CONSIDERING ON THE IMPORTANCE OF ELEMENTS OF THE SYSTEMS WITH SERIES

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Abstract. The paper presents the quantitative evaluation of the importance of the system's elements with series structure, taking into account two cases of specification of reliability parameters: constant; time dependent. The exemplification of importance of elements is done using charts and tables, considering a system with series connection of 3 elements.

Keywords: system, reliability, series

1. Introduction

The determination of the importance of the elements of a system represents a major request in the process of optimizing and improvement of its reliability. Once the reliability chart is drown up, the problem of determining the „weaker" or less reliable elements arises, so that the necessary or to improve the system's reliability can be taken. The analyst's task is not easy, considering the complexity and the scale of industrial systems. If for the simples systems the identification and quantification of the importance of the elements represents a relatively easy problem, in the case of large scale systems a series of mathematical approximation is required.

Taking into account the fact that the systems function in the "space of success", the importance of the elements refers to their contribution to the working order of the system (not to be confounded for the importance of events in the fault tree, where it is quantificated their importance in generating the system's damage).

The study of the importance of elements varies from one case to another, depending on the way their reliability is specified: constant or time dependent.

For optimizing the importance of the systems' elements, it is necessary to consider their cost, which is not the case with this analysis.

2. The importance of the system's elements with constant reliability parameters

The evaluation of the importance of an element of the system is being done with a general relation, as follows:

$$I_{E_i} = \frac{\partial R_S}{\partial R_i}, \tag{1}$$

where R_S represents the system's reliability;

R_i – the reliability of element "i".

In order to exemplify the evaluation of the importance of elements, we consider a system formed from 3 elements, with constant reliabilities, $R_1 = 0.7$, $R_2 = 0.8$ and $R_3 = 0.9$, in series connected (figure 1).

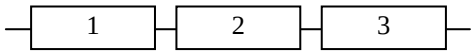

Figure 1. System of 3 elements, in series conected

The system's reliability in this case has the expression [1]:

$$R_S = R_1 R_2 R_2 . \tag{2}$$

For the system in question, the importance of the system's elements has the values [1, 2]:

$$\begin{aligned} I_{E_1} &= \frac{\partial R_S}{\partial R_1} = \frac{\partial (R_1 R_2 R_3)}{\partial R_1} = R_2 R_3 = 0.8 \cdot 0.9 = 0.72, \\ I_{E_2} &= \frac{\partial R_S}{\partial R_2} = \frac{\partial (R_1 R_2 R_3)}{\partial R_2} = R_1 R_3 = 0.7 \cdot 0.9 = 0.63, \\ I_{E_3} &= \frac{\partial R_S}{\partial R_3} = \frac{\partial (R_1 R_2 R_3)}{\partial R_3} = R_1 R_2 = 0.7 \cdot 0.8 = 0.56. \end{aligned} \tag{3}$$

Based on the relations (3), we conclude that the most important element is the „weakest" element of the system, meaning element 1, with the $R_1 = 0.7$, which also results from figure 2.

In order improve this system's reliability, we must act on this element. So, if an increase of an element's reliability, one at a time, is accepted with the difference ΔR_i = 10 %, then through the increase of the reliability of every component

element, one at a time, leads to the increase of the reliability of the system R_S according to table 1, an aspect shown also by the diagrams in figure 3. We can infer that the most significant importance in the improvement of the reliability of the system belongs to element 1, which is also the „weakest".

For element 3, with the reliability of 0.9, we consider an increase of reliability with $\Delta R_3 = 0.09$ and not $\Delta R_3 = 0.1$, in which case the element would be transformed into an ideal element, flawless, with the reliability $R_3 = 1$.

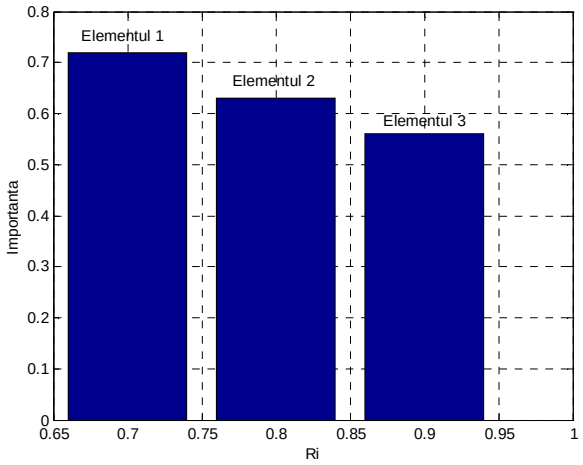

Figure 2. The variation of the importance of elements in the system with 3 elements, in series connected, with constant reliability parameters

Table 1. The dependence between the reliability parameters of the elements and reliability of the system with series structure of 3 elements

Element 1	Element 2	Element 3	Reliability of the system R_S	The increase of system's reliability ΔR_S [%]
0.7	0.8	0.9	0.504	-
0.8	0.8	0.9	0.576	12.55
0.7	0.9	0.9	0.567	11.25
0.7	0.8	0.99	0.554	10.00

3. The importance of the elements of the system with time dependent reliability parameters

For the system with series structure with elements with time dependent reliability, the importance of elements will bee, in turn, time independent and it is determined with a general relation [1, 2]:

$$I_{E_i}(t) = \frac{\partial R_S(t)}{\partial R_i(t)}, \tag{4}$$

where $R_S(t)$ and $R_i(t)$ represent the system's reliability and, respectively, the reliability of element "i", both dependent of the running time t.

