International Conference on Economic Engineering and Manufacturing Systems
Braşov, 25 – 26 October 2007

ON THE DISTRIBUTED CONTROL FOR THE COIL FUNCTION OF A TENTACLE ROBOT

Mircea IVĂNESCU, Mihaela Cecilia FLORESCU
University of Craiova, Romania

Abstract: The paper focuses on the control problem of a hyperredundant robot that performs the coil function of the grasping. First, the dynamic model of a tentacle arm with continuum elements produced by flexible composite materials in conjunction with active-controllable electro-rheological fluids is analysed. Secondly, both problems, i.e. the position control and the force control are approached. The difficulties determined by the complexity of the non-linear integral-differential equations are avoided by using a very basic energy relationship of this system. Energy-based control laws are introduced for the position control problem. A force control method is proposed, namely the DSMC method in which the evolution of the system on the switching line by the ER fluid viscosity is controlled. Numerical simulations are also presented.

Keywords: distributed parameter systems, grasping, tentacle robots, and force control

1. Introduction

The dynamic models of the tentacle manipulators are very complex. [10] proposes a dynamic model for hyper-redundant structures as an infinite degree-of-freedom continuum model and some computed torque control systems are introduced and a sequential distributed control is suggested for a tentacle manipulator actuated by electro-rheological fluids.

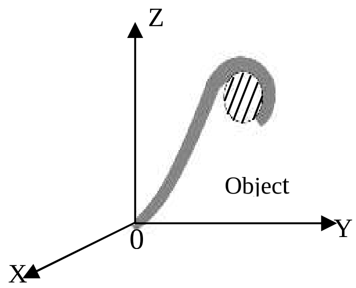

Figure 1. Tentacle arm

In this paper, the problem of a class of hyperredundant arms with continuum elements that performs the grasping function by coiling is discussed. This function is often met in the animal world as in the elephant’s trunk, the octopus tentacle or the constrictor snakes. First, the dynamic model of the system is inferred. Energy-based control laws are introduced for the position control problem. A force control method is proposed, namely the DSMC method, implying the evolution of the system on the switching line by ER fluid viscosity control.

2. Background

2.1. The technological model

The paper presents a class of tentacle arms that can achieve any position and orientation in 3D space, and that can perform a coil function for the grasping (figure 1). The arm has a high degree of freedom structure or a continuum structure. Technologically, these arms are based on the use of flexible composite materials in conjunction with active controllable electro-rheological (ER) fluids.

The general form of the arm is shown in figure 2. It consists of a number (N) of elements, cylinders made of fiber-reinforced rubber. There are four internal chambers in the cylinder, each of them containing the ER fluid with an individual control circuit. The last m elements ($m < N$) represent the grasping terminal. These elements contain a number of force sensors distributed on the surface of the cylinders. The sensor network is constituted by a number of impedance devices [6] that define the dynamic relationship between the grasping element displacement and the contact force.

2.2. The Theoretical Model

The core of the tentacle model is a 3-dimensional backbone curve C that is parametrically described by a vector $r(s) \in R^3$ and an associated frame $\phi(s) \in R^{3\times 3}$ whose columns create the frame bases (figure 4).

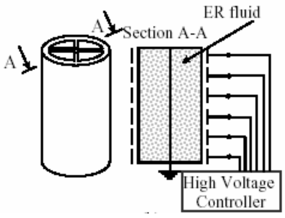

Figure 2. The arm form

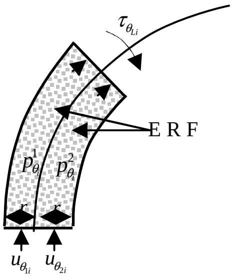

Figure3. The arm element

The independent parameter *s* is related to the arc-length at the origin of the curve *C*, $s \in [0, L]$, where: $L = \sum_{i=1}^{N} l_i$, where l_i represent the length of the elements *i* of the arm in the initial position. We used a parameterization of the curve *C* based upon two "continuous angles" $\theta(s)$ and $q(s)$ (figure 4). At each point $\bar{r}(s,t)$, the robot's orientation is given by a right-handed orthonormal basis vector $\{\bar{e}_x,\ \bar{e}_y,\ \bar{e}_z\}$ and its origin coincides with point $\bar{r} = \bar{r}(s,t)$, where the vector e_x is tangent and vector e_z is orthogonal to the curve C.

