USING SHORT STUBS FOR MATCHING THE IMPEDANCE OF A LOAD TO THE IMPEDANCE OF A TRANSMISSION LINE-ANALYTICAL SOLUTION AND COMPUTER CODE

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Abstract. This paper presents an analytical solution and a computer code for solving the problem of how to match a load impedance to the impedance of a transmission line using three short stubs.

Matching the impedance of a network to the impedance of a transmission line is necessary first, because it makes all the incident power to be delivered to the network, and second, it is necessary for a better behave of the generator, because, usually the generator is designed to work into an impedance close to common transmission line. In this case the load impedance has no reactive part which can pull the generator frequency, and the SWR on the line is close to the unity, so the line connecting the generator to the load is non resonant. Stubs are used for producing a pure reactance at the attachment point, reactance that varies with their length. For matching any impedance load to the impedance of the transmission line 3 stubs, that have fixed positions on the transmission line, can be used. This paper is focused on how to determine their lengths for solving the problem of matching the impedances.

An analytical solution of this problem is given and, based on it, a computer code in MATHCAD is developed too. The computer code offers the lengths these stubs must be regulated to, when the load impedance and the impedance of the transmission line are known. There exist four such solutions, so four combinations of lengths, and the computer code gives all these solutions.

Keywords: short stubs, transmission line, impedance, matching impedance

1. Introduction

In the field of microwaves (see[1, 2]) there are used high frequencies and so little wavelengths λ ($\lambda = c/f$, where c is the wave propagation velocity and f the frequency). The wavelength of a wave is the distance we have to move along the transmission line for the sinusoidal voltage to repeat its pattern, the spatial period of the wave. Because the circuits are often bigger than the wavelength there can't be neglected the propagation of the signals along them. That's why in microwaves even the simplest circuits must be considered as transmission lines, in which the propagation phenomena are not momentary but they are produced with a finite velocity, and so along them there do appear the dephasation.

In any transmission system, a source sends energy to a load (the forward wave). The power has to go somewhere. If it isn't absorbed it must be reflected. The difference between forward and reflected power flow is the power delivered to the load.

The transmission network is designed in the ideal case such that the characteristic impedances of the source, the transmission line and the load are all identical. When the transmission line impedance does not match that of the load, part of the transmitted waveform is reflected to the source (the backward wave). This reflected wave adds to the transmitted one and we get the Standing Wave. So because the amplitude vary as a function of position along the transmission line the Standing Wave Ratio (SWR) (the ratio between the maximum and minimum amplitudes of the total waveform-which occurs at 1/4 wave length away from the maximum), will in this case be greater than one.

The SWR is a sensitive indicator of mismatch on a line. If the impedance match is perfect there is no reflected wave, so the amplitude of the total waveform is constant along the transmission line. In this case SWR = 1 and this situation tells us that maximum power is transferred to the load.

The impedance we measure also varies along the transmission line because it is the total voltage on the line divided by the total current on the line. We have the following total line voltage:

$V = V_+ + V_-$ and similarly: $I = I_+ + I_-$. On a lossless line the current in the forward wave is in phase with the voltage but the current in the backward wave is oppositely. So even if the forward wave voltage and forward wave current are fixed (their ratio is the characteristic impedance of the line), because of the fact that the backward wave phaser swing around, the impedance varies along the line (see [2, 3, 4]).

So the impedance measured at a point along a transmission line depends not only on what is connected to the line, but also on the properties of the line, and where the measurement is made physically along the line, with respect to the load. The total backward wave amplitude divided to the total forward wave amplitude gives a complex dimensionless number, named gamma, (the complex reflection coefficient) which is related to how much is reflected at the end of the line and how far the reflecting point is.

There exists a relation between the complex dimensionless number gamma at any point along the line to the normalized load impedance $z_l = Z_l/Z_0$ (a complex dimensionless quantity too):

$$z_l = \frac{1+\Gamma}{1-\Gamma}, \quad \text{or} \quad \Gamma = \frac{z_l - 1}{z_l + 1}.$$

The normalized impedance at the point P is what a generator would see if we cut the line at this point P and connect the remaining transmission line and its load to the generator terminals.

Gamma is related to the SWR by the relation:

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

It is well known that the complex amplitude of a wave may be defined in three ways: voltage amplitude, current amplitude, or normalized amplitude. It is represented, in each situation, by a complex phaser whose length is proportional to the size of the wave and whose phase angle tells us the relative phase with respect to the origin or zero of the time variable. That's why we use complex arithmetic and algebra: to express both amplitude and phase angel information with a single symbol.

