THE ESTIMATION OF MASS MOMENT OF INERTIA ON AN EQUIPMENT - MODELLING AND OPTIMISATION

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Abstract. This article deals with an approach for estimation of a mass moment of inertia of existing mechanical working excavator equipment. The mechanically working equipment is a part of one mechanical system of an excavator with no documentation or incomplete documentation. An experiment was carried out with real – life excavator working equipment. The experimental data present a period of variation of the excavator working equipment. The period of variation is obtained by preliminary data processing of the experimental data. Confidence intervals of the mass moment of inertia and some other parameters of the mechanical system are derived. The non-linear least squares optimisation routine by the use of Levenberg – Marquartd method is applied for estimation of the value of mass moment of inertia for different positions of the excavator working equipment. All investigations are made in MATLAB environment.

Keywords: mass moment of inertia, modelling, optimisation, estimation

1. Introduction

Dynamical investigation for existing working equipment of excavator is an important task of the engineering practice. The synthesis of the mathematical model for this type rotating mechanism needs some data for mass moment of inertia subject to different positions of the equipment [1, 2].

The aim of this article is to formulate an approach for estimation the mass moment of inertia as a part of a mechanical system. The period of the variation for excavator working equipment is obtained based on the experimental data. The procedure for estimation of the mass moment of inertia includes the following steps:

- Carrying out of real – life experiment and data collection.
- Preliminary data processing.
- Estimation of the initial values and confidence intervals for the mass moment of inertia and other parameters of the mechanical system.
- Formulation of optimisation problem subject to Levenberg – Marquartd method and the least squares criterion for estimation of the mass moment of inertia with parameter boundaries.

STATGRAPHICS package is used for preliminary data processing [7]. All researches in the article are done in MATLAB environment [4, 5, 6].

2. Setting of the experiment and experimental data

The setting of experiment for mass moment of inertia estimation is shown in figure 1. Initially the excavator takes a position in a horizontal plane xOy (figure 1a). The working excavator equipment is orientated with direction to midnormal of the angle xOy.

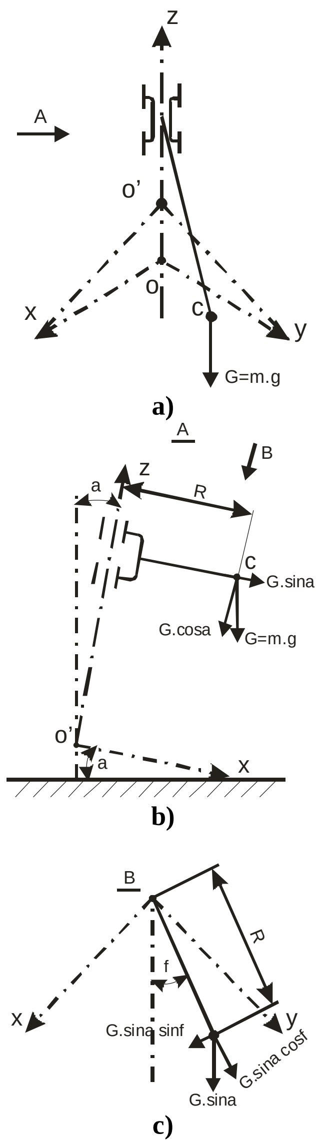

Figure 1. Setting of the experiment for mass moment of inertia determination

The rear part of the excavator is lifted and its mass centre O is shifted to point O' and at the same time the axis of the working equipment tilt in relation to vertical O'-A on an angle-α (figure 1b). Force, as a result of the working equipment mass, applied in the mass centre C has two components $G \cdot \cos(\alpha)$ and $G \cdot \sin(\alpha)$ respectively. On the next step the working equipment rotates on an angle φ (figure 1c) in relation to neutral position and as a result the force $G \cdot \sin(\alpha)$ divides by two components and gives birth to torque as follows:

$$T(t) = m \cdot g \cdot R \cdot \sin(\alpha) \cdot \sin(\varphi) \qquad (1)$$

where: m - mass of the working equipment; g - acceleration of gravity; R - distance of the mass centre of the working equipment to the axis of rotation; α - angle of the slope of axis of rotation in relation to vertical; φ - initial angle of deviation of the working equipment.

