DYNAMICAL MODELING OF MECHANICAL OSCILLATIONS IN MAGNETIC FIELD

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Abstract. The paper deals with the problems of magneto-mechanical system consisting of elastic cantilever with ferromagnetic mass at its free end posed in a force potential field of the permanent magnet. The basic relations of the mechanical oscillations are worked out. A quality analysis of the dynamical model is done and the possibilities for influence and control of the magneto-mechanical system are shown.

Keywords: oscillations, natural frequency, permanent magnet, MEMS

1. Introduction

An important attention is paid to the vibrations in the developing of the modern high technological devices for the micromechanical and nanotechnological needs. They are in straight dependence of the dimensions of the microelements [1]. The cantilevers are one of the most frequently applied elements in the micro-electromechanical systems (MEMS). Except as passive elements [2] with constant frequency, it is being searched in these devices for some design decisions for developing of micro-cantilevers with controlled frequency of the oscillations [3], which could be variable in the time. The characteristics of such kind cantilevers are different from these with nonlinear or step shaped curves [4] because of the fact that the change of the elastic constant is due to the change of some characteristics of the potential energy of the environment in which the oscillations are accomplished. The characteristics of the potential field of the environment are influenced by adding electrical charges [5], magnetic or electromagnetic sources. The change of the natural frequency of the system is used for accomplishing of micro positioning, measurement and generation of vibrations with different frequencies, designing of electro dynamical smart vibroisolators [6]. The HHD actuators move its read-write heads in the magnet environment. The vibration in these devices must be eliminated because of the needs of high level of precision positioning [7].

The potential field characteristics could be controlled in determined limits by arbitrary given time dependent low. Actually, this is frequency control of the oscillated spring-mass-magnet system. Since the actuator is treated as a part of micro of nanostructure of electronic element, HDD or microsensor actuator, the possibilities of application of pure mechanical controlled mechanisms are very restricted.

The fundamentals of the vibrations of a spring-mass-magnet system are considered in the paper. The aim of the investigations is the analysis of the options of control of the parameters of the oscillations, the position function to be investigated and to determine the main relations for choice of the parameters of the spring-mass-magnet system.

2. Simplified dynamical model of the spring-mass-magnet system

An elastic cantilever 1 with length l and constant rectangular cross section $S = a \cdot b$, where a is the width and b is the thickness is given (Figure 1). A core ends at the free cantilever with mass m of ferromagnetic material. Below the core a permanent magnet 2 is placed. There is an air gap h between the faces of the permanent magnet and the core when the cantilever is in horizontal position.

If the mass distribution of the cantilever along its length is neglected and some other simplifications are made, then the motion of the core could be described by the model shown in Figure 1b.

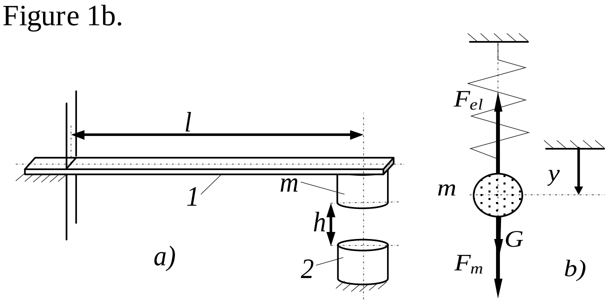

Figure 1. A cantilever in magnetic field
a) sketch; b) dynamical model

On the mass m the elastic force

$$F_{el} = -c \cdot y \tag{1}$$

acts, as well as the weight

$$G = m \cdot g \tag{2}$$

and the magnetic force

$$F_m = \frac{k}{(h-y)^2} \tag{3}$$

are applied. Here c = $3E \cdot J/l^3$ is the cantilever stiffness, E – module of elasticity, $J = a \cdot b^3/12$ is the inertia moment of the cantilever cross section, g is the gravity acceleration, k is a constant of the permanent magnet [8], depending of its properties. According to Coulomb's low, k shows the influence of some imaginary magnet masses [9, 10].

The magnet force is linearised by MacLaurin expansion to the second order and it is got

$$F_m \approx \frac{k}{h^2} + \frac{2k}{h^3} \cdot y . \tag{4}$$

From the Newton's second low of motion the equation

$$m \cdot \ddot{y} + \left(c - \frac{2k}{h^3} \right) \cdot y = \frac{k}{h^2} + m \cdot g \tag{5}$$

follows, which after dividing by m of both hand sides is transformed in

$$\ddot{y} + k_1^2 \cdot y = f \tag{6}$$

where

$$k_1 = \sqrt{\frac{c}{m} - \frac{2k}{h^3 \cdot m}} = \sqrt{k^2 - \frac{2k}{h^3 \cdot m}} \tag{7}$$

is the natural frequency of the system, here called magnetic-mechanical natural frequency, and

$$k = \sqrt{\frac{c}{m}} \tag{8}$$

is the natural frequency of the system without permanent magnet called here mechanical natural frequency, $f = \frac{k}{m \cdot h^2} + g$.