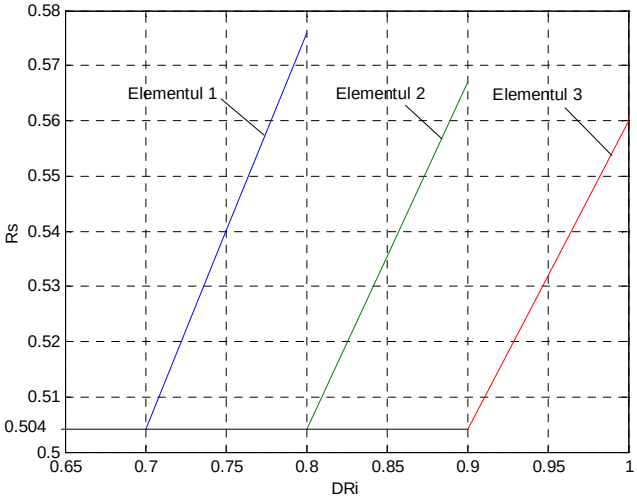

Figure 3. The increases of the reliability of the system with 3 elements, in series connected, through the increase of the reliability of elements

Considering the system of 3 elements in series connected (figure 1), with probability densities of the lifetime that follow a Weibull [3, 4] distribution with shape parameter $\beta = 3$ and scale parameter $\eta_1 = 1000$, $\eta_2 = 1500$ and, respectively $\eta_{13} = 3000$ then the reliabilities of the 3 elements of the system have the expressions [5].

$$R_1(t) = e^{-\left(\frac{t}{\eta_1}\right)^{\beta}}, \quad R_1(t) = e^{-\left(\frac{t}{\eta_2}\right)^{\beta}}, \quad R_1(t) = e^{-\left(\frac{t}{\eta_3}\right)^{\beta}}. \tag{5}$$

To determine the importance of the 3 elements of the system we must consider the system's lifetime. So, for a lifetime of $t = 1000$ hours the important of the 3 elements of the system will have the values from equation (6).

Based on the relations (6), we come to the conclusion that the most important element is element 1, which, after the determination of its reliability according to the relation (4), presents the lowest reliability.

As it is presented above, the importance of the system's elements with time dependent reliabilities varies depending on the system's

lifetime. This aspect is shown in diagrams in figure 4, which were evaluated for the system's mission of a t = 2000 hours.

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\begin{aligned}
I_{E_1}(t) &= \frac{\partial R_S(t)}{\partial R_1(t)} = \frac{\partial(R_1(t)R_2(t)R_3(t))}{\partial R_1(t)} = \\
&= R_2(t)R_3(t) = e^{-\left(\frac{t}{\eta_2}\right)^{\beta}} e^{-\left(\frac{t}{\eta_3}\right)^{\beta}} = \\
&= e^{-\left(\frac{1000}{1500}\right)^{3}} e^{-\left(\frac{1000}{3000}\right)^{3}} = 0.716, \\
I_{E_2}(t) &= \frac{\partial R_S(t)}{\partial R_2(t)} = \frac{\partial(R_1(t)R_2(t)R_3(t))}{\partial R_2(t)} = \\
&= R_1(t)R_3(t) = 0.3545, \\
I_{E_3}(t) &= \frac{\partial R_S(t)}{\partial R_3(t)} = \frac{\partial(R_1(t)R_2(t)R_3(t))}{\partial R_3(t)} = \\
&= R_1(t)R_2(t) = 0.2735.
\end{aligned}
\tag{6}
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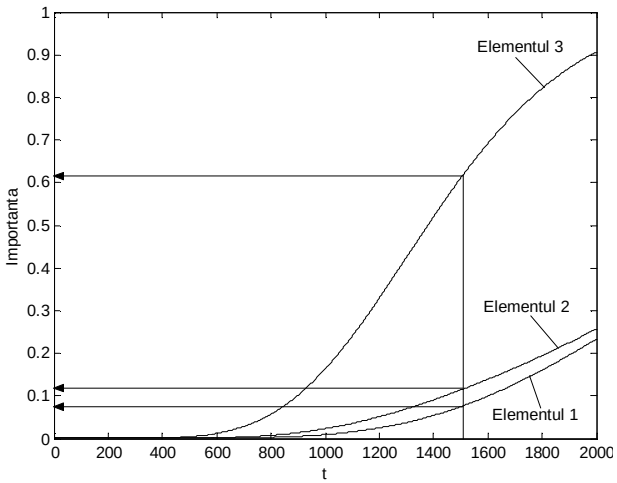

Figure 4. The variation of the importance of the elements of the system, with 3 elements, in series connected, with time dependent reliabilities

4. Conclusions

By identifying and quantification of the importance of the elements of the systems with series structure we conclude that the most important elements in ascertaining the systems operation are the less reliable elements. Onto these elements we must act for the improvement of reliability or to achieve specified reliabilities of the system. The importance of the systems' elements, which takes into account their reliability parameters, must be correlated with the aspects regarding the cost of elements.

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