On a lossless transmission line the waves propagate along the line without change of amplitude and that's why $|\Gamma|$ doesn't depend on the position along the line. The phase angle of the wave complex amplitude varies as we move along the transmission line (for the positive traveling waves the phase decreases with increasing distance from the generator, whereas for the negative traveling waves the phase advances with increasing distance from the generator) (see[5]).

We can use Smith charts to determine input impedances and how values of complex impedance affect the complex reflection coefficient, for example what complex reflection coefficient would result from connecting a particular load impedance to a system having a given characteristic. The Smith chart is more than a chart it is a graph method needed to describe the characteristics of microwave and it represents a special bidimensional graph for the coexistence of complex impedance and complex reflection coefficient information. It is an invaluable aid for the design of impedance-matching networks.

We can also treat the problems of impedance-matching networks finding analytical solutions as we proceed in this paper.

2. Using stubs for impedance-matching transmission lines

Matching the impedance of a network to the impedance of a transmission line has two principal advantages. First all the incident power is delivered to the network. Second the generator which is usually designed to work into an impedance close to common transmission line impedances would pull the generator frequency, and also the line that connects the generator to the load would not be resonant.

One can match the impedance of a load to the impedance of a transmission line using for example stubs (see [2, 5]). These are shorted or open circuit lengths of transmission line intended to produce a pure reactance at the attachment point, for the line frequency of interest. Any value of reactance can be made, as the stub length is varied from zero to half a wavelength. It is important to notice that if we add a half wavelength to the stub length the reactance of the stub comes back to the same value, so that's why in practice the stubs are made in the range 0 to 0.5 wavelengths long.

Because it is difficult to make a perfect nonradiating open circuit s there are always some effects on the line, there are more often used the short circuit stubs as they have less radiation from the ends.

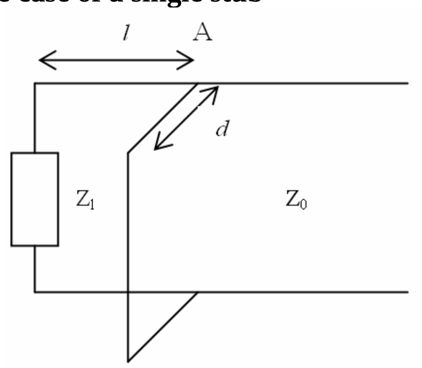
A stub may be placed in series with one of the transmission line conductors but when it is difficult to do this the stub and the transmission line are connected in parallel. There are not big differences between the two cases, but we consider in this paper the case of short stubs connected in parallel with the transmission line and we follow the ideas given in [6].

There can be used one, two, three or more such stubs.

The single stub tuner is perhaps the most widely used matching circuit. It can match any load, but if the load impedance changes, to adjust a single stub tuner is very difficult, because the stub must be removed, the line must be fixed at the break and a new position and a new length have to be determined.

Two stubs attached to the line at fixed points of attachment may be tuned by altering their lengths but, as we would see from the following paragraphs, solution does not exist for all situations (load impedances). That's why usually there are used three stubs which are generally spaced to unequal intervals. Triple stubs can match any load.

2.1. The case of a single stub



For this case the Smith chart can be used for example to find the stub length, but also an analytical solution may be found.

We have the relation:

$$Y_A = Y_l + Y_{st}\ , \tag{1}$$

where Y_A = $1/Z_A$ is the admittance at the stub location A, Y_l the wave admittance at distance l from the load and Y_{st} the admittance of the stub (a short-circuited stub).

We have the following expressions:

$$Y_{st} = -iY_0 ctg\beta d\ , \tag{2}$$

$$Y_l = Y_0 \frac{1-\Gamma_l}{1+\Gamma_l}, \quad \Gamma_l = |\Gamma_l| e^{-2i\beta l}, \tag{3}$$

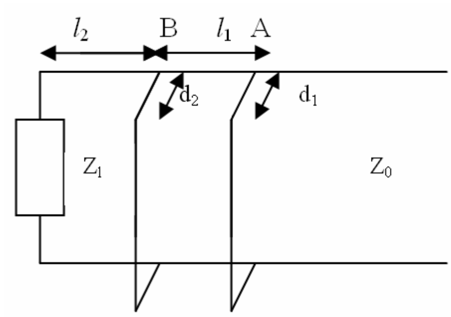
where β represents the wave number.