The solution of the linear differential equation (6) is sufficiently investigated in the introductory curses of vibrations [11] and could be expressed in the view

$$y = A \cdot \cos(k_1 \cdot t + \alpha) + \frac{f}{k_1^2} \tag{9}$$

where $A = y_0 \cdot k_1^2 / k_1^4$ is the amplitude, and $\alpha = \arctan \frac{y_0 \cdot k_1}{f - y_0 \cdot k_1^2}$ is the phase angle.

On the linearizing of the magnetic force the approximate value of the static deformation y_{st} due to the magnetic force action and the weight is evaluated from the equation

$$c \cdot y_{st} = \frac{k}{h^2} + \frac{2k}{h^3} \cdot y_{st} + m \cdot g \tag{10}$$

which yields

$$y_{st} = \frac{k \cdot h - m \cdot g \cdot h^3}{c \cdot h^3 - 2k} . \tag{11}$$

It is seen by formula (7) that the magnetic field influences the natural frequency and in this case can decrease it. Maximal frequency values can be reached with smallest magnetic force, which corresponds to maximum air gap.

If it is assumed, that the magnetic field influence is expressed by some imaginary additional magnet mass m_m, it can be evaluated by the help of formula (7), from which the equation follows

$$\frac{c}{m + m_m} = \frac{c}{m} - \frac{2k}{h^3 \cdot m} \tag{12}$$

and it is got the solution

$$m_m = \frac{2k}{c \cdot h^3 - 2k} \cdot m . \tag{13}$$

3. Quality investigation of the dynamical model

The simplified dynamical model (5) describes approximately the system behavior when the oscillations are small in the area of the $y = 0$, where the MacLaurin's expansion is valid. Taking in account the above made simplifications the exact differential equation is

$$m \cdot \ddot{y} + c \cdot y = \frac{k}{(h-y)^2} + m \cdot g . \tag{14}$$

The solution of this differential equation can be presented by elliptical integrals, which enforce numerical methods to be used. Therefore, this solution is unsuitable for generalized analysis. A phase portrait of the equation (14) with zero initial velocity and given initial y_0 can be obtained by the Energy Conservation Theorem of the considered conservative spring-mass-magnet system [12]

$$E_k + E_p = 0 \tag{15}$$

where

$$E_k = \frac{m \cdot v^2}{2} = \frac{m \cdot \dot{y}^2}{2} \tag{16}$$

is the kinetic energy, and

$$E_p = \frac{c \cdot y^2}{2} - \frac{c \cdot y_0^2}{2} - \frac{k}{h-y} + \frac{k}{h - y_0} + m \cdot g \cdot (y - y_0) \tag{17}$$

is the potential energy. On substituting in (15) the expression it is found

$$\frac{m\cdot\dot{y}^2}{2}-\frac{c\cdot y^2}{2}-\frac{c\cdot y_0^2}{2}-\frac{k}{h-y}+ \\ +\frac{k}{h-y_0}+m\cdot g\cdot(y-y_0)=0 \qquad (18)$$

which could be worked out from the Theorem of Kinetic Energy Change if instead of potential energy the work or the potential forces is expressed.

The phase portrait of the differential equation (14) for the appropriate practical values of the permanent parameters is shown in Figure 2.

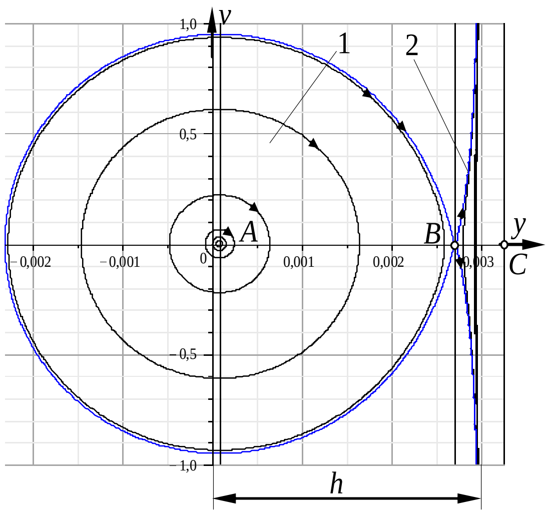

Figure 2. Phase portrait of the no dissipative sprig-mass-magnet system

Two characteristic zones can be distinguished in the figure. Steady state periodical motion with nonzero equilibrium position denoted in figure as zones 1 and no periodical limited motion denoted by zones 2. The zone of the periodical motion confirms the solution of the approximate differential equation (5). As it is seen there is a deviation from the origin, which can be explained by the nonhomogeneity of the differential equation. The conditions for noncyclical motion could be getting by determining the equilibrium points of the system (14). These points follows from the system

$$\begin{cases} v=\dot{y}=0 \\ \dot{v}=\dfrac{k}{m\cdot(h-y)^2}-k^2\cdot y+g=0 \end{cases} \qquad (19)$$

There is 3 real roots of the system (19) denoted as points *A*, *B* and *C* in Figure 2. The point A corresponds to the harmonic oscillations with relatively small amplitude. The second point *B* gives the critical initial deflection which is the limit between periodical and non periodical motion. The third point *C* is out of possible values of *y* ($y_C > h$) so this solution has not physical sense. The comparisons of some numerical solutions of the differential equations (5) and (14) show that the equation (5) is valid in the interval for $y_0 \le 2h/3$. It is seen that for values of y_0 biggest than $2h/3$ the amplitude and the frequency decrease.

