determination of fatigue life of machine components. This is a new and more accurate approach in computing the fatigue life of gears operating in oil. The model which covers a net of cracks gives results much closer to the experimental data.

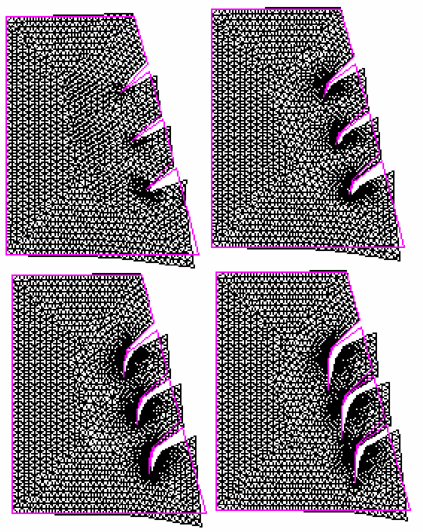

Figure 7. Phases of cracks growth (corresponding to the stresses from figure 6)

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