The torque as a function of the angle φ influences a damping variation movement of the working equipment in relation to axis of rotation with a period of swinging depending on its mass moment of inertia. The experiment is carried out subject to the following conditions:

- Flexible connections coupling of the rotating cylinder are disconnected.
- Part of the hydraulic liquid is drained out.
- Some residual hydraulic liquid presents in cylinders during the experiment.
- Working motor of the excavator and influence noises from equipment vibrations.
- The sensor does not recognize the direction of rotation (plus – minus) and because of this, the half-periods of swings have only positive sign.

The experimental data presenting the time course of the velocity of the working equipment for excavator BX 025 – MT are shown in figures 2 – 4 subject to three cases: equipment with maximum carried bucket without load, with load and gathered bucket without load. The value of the angle of the slope of axis of rotation in relation to vertical is α = 7° (0.1221 rad) and the value of the angle φ of deviation of the working equipment depends of three cases of study (figures 2 – 4).

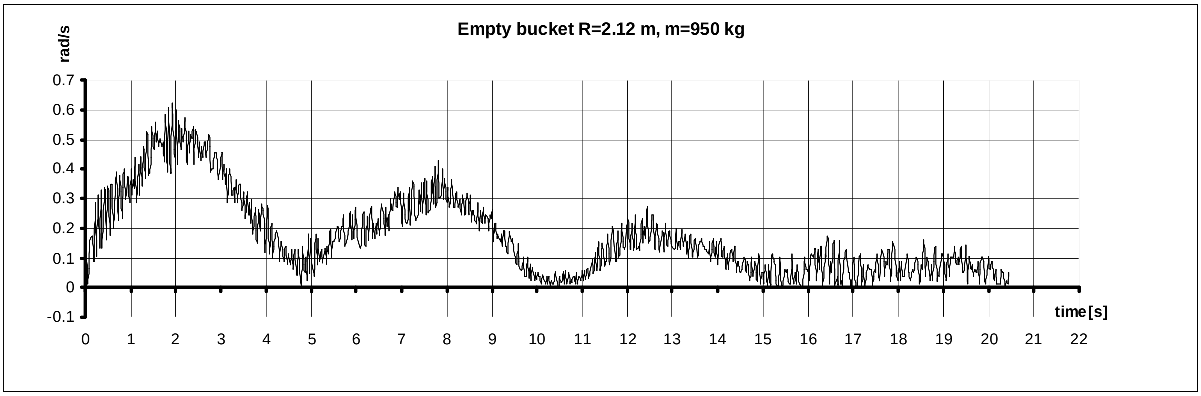

Figure 2. Equipment with maximum carried bucket without load

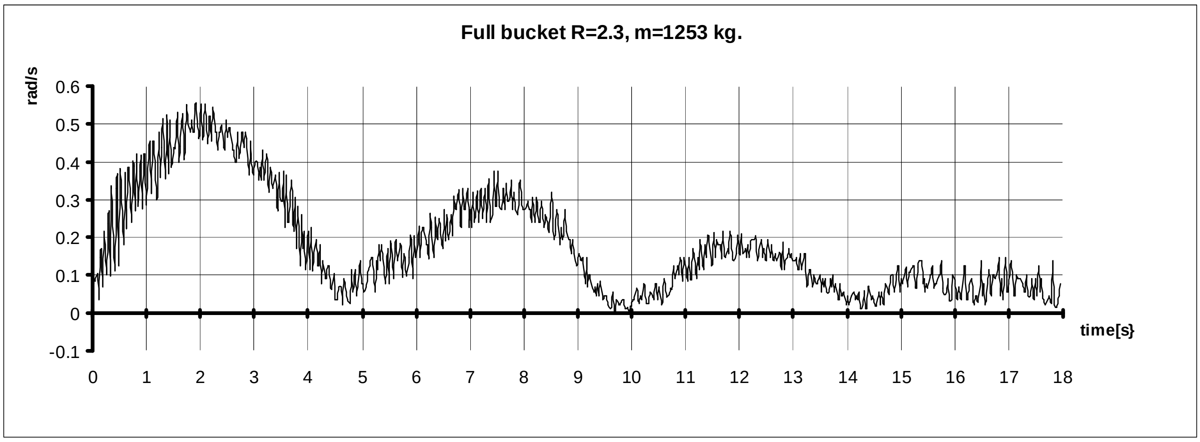

Figure 3. Equipment with maximum carried bucket with load

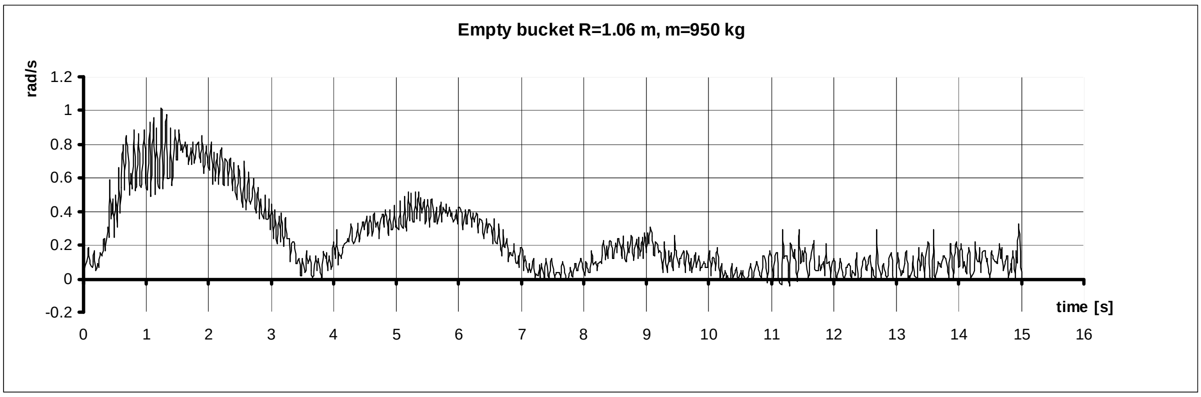

Figure 4. Equipment with maximum gathered bucket without load

3. Preliminary data processing

