

MATHEMATICAL METHODS TO DETERMINATE THE UNFOLDING OF THE CYLINDERS

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Abstract. This paper establishes the intersection curves between cylinders, by using the Mathematic 5.1. program. This thing can be obtained by introducing the curves equations, which are inferred, in Mathematic 5.1 program. This paper takes into discussion two cylinders.

Keywords: unfolding, Mathematic 5.1, intersection curve, cylinder, projection vertical

1. Introduction

The calculation of the unfolding has a wide applicability, especially in the connection of the pipes with equal or different diameters. This calculation can be done by using several methods among which: the use of classical methods, the use of analytical methods, the use of mathematical methods with a view to automating their drawing and numerical methods using the “spline”. For illustration, we took two cylinders with the following diameters: C of diameter $D = 48$ mm and C_1 of diameter $D_1 = 30$ mm. We know the angle $\varphi = 45^\circ$, too.

2. The mathematical method of establishing the intersections curves

The projection of the intersection curves necessitates the solving of the following phases:

- the writing of the curves equations resulted from the intersections of the areas that can be unfolded;
- the writing of the transformations equations by the unfolding of the intersection curve.

2.1. The calculation of the intersection curve γ_1 of the cylinder C and γ_2 of the cylinder C_1 .

In accordance with figure 1 we take the cylinder C , of diameter D , and its reference system $Oxyz$ and the cylinder C_1 , of diameter D_1 , and its reference system $O_1x_1y_1z_1$, where $y \equiv y_1$ and $O \equiv O_1$.

The cylinders equations expressed in the chosen reference systems are:

$$x^2 + y^2 = R^2 \quad (1)$$

$$y_1^2 + z_1^2 = R_1^2 \quad (2)$$

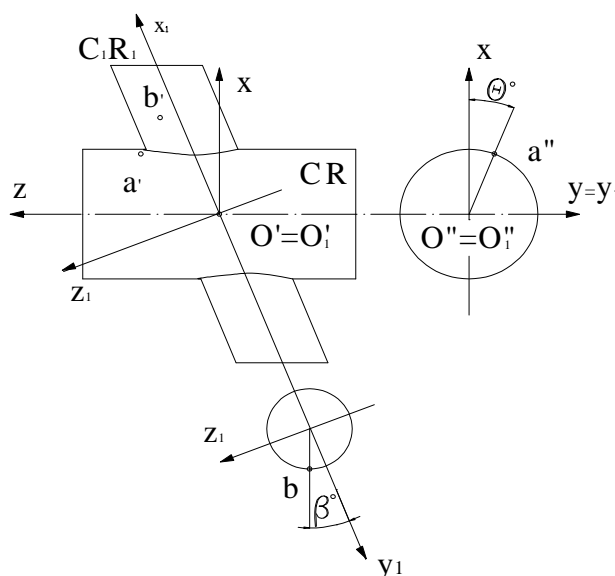


Figure 1. The geometrical elements of the cylinders

The two reference systems are rotated, one given another, by the angle φ . The transformation formula of the coordinates, to passing from the system $Oxyz$ into $O_1x_1y_1z_1$ and viceversa are:

$$x_1 = x \cos \varphi + z \sin \varphi \quad (3)$$

$$z_1 = z \cos \varphi - x \sin \varphi \quad (4)$$

$$x = x_1 \cos \varphi - z_1 \sin \varphi \quad (5)$$

$$z = x_1 \sin \varphi + z_1 \cos \varphi \quad (6)$$

We relate the equations of the both cylinders to system $Oxyz$ and by eliminating the variable y , we obtain the equation of the vertical projection of the intersection:

$$z^2 - 2x \cdot \operatorname{tg} \varphi \cdot z + \frac{R^2 - R_1^2}{\cos^2 \varphi} - x^2 = 0 \quad (7)$$

The equation of the transformation curve γ_1 , border of the cylinder C , is obtained by applying the transformations (8, 9) to the equation (7).

$$x = R \cos \theta = R \cos \frac{x_d}{R} \quad (8)$$

$$z = z_d \quad (9)$$

where x_d and z_d are the coordinates of the point A in unfolding. This point A is indicated by its projections a' and a'' .