4. Investigating the possibilities of influence of the oscillation parameters and position function

The approximate model gives good results for the so given task and here will be analyzed the expression of the natural frequency (7). For h → ∞, $k_1 = k = \sqrt{c/m}$ is upper limit of the natural frequency, which coincides with the mechanical natural frequency *k*.

The minimum value of *h*, which gives harmonic solution of (6) follows from the expression (7), which has to be positive, or

$$\frac{c}{m}-\frac{2k}{h_n^3\cdot m}\ge 0 \qquad (20)$$

and it is obtained

$$h \ge h_n = \sqrt[3]{\frac{2k}{c}}\,. \qquad (21)$$

The influence coefficient of the initial air gap of the natural frequency of the system is

$$k_{1_h}=\frac{\partial k_1}{\partial h}=\frac{3k}{k_1\cdot m\cdot h^4}\,. \qquad (22)$$

The obtained result shows that the coefficient k_{1_h} is inverse proportional of fourth order of h and proportional to the imaginary magnet mass. By the help of Figure 3 it is concluded that the influence of *h* of the natural frequency is effective in small interval near critical values.

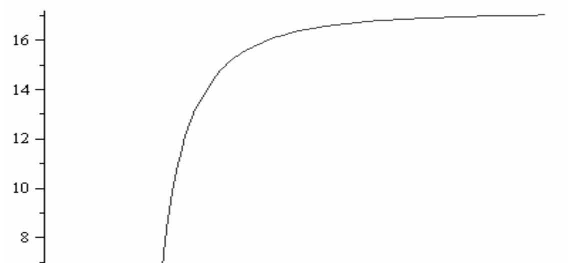

Figure 3. The relation $h= h(k_1)$

The position function of the cantilever is worked out by solving the equation

$$c \cdot y_{st} = \frac{k}{\left(h - y_{st}\right)^2} + m \cdot g \qquad (23)$$

and it is found

$$h = \frac{-c \cdot y_{st} + G \cdot y_{st} \pm \sqrt{\left(c \cdot y_{st} - G \cdot y_{st}\right) \cdot k}}{G - c \cdot y_{st}}. \qquad (24)$$

In Figure 4 the graphs show that for a relatively big interval of change of *h* the relation is strongly nonlinear. When the interval of *h* is properly small some approximately inverse proportional dependence could be obtained (Figure 4b).

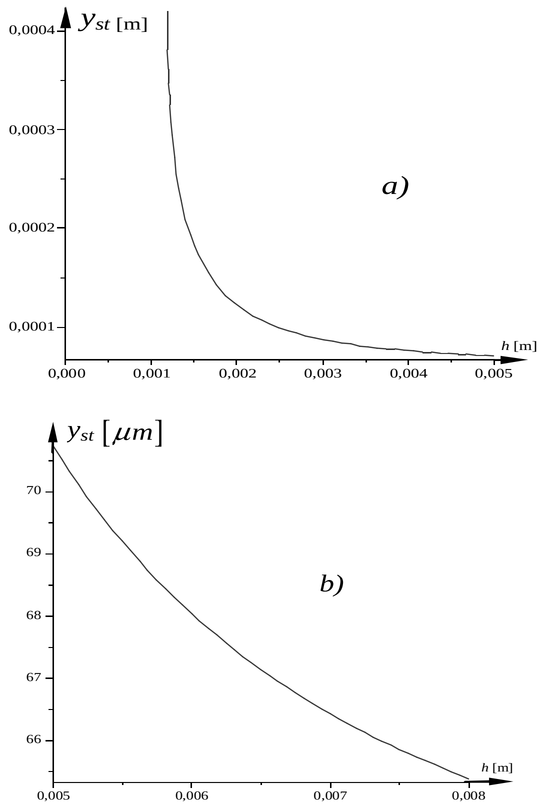

Figure 4. Position function $h = h(y_{st})$
a) big interval of $0 \leq h \leq 5$ mm;
b) small interval $0 \leq h \leq 8$ μm

5. Conclusions

In the so proposed model, some simplifications has been made, the more important of which are: the neglecting of dissipations, the neglecting of the mass distribution along the cantilever length, neglecting of realistic distribution of the force lines of the magnetic field. In spite of this the approximate model gives possibilities for an initial orientation when have to choice the parameters of the system in the frames of determined sphere of tasks.

At the base of the obtained results the parameters of the oscillating system can be approximately evaluated. There can be taken a choice of the limit value of the initial deformations initial air gaps and critical values of the deformation and the dimensions. Ever more it is possible to be taken a relevant choice of measure for extinguishing of the vibrations or for change of its frequency in a certain direction.

The model discovers also the disadvantages of the system, the more important of which are: the natural frequency of the system can only be decreased, the controlling of the frequency can be established mechanically by change of initial air gap; except positioning there is some small rotation at the and of the cantilever.

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