The aim of preliminary data processing is to determine the general trend of the time series without changing the period of swing. As if is shown in figures 2 – 4, a noise presents in the experimental data. Exponential smoothing technique is applied to the experimental data. The formula of this method is written as

$$S_t = \alpha \cdot Y_t + (1-\alpha)\cdot S_{t-1} \quad (2)$$

where α is a smoothing constant that is selected in the range $0 < \alpha < 1$, Y_t – the observed value at time t, S_t – the smoothed value at time t.

Root mean square prediction error (RMSPE) criterion is used to determine the value of the smoothing constant which minimized the criterion. RMSPE is defined as

$$\text{RMSPE} = \sqrt{(\text{Mean prediction error})^2 + (\text{Standard deviation of prediction error})^2} \quad (3)$$

The smoothed data in case of maximum carried bucket without load are shown in figure 5.

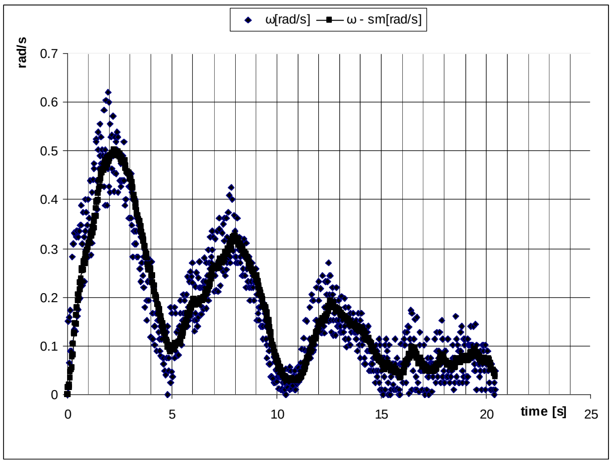

Figure 5. Time course of the smoothing experimental data

The following notations are used in figure 5:

ω [rad/s] – angular velocity,

ω - sm [rad/s] – smoothed angular velocity.

4. Estimation of the unknown mass moment of inertia by least squares method

The estimation of the mass moment of inertia is done by two stages. Confidence intervals of the input parameters for optimisation procedure are estimated at the first stage. The optimisation procedure with least squares criterion and constrains of the parameters is solved at the second stage. The least squares method has many applications for the practical reasons:

- It is simply to calculate this criterion.
- It is easy to understand and physically easy to interpretation.
- There is no necessity of apriority information.
- Its applications are successful in both cases for linear and non-linear optimisation problems.