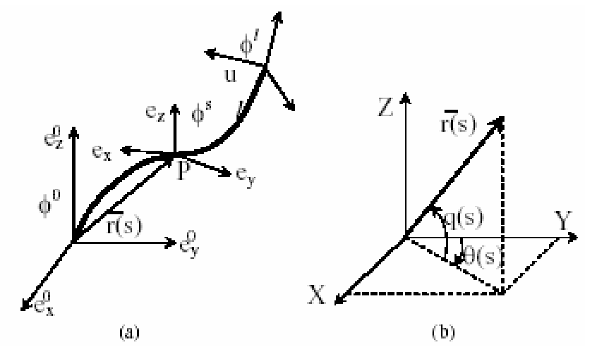

Figure 4. The 3-dimensional backbone model

The position vector on curve *C* is given by

$$\bar{r}(s,t) = [x(s,t) \quad y(s,t) \quad z(s,t)]^T \tag{1}$$

where $x(s,t) = \int_0^S \sin\theta(s',t)\cos q(s',t)ds'$,

$$y(s,t) = \int_0^S \cos\theta(s',t)\cos q(s',t)ds',$$

$$z(s,t) = \int_0^S \sin q(s',t)ds', \text{ with } s' \in [0, \quad s].$$

For an element *dm*, the kinetic and gravitational potential energy will be:

$$dT = \frac{1}{2}dm\left(v_x^2 + v_y^2 + v_z^2 + v_u^2\right), \qquad dV = dm \cdot g \cdot z,$$

where $dm = \rho ds$.

We shall consider $F_\theta(s,t)$, $F_q(s,t)$ the distributed forces on the length of the arm that determine motion and orientation in the θ- and *q*-plane. From [9], the mechanical work is:

$$L = \int_0^l \int_0^t \left(F_\theta(s,\tau)\dot{\theta}(s,\tau) + F_q(s,\tau)\dot{q}(s,\tau)\right)d\tau ds \tag{2}$$

where $\dot{\theta}$, $\dot{q}$ denote: $\dot{\theta}(s,t) = \frac{\partial\theta}{\partial t}(s,t)$, $\dot{q}(s,t) = \frac{\partial q}{\partial t}(s,t)$.

3. Dynamic Model

In this paper, the manipulator model is considered a distributed parameter system defined on a variable spatial domain Ω = [0, *L*] and the spatial coordinate *s*.

The distributed parameter model is

$$\begin{aligned}
&\rho\int_0^S\int_0^S \big(\ddot{q}'(\sin q'\sin q''\cos(q'-q'') + \cos q'\cos q'') - \\
&-\ddot{\theta}'\cos q'\sin q''\sin(\theta''-\theta') + \\
&+(\dot{q}')^2(\cos q'\sin q''\cos(\theta'-\theta'') - \sin q'\cos q'') + \\
&+(\dot{\theta}')^2\cos q'\sin q''\cos(\theta'-\theta'') - \\
&-\dot{q}'\dot{q}''\sin(q''-q')\big)ds'ds'' + \rho g\int_0^S \cos q' ds' = F_q
\end{aligned} \tag{3}$$

$$\begin{aligned}
&\rho\int_0^S\int_0^S \big(\ddot{q}'\sin q'\cos q''\sin(\theta''-\theta') + \ddot{\theta}'\cos q'\cos q''\cos(\theta''-\theta') - \\
&-(\dot{q}')^2\cos q'\cos q''\sin(\theta''-\theta') + (\dot{\theta}')^2\cos q'\cos q''\sin(\theta''-\theta') - \\
&-\dot{\theta}'\dot{q}'\sin q'\cos q''\cos(\theta''-\theta')\big)ds'ds'' = F_\theta
\end{aligned} \tag{4}$$

where we used the notations: $\dot{q}' = \partial q(s',t)/\partial t$, $\ddot{q}' = \partial^2 q(s',t)/\partial t^2$, $F_q = F_q(s,t)$, $s \in [0, L]$, $s' \in [0, s]$.

The state of this system at any fixed time t is specified by the set ($\omega(t,s)$, $v(t,s)$), where $\omega = [\theta\ q]^T$ represents the generalized coordinates and *v* defines the momentum densities. The set of all functions $s \in \Omega$ that ω, *v* can take on at any time is the state function space Γ(Ω). We shall assume that $\Gamma(\Omega) \subset L_2(\Omega)$.