The matching condition is that $Y_A = Y_0$, and after some calculus (equating real and imaginary parts of the resulting equation) the solution is obtained. It consists of two values: the values for d – the stub length- and l – the distance from the load for the stub position.

As we see from the above paragraph each load to be matched asks for its proper value for l and so arises the biggest inconvenient of this matching circuit if different load impedances have to be matched.

An alternative method uses two stubs but it has the inconvenient that it doesn't mach any load, as we would see.

2.2. The case of two stubs



We consider that the two short stubs are connected in parallel at points A and B, A situated at distance l_1 from B, and B at distance l_2 from the load we have, A and B having fix positions, as in the above figure.

We have:

$$y_A = y_{l1} + y_{st_1}\ , \tag{4}$$

and similarly

$$y_B = y_{l2} + y_{st_2}\ , \tag{5}$$

where $y_A = Y_A/Y_0$ and $y_B = Y_B/Y_0$ are the normalized admittances at the connection points, $y_{st_i} = -ictg\beta d_i$, i = 1, 2, the normalized stubs admittances, d_i being the corresponding stub length,

$$y_{l1} = \frac{y_B + itg\beta l_1}{1 + iy_B tg\beta l_1}, \quad y_{l2} = \frac{y_l + itg\beta l_2}{1 + iy_l tg\beta l_2} \tag{6}$$

$y_l = g_l + ib_l$, g_l the normalized conductance of the load and b_l the normalized susceptance of the load.

The matching condition is

$$y_A = 1\ . \tag{7}$$

As we can see if the normalized admittance of the load is known we can evaluate y_{l2}.

Denoting by

$g_B = \mathrm{Re}(y_{l_2})$ and by $b_B = \mathrm{Im}(y_{l_2})$

we can write that $y_B = g_B + ib_B - ictg\beta d_2$, and further

$$y_B = g_B + i(b_B - ctg\beta d_2)\ . \tag{8}$$

From the matching condition we deduce two equations for real numbers and after solving them we obtain the distances d_1 and d_2. These equations are:

$$ctg\beta d_2 = b_B - b\,, \tag{9}$$

$$ctg\beta d_1 = \frac{1 - btg\beta l_1 - g_B}{g_B tg\beta l_1}\,, \tag{10}$$

where $b = ctg\beta l_1 \pm \sqrt{g_B(g_1 - g_B)}$, and

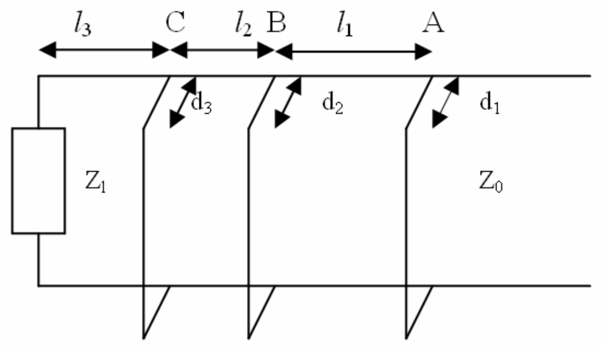
$$g_1 = 1 + ctg^2\beta l_1 = \frac{1}{\sin^2\beta l_1}\,. \tag{11}$$

There are obtained solutions only when the existence condition is satisfied, in fact when

$$g_1 \geq g_B\,, \tag{12}$$

and there are more than one solution for these situations.

2.3. The case of three stubs



For the case of three short stubs situated at distances l_1, l_2, l_3 as in figure bellow, the third stub is used for ensure the condition:

$$g_1 \geq g_B\,,$$

where g_B represents the conductance in point B.

In this case the total admittance in B consists of: the total admittance in C, that one that corresponds to line of length l_2, and of course that from the second stub:

$$y_B = \frac{y_C + itg(\beta l_2)}{1 + iy_C tg(\beta l_2)} - ictg\beta d_2\,. \tag{13}$$

For the point C we have:

$$y_C = \frac{y_l + itg(\beta l_3)}{1 + iy_l tg(\beta l_3)} - ictg\beta d_3 = g_C + ib'\,, \tag{14}$$

where,

$$b' = b_C - ctg\beta d_3\,,$$

$$g_C = \mathrm{Re}\left(\frac{y_l + itg(\beta l_3)}{1 + iy_l tg(\beta l_3)}\right), \quad b_C = \mathrm{Im}\left(\frac{y_l + itg(\beta l_3)}{1 + iy_l tg(\beta l_3)}\right). \tag{15}$$

Because the normalized admittance of the load is known, and also the parameters β and l_3, we can use a computer to evaluate g_C and b_C.