4.1. Initial estimate of the input parameters

Initial estimates of the input parameters for optimisation procedure: J – mass moment of inertia; m - mass of the working equipment; R – distance of the mass centre of the working equipment to the axis of rotation; T_{FR} – moment of friction; K – coefficient; α - angle of the slope of axis of rotation in relation to vertical; φ – initial angle of deviation of the working equipment, are represented in following cases:

1. Equipment with maximum carried bucket without load $J = 9074.14$ kg·m^2, $m = 1253$ kg, $R = 2.3$ m subject to following interval constrains

$J = 5000 \div 7500$ kg·m^2

$m = 850 \div 1050$ kg

$R = 1.95 \div 2.3$ m

$T_{FR} = 80 \div 300$ N·m

$K = 1 \div 2500$ N·m

$\varphi = 1.027 \div 0.923$ rad ($59^0 \div 53^0$).

2. Equipment with maximum carried bucket with load J = 9074.14 kg·m^2, m = 1253 kg, R = 2.3 m subject to following interval constrains
   J= 7000 ÷ 10500 kg·m^2
   m = 1100 ÷ 1300 kg
   R = 2.1 ÷2.4 m
   T_{FR} = 80 ÷ 300 N·m
   K = 1 ÷ 2500 N·m
   φ = 0.95 ÷ 0.844 rad (54^0 ÷ 48^0).
3. Equipment with maximum gathered bucket without load J = 1577.37 kg·m^2, m = 950 kg, R = 1.06 m subject to following interval constrains
   J = 1000 ÷ 2000 kg·m^2
   m = 850 ÷ 1050 kg.
   R = 0.8 ÷1.2 m
   T_{FR} = 50 ÷ 300 N·m
   K = 1 ÷ 2500 N·m
   φ = 1.1 ÷ 0.99 rad (63^0 ÷ 57^0).

4.2. Formulation of the optimisation procedure for estimation of the mass moment of inertia

Mathematical model for describing of the process of swinging is used for estimation of the moment of inertia of the equipment in relation to axis of rotation O'-Z (figure 1b). The model is presented in a state-space form. State space vector $X^T = [X_1 \;\; X_2]^T$ is defined as

$$\mathbf{X}_1 = \varphi(t)$$
$$\dot{\mathbf{X}}_2 = \frac{d\varphi}{dt} = \dot{\varphi}, \qquad (4)$$

where $\dot{\varphi}$ is the angular velocity of the equipment.

The state space equation and equation of the measurable output are

$$\frac{d\mathbf{X}}{dt} = \mathbf{A}(\mathbf{X})\cdot\mathbf{X} + \mathbf{B}\cdot\mathbf{T}_{FR}$$
$$\mathbf{X}_2 = \mathbf{C}\cdot\mathbf{X} \qquad (5)$$

Matrices **A**, **B** and **C** are

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\dfrac{\mathbf{T}(\mathbf{X}_1)}{J\cdot\mathbf{X}_1} & -\dfrac{K}{J} \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 0 \\ -\dfrac{1}{J} \end{bmatrix}, \qquad (6)$$
$$\mathbf{C} = [0 \quad 1]$$

where K is a complex parameter that takes into account the moment of friction within hydraulic cylinders and hydraulic losses through pipes.

The optimisation problem could be formulated as follows

$$\min_{\theta\in R^n} F(x) = \sum_{j=1}^{N}\left(y(\dot{\varphi}(j)) - \dot{\varphi}_M(j,\theta)\right)^2 dt, \qquad (7)$$
$$\theta\in[\theta_{min},\theta_{max}],$$

where y is a vector of the experimental data representing the angular velocity $\dot{\varphi}$ and $\dot{\varphi}_M(j,\theta)$ representing the numerical solution of the dynamic model whit vector of unknown parameters θ at the point j of the experimental data.

For the mechanical system (5 – 6) the optimisation problem is defined as a task with coordinate constrains as follows $\theta\in[\theta_{min},\theta_{max}]$. An important optimization method is a Marquardt – Levenberg method and therefore uses a search direction, which is a cross between the Gauss-Newton direction and the steepest descent (controlled by the parameter of the Marquardt). The vector θ includes the input parameters J, m, R, T_{FR}, K, φ described above. The interval boundaries of these parameters are used is addition for optimisation procedure. The model (5 – 6) is solved by SIMULINK package. Block–diagram of the optimisation procedure is shown in figure 6.