The control forces have the distributed components along the arm, $F_\theta(s,t)$, $F_q(s,t)$, $s \in [0, L]$, that are determined by the lumped torques,

$$F_\theta(s,t) = \sum_{i=1}^{N}\delta(s - il)\tau_{\theta i}(t) \tag{5}$$

$$F_q(s,t)=\sum_{i=1}^{N}\delta(s-il)\tau_{qi}(t) \quad (6)$$

where δ is Kronecker delta, $l_1 = l_2 = \ldots = l_N = l$, and

$$\tau_{\theta i}(t)=\left(p_{\theta i}^1-p_{\theta i}^2\right)S\cdot d/8 \quad (7)$$

$$\tau_{qi}(t)=\left(p_{qi}^1-p_{qi}^2\right)S\cdot d/8,\ i=1,2,\ldots,N \quad (8)$$

In (7), (8), $p_{\theta i}^1$, $p_{\theta i}^2$, p_{qi}^1, p_{qi}^2 represent the fluid pressure in the two chamber pairs, θ, q and S, d are section area and the diameter of the cylinder, respectively (figure 3).

The pressure control of the chambers is described by the following equations, according to [5]

$$a_{ki}(\theta)\frac{dp_{\theta i}^k}{dt}=u_{\theta ki} \quad (9)$$

$$b_{ki}(q)\frac{dp_{qi}^k}{dt}=u_{qki};\ k=1,2;\ i=1,2,\ldots,N \quad (10)$$

where a_{ki}, b_{ki} are the coefficients determined by the fluid parameters and the geometry of the chambers and $a_{ki}(0)>0$, $b_{ki}(0)>0$, where $k=1,2$; $i=1,2,\ldots,N$; $\theta,q\in\Gamma(\Omega)$.

4. Control Problem

The control problem of the grasping function by coiling is constituted from two problems: the position control of the arm around the object-load and the force control of grasping.

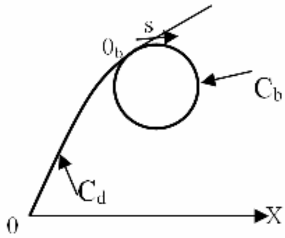

Figure 5. The coiling function

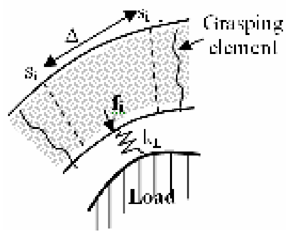

Figure 6. The contact between load and arm element

4.1. Position control

We consider that the initial state of the system is given by $\omega_0=\omega(0,s)=[\theta_0,\quad q_0]^T$, $\nu_0=\nu(0,s)=[0,\quad 0]^T$, where $\theta_0=\theta(0,s)$, $q_0=q(0,s)$, $s\in[0, L]$ corresponding to the initial position of the arm defined by the curve C_0:

$$C_0:(\theta_0(s),\quad q_0(s));\ s\in[0,\quad L] \quad (11)$$

The desired point in $\Gamma(\Omega)$ is represented by a desired position of the arm, the curve C_d that coils the load:

$$C_d:(\theta_d(s),\quad q_d(s));\ s\in[0,\quad L] \quad (12)$$

In a grasping function by coiling, only the last m elements ($m < N$) are used. Let l_g be the active grasping length $l_g=\sum_{i=m}^{n} l_i$. Let C_b be the curve defines the boundary of the load and we denote by O_b the origin of the coiling function, where O_b is the intersection between the tangent from origin O and the curve C_L (figure 5). This curve can be expressed using the coordinates $(\theta,\quad q)\in\Gamma(\Omega)$.

$$C_b:\left(\theta_b\left(s^*\right),\quad q_b\left(s^*\right)\right);\ s^*\in[0,\quad L_b] \quad (13)$$

where L_b is the length of the coiling measured on the boundary C_b and $s=L-l_g+s^*$. We define the position error by $e_p(t)$

$$e_p(t)= \\ =\int_{L-l_g}^{L}\left(\left(\theta(s,t)-\theta_b(s)\right)+\left(q(s,t)-q_b(s)\right)\right)ds \quad (14)$$

It is difficult to measure practically the angles θ, q for all $s\in[0, L]$. These angles can be evaluated or measured at the terminal point of each element. In this case, the relation (14) becomes

$$e_p(t)=\sum_{i=m}^{N}\left(\left(\theta_i(t)-\theta_{bi}\right)+\left(q_i(t)-q_{bi}\right)\right) \quad (15)$$

The error can also be expressed with respect to the global desired position C_d

$$e_p(t)=\sum_{i=1}^{N}\left(e_{\theta i}(t)+e_{qi}(t)\right) \quad (16)$$

The position control of the arm means the motion control from the initial position C_0 to the desired position C_b in order to minimize the error.