The corresponding conductance to use in formula (12) is (the first two stubs see the load of conductance):

$$g_B = \frac{g_C\left(1 + tg^2(\beta l_2)\right)}{\left(b' tg(\beta l_2) - 1\right)^2 + g_C^2 tg^2(\beta l_2)}\,. \tag{16}$$

This is used to find d_3.

So d_3 is adjusted to ensure the condition: $g_B \leq g_1$.

In order to manipulate this condition more easily we introduce a new parameter, noted f, which satisfies the condition $f \leq 1$, such as: $g_B = fg_1$

Replacing this expression in (16) we obtain:

$$fg_1 = \frac{g_C\left(1 + tg^2(\beta l_2)\right)}{\left(b' tg(\beta l_2) - 1\right)^2 + g_C^2 tg^2(\beta l_2)}\,, \tag{17}$$

and denoting by $g_2 = \frac{1}{\sin^2(\beta l_2)}$ further we get the equation:

$$\left(b' - ctg(\beta l_2)\right)^2 = \frac{g_2 g_C}{fg_1} - g_C^2\,. \tag{18}$$

We deduce from the above equation the two solutions for b':

$$b' = ctg(\beta l_2) \pm \sqrt{\frac{g_2 g_C}{g_1 f}\left(1 - \frac{f}{F}\right)}\,, \tag{19}$$

where

$$F = \frac{g_2}{g_1 g_C}\,. \tag{20}$$

Because sometimes one can choose a smallness value for f such as $f > F$ we shall consider in relation (19) instead of f another parameter, noted r, given by the following condition:

$$r = \begin{cases} f & if\ f \leq F \\ F & otherwise \end{cases}. \tag{21}$$

We get:

$$b' = ctg(\beta l_2) \pm \sqrt{\frac{g_2 g_C}{g_1 r}\left(1 - \frac{r}{F}\right)} \tag{22}$$

the two solutions for b' (b_1', b_2'), and further the two solutions for d_3, given by the equation:

$$ctg\beta d_3 = b_C - b' \tag{23}$$

$$d_{31} = arctg\left(\frac{1}{b_C - b_1'}\right)$$

and

$$d_{32} = arctg\left(\frac{1}{b_C - b_2'}\right) \tag{24}$$

In order to bring these solutions within the minimum interval [0, λ/2] we reduce them modulo λ/2.

These two solutions replaced in relation (14) allow us to know two complex values for the admittances in point C: y_{C1}, y_{C2}.

For the first value we obtain:

$$y_{C1} = g_{C1} + ib_{C1} - ictg\beta d_{31} = g_{C1} + ib_1', \qquad (25)$$

$$y_{B1} = \frac{y_{C1} + itg(\beta l_2)}{1 + iy_{C1}tg(\beta l_2)} - ictg\beta d_2, \qquad (26)$$

$$y_{B1} = g_{B1} + ib_{B1} - ictg\beta d_2, \qquad (27)$$

$$g_{B1} = \frac{g_{C1}\left(1 + tg^2(\beta l_2)\right)}{\left(b_1' tg(\beta l_2) - 1\right)^2 + g_{C1}^2 tg^2(\beta l_2)} = \mathrm{Re}(y_{B1}) \qquad (28)$$

The lengths of the first and the second stub can be obtained from the equations:

$$ctg\beta d_{21} = b_{B1} - b_1, \qquad (29)$$

$$ctg\beta d_{11} = \frac{1 - b_1 tg\beta l_1 - g_{B1}}{g_{B1} tg\beta l_1}, \qquad (30)$$

where

$$b_1 = ctg\beta l_1 \pm \sqrt{g_{B1}(g_1 - g_{B1})}. \qquad (31)$$

As we can see b_1 has in this case two values, b_1^1 and b_1^2 so we deduce two values for the length of stub number two and two values for the length of the first stub.

We get:

$$d_{21}^1 = \frac{arctg\left(\frac{1}{b_{B1} - b_1^1}\right)}{\beta}, \quad d_{21}^2 = \frac{arctg\left(\frac{1}{b_{B1} - b_1^2}\right)}{\beta}, \qquad (32)$$

for the length of the second stub and

$$d_{11}^1 = \frac{arctg\left(\frac{g_{B1} tg\beta l_1}{1 - g_{B1} - b_1^1 tg\beta l_1}\right)}{\beta}, \quad d_{11}^2 = \frac{arctg\left(\frac{g_{B1} tg\beta l_1}{1 - g_{B1} - b_1^2 tg\beta l_1}\right)}{\beta} \qquad (33)$$

for the first stub.