In this case the following equation is obtained:

$$z_d^2 - 2Rz_d \cos \frac{x_d}{R} \cdot \operatorname{tg} \varphi + \left[\frac{R^2 - R_1^2}{\cos^2 \varphi} - R^2 \cos^2 \frac{x_d}{R} \right] = 0 \quad (10)$$

Then:

`Plot[{24*Cos[x/24]*Tan[45 Degree] + Sqrt[225-576*(sin[x/24]^2)/Cos[45 Degree], 24*Cos[x/24]*Tan[45Degree] - Sqrt[225- 576* (sin[x/24]^2)/Cos[45 Degree]], {x,-24* Arcsin[15/24],24* Arcsin[15/24]}`

$$z_{d1,2} = R \cos \frac{x_d}{R} \operatorname{tg} \varphi \pm \pm \frac{1}{\cos \varphi} \sqrt{R_1^2 - R^2 \sin^2 \frac{x_d}{R}} \quad (11)$$

$$x_d \in \left[-R \cdot \arcsin \frac{R_1}{R}, R \cdot \arcsin \frac{R_1}{R} \right]$$

We obtain the figure 2, by introducing the relations (11) into Mathematic 5.1 program.

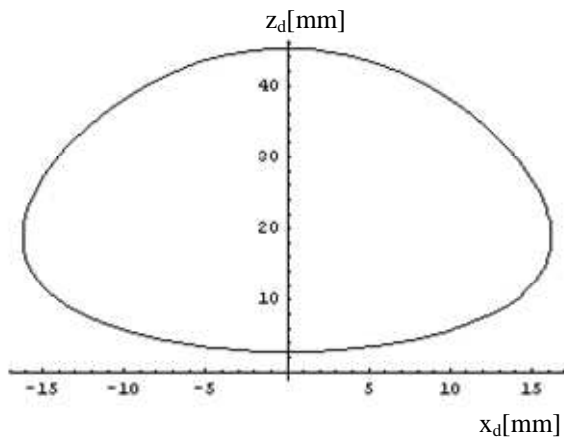


Figure 2. The unfolding of the intersection curve γ_1 of the cylinder C

The equation of the transformation curve γ_2 , border of the cylinder C_1 , is obtained by applying the transformations (12, 13) to the equation (7):

$$x_1 = x_{d1} \quad (12)$$

$$z_1 = R_1 \sin \alpha = R_1 \sin \frac{z_{d1}}{R} \quad (13)$$

where x_{d1} and z_{d1} are the coordinates of the point B(b' , b) in unfolding.

The following equation is obtained:

$$x_{d1}^2 + 2R_1 \sin \frac{z_{d1}}{R_1} x_{d1} - R_1^2 \sin^2 \frac{z_{d1}}{R_1} - \frac{R^2 - R_1^2}{\cos^2 \varphi} = 0 \quad (14)$$

Then:

$$x_{d1} = -R_1 \sin \frac{z_{d1}}{R_1} \pm \pm \frac{1}{\cos \varphi} \sqrt{R^2 - R_1^2 \cos^2 \frac{z_{d1}}{R_1}} \quad (15)$$

$$z_{d1} \in [0, 2\pi R_1] \quad (16)$$

The figure 3 is obtained by introducing the relations (15, 16) into Mathematic 5.1 program.

`Plot[{-15*Sin[z/15] + Sqrt[576-225*(cos[z/15]^2)/Cos[45 Degree], -15*Sin[z/15]-Sqrt[576-225*(Cos[z/15]^2)/Cos[45Degree]], {z,0,2*PI*15}}`

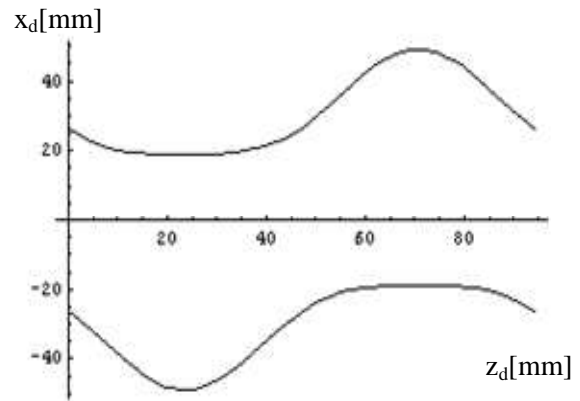


Figure 3. The unfolding of the intersection curve γ_2 of the cylinder C_1

2.2. The calculation of the intersection curve γ_1 of the cylinder C and γ_2 of the cylinder C_1 , using another mathematical method.