Theorem 1. The closed-loop control system of the position (3)-(10) is stable if the fluid pressure control law in the chambers of the elements given by:

$$u_{\theta ji}(t)=-a_{ji}(\theta)\left(k_{\theta i}^{j1}\dot{e}_{\theta i}(t)+k_{\theta i}^{j2}\ddot{e}_{\theta i}(t)\right) \quad (17)$$

$$u_{qji}(t)=-b_{ji}(\theta)\left(k_{qi}^{j1}\dot{e}_{qi}(t)+k_{qi}^{j2}\ddot{e}_{qi}(t)\right) \quad (18)$$

where $j = 1,2$; $i = 1,2,\ldots,N$, with initial conditions:

$$p_{\theta i}^1(0)-p_{\theta i}^2(0)=\left(k_{\theta i}^{11}-k_{\theta i}^{21}\right)e_{\theta i}(0) \quad (19)$$

$$p_{qi}^1(0)-p_{qi}^2(0)=\left(k_{qi}^{11}-k_{qi}^{21}\right)e_{qi}(0) \quad (20)$$

$$\dot{e}_{\theta i}(0)=0,\ \dot{e}_{qi}(0)=0,\ i=1,2,\ldots,N \quad (21)$$

and the coefficients $k_{\theta i}$, k_{qi}, $k_{\theta i}^{mn}$, k_{qi}^{mn} are positive and verify the conditions

$$k_{\theta i}=\frac{Sd}{8}\left(k_{\theta i}^{11}-k_{\theta i}^{21}\right),\quad k_{qi}=\frac{Sd}{8}\left(k_{qi}^{11}-k_{qi}^{21}\right),\qquad (22)$$

$$k_{\theta i}^{11}>k_{\theta i}^{21};\ k_{\theta i}^{12}>k_{\theta i}^{22};\ k_{qi}^{11}>k_{qi}^{21};\ k_{qi}^{12}>k_{qi}^{22},\qquad (23)$$

4.2. The Force control

The contact between an element and the load is presented in figure 6. It is assumed that the grasping is determined by the chambers in the θ-plane.

The relation between the fluid pressure and the grasping forces can be inferred for a steady state,

$$\int_0^l k\frac{\partial^2\theta(s)}{\partial s^2}ds+\int_0^l f(s)\tilde{T}\tilde{\theta}(s)\int_0^s \tilde{T}^T\tilde{\theta}(s)ds= =(p_1-p_2)S\frac{d}{8}\qquad (24)$$

where

$$\tilde{T}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix};\ \tilde{\theta}(s)=\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}\qquad (25)$$

and $f(s)$ is the orthogonal force on the curve C_b, $f(s)$ is $F_\theta(s)$ in θ-plane and $F_q(s)$ in q-plane, respectively.

A spatial discretization $s_1, s_2, \ldots, s_{l1}$ is introduced and $\Delta=s_{i+1}-s_i$, $\theta_i=\theta(s_i)$, $i=1,2,\ldots,l_1$. For small variation $\Delta\theta_i$ around the desired position θ_{id}, in the θ-plane, the dynamic model (3) can be approximated by the following discrete model [7],

$$m_i\Delta\ddot{\theta}_i+c_i\Delta\dot{\theta}_i+H_i(\theta_{id}+\Delta\theta_i,\ \theta_{id},\ q_d)- -H(\theta_{id},\ q_d)=d_i(f_i-F_{ei})\qquad (26)$$

where $m_i=\rho S\Delta$, $i=1,2,\ldots,l_1$, $H(\theta_{id},\ q_d)$ is a nonlinear function defined in the desired position $(\theta_{id},\ q_d)$, $c_i=c_i(\nu,\ \theta_i,\ q_d)$, $c_i>0$, $\theta, q\in\Gamma(\Omega)$ and ν is the viscosity of the fluid in the chambers.

$$H_i(\theta_{id}+\Delta\theta_i,\ \theta_{id},\ q_d)-H(\theta_{id},\ q_d)\cong \cong\left.\frac{\partial H_i}{\partial\theta}\right|_{\substack{\theta=\theta_d\\ q=q_d}}\Delta\theta_i=h_i(\theta_{id},\ q_d)\cdot\Delta\theta_i\qquad (27)$$

and F_{ei} is the external force due to the environment.