After their evaluation we reduce them modulo λ/2, from the same reasons as in case of d_3.

For the second value of d_3, d_{32}, doing as before, we deduce two new values for the length of stub number two and two values for the length of the first stub.

So the problem has for solutions.

3. Computer code and numerical results

The analytical solution from the above paragraph, for the case of three short stubs, can be the source for a computer code as the expressions found for the stubs lengths can be fast and easily evaluated using a computer. We use MATHCAD to make such a computer code and with it we have obtained some numerical results. The computer code needs as input dates:

- the impedance of the transmission line: Z_0;
- the load impedance Z_l;
- the distances l_1, l_2, l_3 between the stubs, and between the last stub and the load;
- the smallness parameter $f < 1$;
- the wave length λ to deduce the wave number (or directly the wave number) β (β = 2π/λ).

The main steps of the computer code are:

1. The evaluation of the normalized admittance and of the real numbers given by relation (15);
2. The evaluation of g_1, g_2, F, r;
3. Getting the two solutions for b', with (22);
4. For each of the solutions obtain at the step before we get a length for stub three;
5. Reducing these lengths modulo λ/2;
6. The evaluation of y_{C_1}, y_{C_2}, g_{B_1}, g_{B_2}, b_{B_1}, b_{B_2};
7. Getting the solutions for b_1, first two solutions for the case of g_{B_1}, and then other two for g_{B_2}, using (31);
8. The evaluation of the lengths for the second and the first stub, using (32), and (33).

As outputs we get the stubs lengths (all the forth solutions): two values for the lengths of the third stub, and for each of them two solutions for the other lengths.

We run this computer code for different input dates and we present in the following paragraph the following results obtained.

For the first example we consider that the input dates are:

$Z_0 = 2 + 4i$
$Z_l = 3 + 2i$
$l_1 = 0.583$
$l_2 = 0.583$
$l_3 = 3.5$

$f = 0.9$
$\lambda = 0.35$

We get the following solutions for the stubs lengths:

$d_{31} = 0.1424$
$d_{32} = 0.1106$
$d_{111} = 0.1288$
$d_{211} = 0.1536$
$d_{221} = 0.1449$
$d_{121} = 0.1008$
$d_{212} = 0.119$
$d_{112} = 0.1288$
$d_{222} = 0.0784$
$d_{122} = 0.1008$

For the second example the input dates are:

$Z_0 = 5 + 3i$
$Z_l = 7 + 2i$
$l_1 = 0.075$
$l_2 = 0.1$
$l_3 = 0.2$
$f = 0.9$
$\lambda = 0.6$

For this case the solutions are:

$d_{31} = 0.2058$
$d_{32} = 0.1842$
$d_{111} = 0.2388$
$d_{211} = 0.264$
$d_{221} = 0.2388$
$d_{121} = 0.2064$
$d_{212} = 0.252$
$d_{112} = 0.2388$
$d_{222} = 0.2022$
$d_{122} = 0.2004$

Observation: The distances l_1, l_2, l_3 can be given in terms of number of wavelength, so they may be represented by dimensionless numbers (we can give instead of l_1, l_2, l_3, the numbers n_1, n_2, n_3), using the relations: $l_\mathrm{i} = n_i\lambda$, i = 1, 2, 3. In this case the stubs lengths are also dimensionless numbers, given in terms of wavelength numbers too. For this case the reduction must be made by modulo 0.5, and we can run the program without respect to λ, for all cases.

References

1. Constantin, P., Birca-Galateanu, etc. *Industrial electronics.* “Didactică şi Pedagogică” Publishing House, Bucharest, 1983 (in Romanian).
2. Lojewski, G: *Microwave devises and circuits.* “Tehnică” Publishing House, Bucharest, 2005, ISBN 973-31-2263 (in Romanian).
3. Sandu, D.D: *Electronics devices and circuits.* “Didactică şi Pedagogică” Publishing House, Bucharest, 1975 (in Romanian).
4. Dumitrescu, I., etc. *Electronics and electrical machines.* “Didactică şi Pedagogică” Publishing House, Bucharest, 1983 (in Romanian).
5. Saad, TH. (editor): *Microwave Engineer’s Handbook,* Artech House, New York, ISBN 978-0890060032 1971.
6. Orfanidis, S.J.: *Electromagnetic Waves and antennas,* Rutgers University, 2004. Available: http://www.ece.rutgers.edu/~orfanidi/ewa/