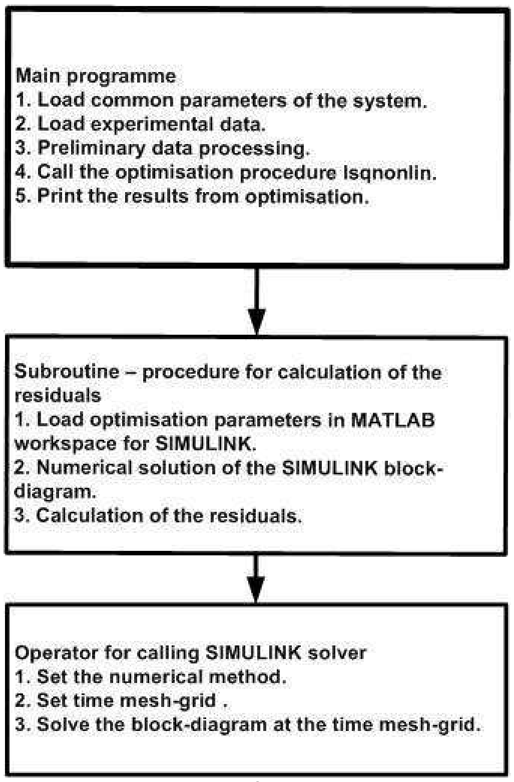

Figure 6. Block–diagram of the optimisation procedure

4.3. Basic results and analysis

The results from the optimization procedure are shown in figures 7 – 9, where the experimental data for angular velocity are denoted by number one and data model fit with number two. The derived estimations of the input parameters are shown in table 1. Validation of the established parameters is done by the comparative analysis for mass moment of inertia by modelling of the equipment within

SolidWorks computer package.

It could be seen from figures 7 ÷ 9 and table 1 that the error between the estimations of the mass moment of inertia by CAD system SolidWorks and the approach introduced in this article is in the range between 1% and 4.2%. It could be said that the results from simulation of the dynamic model (5 – 6) are acceptable in practice because the CAD system does not take into account the mass of the welded seams.