The equation (26) becomes,

$$m_i\Delta\ddot{\theta}_i+c_i(\nu,\ \theta_i,\ q_d)\Delta\dot{\theta}_i+ +h_i(\theta_{id},\ q_d)\cdot\Delta\theta_i=d_i(f_i-F_{ei})\qquad (28)$$

The aim of the explicit force control is to exert a desired force F_{id}. If the contact with the load is modeled as a linear spring with constant stiffness k_L, the environment force can be approximated as:

$$F_{ei}=k_{Li}\Delta w_i\approx k_{Li}\Delta_i\sin\Delta q_i\approx k_L\Delta q_i\qquad (29)$$

The error of the force control may be introduced in the form of

$$e_{fi}=F_{ie}-F_{id}\qquad (30)$$

It may be easily shown that the equation (28) becomes

$$\frac{m_i}{k_L}\ddot{e}_{fi}+\frac{c_i}{k_L}\dot{e}_{fi}+\left(\frac{h_i}{k}+d_i\right)e_{fi}=d_i f_i-\left(\frac{h_i}{k}+d_i\right)F_{id}\qquad (31)$$

Theorem 2. The closed force control system is asymptotically stable if the control law is

$$f_i=\frac{1}{k_L d_i}\begin{pmatrix}\left(h_i+k_L d_i+m_i\sigma^2\right)e_{fi}-\\ -(h_i-k_L d_i)F_{id}\end{pmatrix}\qquad (32)$$

$$c_i>m_i\sigma\qquad (33)$$

Proposition. The DSMC control is ensured if the coefficients c_i of the control system verify the conditions:

$$c_i^2>4m_i(h_i+d_i k_L)\qquad (34)$$

The force control system is developed into two steps. In the first step, according to Theorem 2, the trajectory of the error is controlled by the force f_i. In the second step, the viscosity of the fluid is increased and the trajectory switches directly toward the origin on the switching line (figure 7).

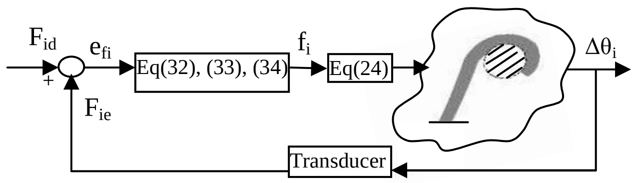

Figure 7. The block scheme of the control system

5. Simulation

A tentacle manipulator (figure 8) with eight elements is considered. The control problem in the θ-plane will be analyzed. The initial position is the defined by $C_0:\left(\theta_0(s)=\frac{\pi}{2}\right)$ and the grasping function is performed for a circular load defined by $C_b:\left(x^*-x_0^*\right)^2+\left(y^*-y_0^*\right)^2=r^2$, where $\left(x^*, y^*\right)$ represent the coordinates in θ-plane. A discretization for each element with an increment $\Delta=l/3$ is introduced.

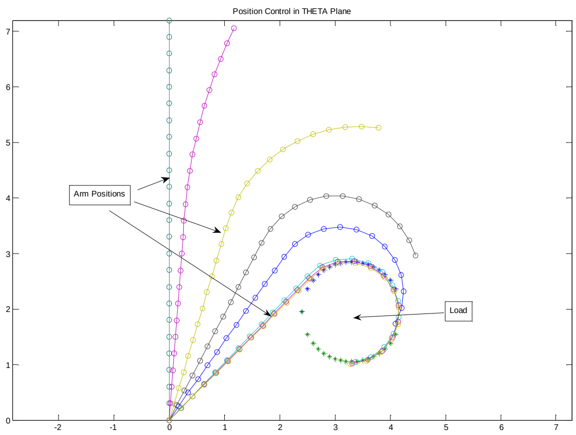

Figure 8. Movement of tentacle manipulator

6. Conclusions

The paper treats the control problem of a tentacle robot with continuum elements that performs the coli function for grasping. The structure of the arm is given by flexible composite materials in conjunction with active-controllable electro-rheological fluids. The dynamic model of the system is inferred by using Lagrange equations developed for infinite dimensional systems.

The grasping problem comprises in two problems: the position control and the force control. The difficulties determined by the complexity of the non-linear integral-differential equations are avoided by using a basic energy relationship of this system and energy-based control laws are introduced for the position control problem. The force control is obtained by using the DSMC method in which the evolution of the system on the switching line is controlled by the ER fluid viscosity. Numerical simulations are also presented.

References

1. Chirikjian, G.S., Burdick, J.W.: *Kinematically optimal hyperredundant manipulator configurations*. IEEE Trans. Robotics and Automation, vol. 11, no. 6, Dec. 1995, p. 794-798, ISSN 1042-296X.
2. Mochiyama, H., Kobayashi, H.: *The shape Jacobian of a manipulator with hyper degrees of freedom*. Proc. 1999 IEEE Int. Conf. on Robotics and Automation, Detroit, May 1999, p. 2837-2842, ISSN 1042-296X.
3. Mochiyama, H., Shimeura, E., Kobayashi, H.: *Direct kinematics of manipulators with hyper degrees of freedom and Serret-Frenet formula*. Proc. 1998 IEEE Int. Conf. on Robotics and Automation, Leuven, Belgium, May 1998, p. 1653-1658, ISSN 1042-296X.