The situation vector of any point, which is placed on the cylinder C (figure 4), is:

$$\vec{\rho} = x \vec{i} + y \vec{j} + z \vec{k} \quad (17)$$

where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors of the axes of the choice reference system.

But x, y, z are the coordinates of a point which is situated on the cylinder:

$$\begin{aligned} x &= R \sin \theta \\ y &= R \cos \theta \\ z &= z \end{aligned} \quad (18)$$

where θ is the angle of the vectorial radius with Oy.

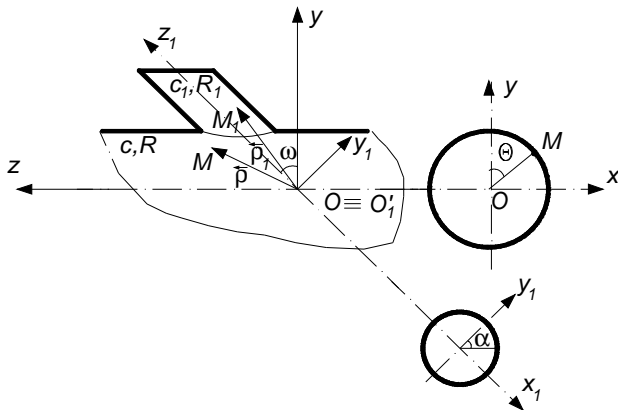


Figure 4. The geometrical elements of the cylinders

Then:

$$\vec{\rho} = R \sin \theta \vec{i} + R \cos \theta \vec{j} + z \vec{k} \quad (19)$$

The situation vector of a point, which is placed on the cylinder C_1 , is:

$$\vec{\rho}_1 = x_1 \vec{i} + y_1 \vec{j}_1 + z_1 \vec{k}_1 \quad (20)$$

where $\vec{i}_1, \vec{j}_1, \vec{k}_1$ are the unit vectors of the rotated system $O_1x_1y_1z_1$. The coordinates $x_1y_1z_1$ of a point placed on the cylinder C_1 are:

$$\begin{aligned} x_1 &= R_1 \sin \alpha \\ y_1 &= R_1 \cos \alpha \\ z_1 &= z_1 \end{aligned} \quad (21)$$

where α is the angle between the vectorial radius with O_1x_1 . The unit vectors of the coordinates axes $O_1x_1y_1z_1$ can be expressed depending on the unit vectors of the system Oxyz thus:

$$\vec{i}_1 = \vec{i}$$

$$\vec{j}_1 = \vec{j} \sin \omega - \vec{k} \cos \omega \quad (22)$$

$$\vec{k}_1 = \vec{k} \sin \omega + \vec{j} \cos \omega$$

Then, the situation vector of any point placed on the cylinder C_1 , expressed given the reference system Oxyz is:

$$\begin{aligned} \vec{\rho}_1 &= R_1 \cos \alpha \vec{i} + \\ &+ (z_1 \cos \omega + R_1 \sin \alpha \sin \omega) \vec{j} + \\ &+ (z_1 \sin \omega - R_1 \sin \alpha \cos \omega) \vec{k} \end{aligned} \quad (23)$$

For all the intersection points of the cylinders C and C_1 , the vectors $\vec{\rho} = \vec{\rho}_1$, therefore their projections on the axes of the reference system are likewise:

$$\begin{aligned} R \sin \theta &= R_1 \cos \alpha \\ R \cos \theta &= z_1 \cos \omega + R_1 \sin \alpha \sin \omega \end{aligned} \quad (24)$$

$$z = z_1 \sin \omega - R_1 \sin \alpha \cos \omega$$

From (24) we obtain:

$$z_1 = R \frac{\cos \theta}{\cos \omega} - R_1 \sin \alpha \cdot \operatorname{tg} \omega \quad (25)$$

Therefore:

$$\begin{aligned} z_1 &= \pm \frac{\sqrt{R^2 - R_1^2 \cos^2 \alpha}}{\cos \omega} - R_1 \sin \alpha \operatorname{tg} \omega \\ \alpha &\in [0, 2\pi] \end{aligned} \quad (26, 27)$$

Is obtained the same figure, like in the previous case, by introducing the relations (26, 27) into Mathematic 5.1 program.

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