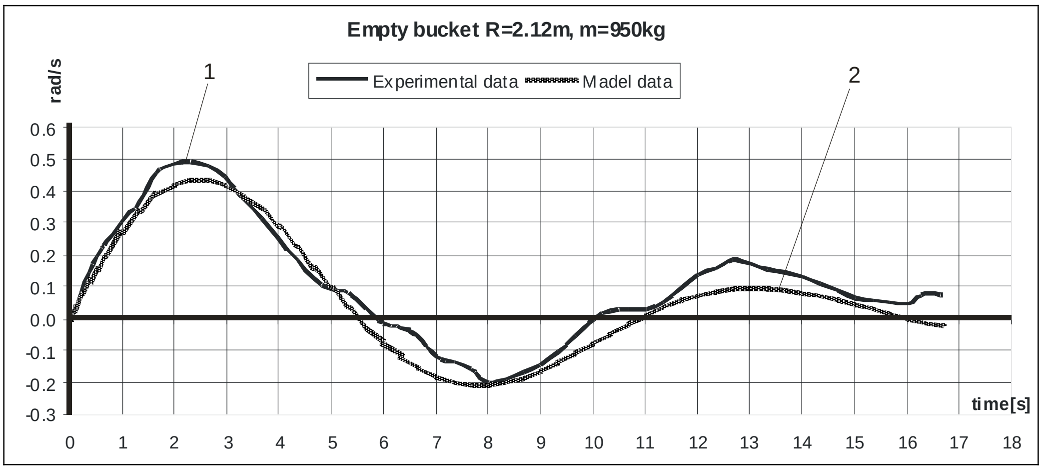

Figure 7. Equipment with maximum carried bucket without load

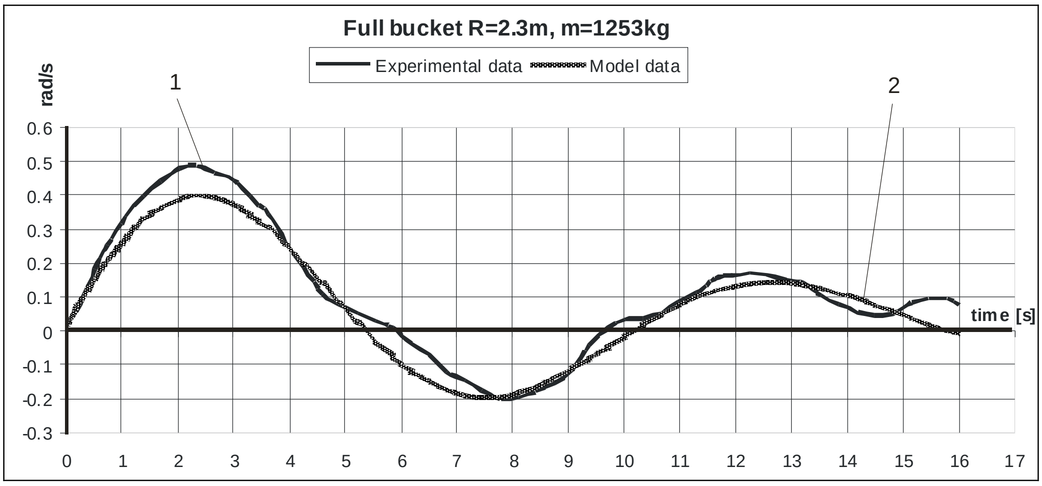

Figure 8. Equipment with maximum carried bucket with load

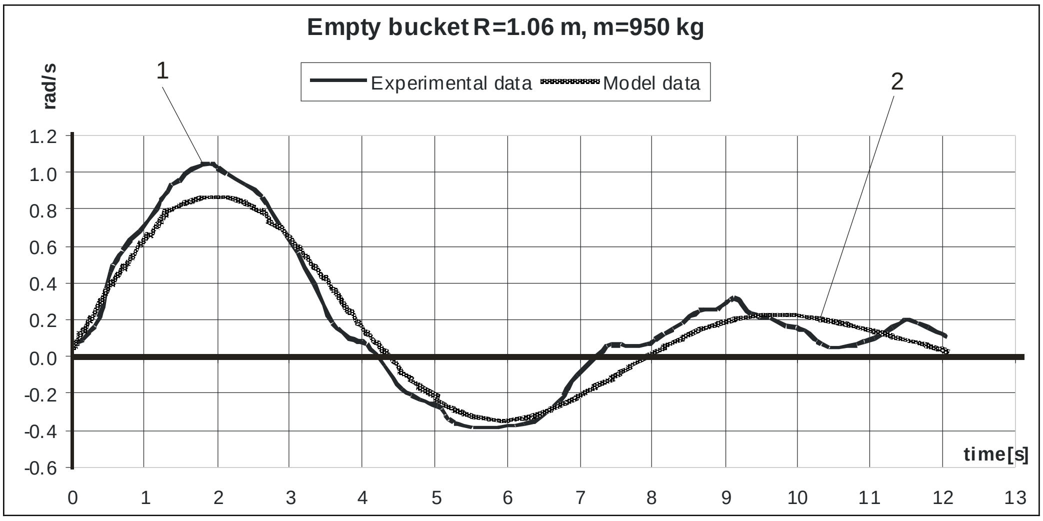

Figure 9. Equipment with maximum gathered bucket without load

Table 1. Results from optimisation and comparative study

No.	Optimisation method based on experimental data						Theoretical Method	Error
	m [kg]	R [m]	φ [rad]	T_{FR} [N·m]	K [N·m]	J [kg·m^2]	SolidWorks	%
1	945	1.97	0.98	172	1321	6602.5	6879.6	-4.2%
2	1254.4	2.24	0.89	274	1746	8867.4	9074.1	-2.9%
3	948.3	1.05	0.105	59	250	1652.7	1574.4	1.05%

5. Conclusion

1. Experiments have been carried out for the determination of fluctuation period of the working equipment of an excavator in order to estimate the mass moment of inertia.
2. A computer model on the base of Solid Works package has been developed. It allows for calculating the mass moment of inertia of the working equipment of excavators.
3. A dynamic model on the base of MATLAB computer package has been developed. This model allows for optimizing the design of excavators working equipment in regard to its dynamic characteristics.
4. A methodology for the determination of moment of inertia of mechanically operating equipment of excavators has been proposed. This methodology could be implemented in practice as it has a high degree of coincidence with experimentally obtained data.

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