THEORETIC-EXPERIMENTAL APPROACH TO COMPUTATION OF DIGGING FORCE

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Abstract. A theoretic-experimental approach to computation of digging force in arbitrary chosen points from implemented tool trajectories of earthmoving machines is proposed. The approach allows automatic collection of statistical information for digging forces and digging processes in real operating conditions. It is based on the developed original algorithms for position and static force analysis of corresponding operating equipment as well as on the obtained experimental data for pressure (force) and working stroke of chosen driving hydrocylinders. An interactive MathCAD program is developed for computation of digging force parameters in backhoe operating equipment of hydraulic excavator. The necessary forces and strokes of bucket and stick hydrocylinders for real operated hydraulic excavator BEN 195 are measured and digging force parameters are calculated.

Keywords: digging force, hydraulic excavator, position and static force analysis

1. Introduction and problem description

Accurate determination of the digging force parameters, acting to the implemented tools of earthmoving machines has a fundamental importance for their optimal design and intelligent exploitation.

Some theoretical, simulation and experimental approaches for determination of the digging force parameters are known. The obtained results by theoretical [1] and simulation [2, 3] approaches are approximate because: 1) they do not consider with sufficient accuracy the random character of the cutting resistance; 2) the real digging process is idealized and schematized; 3) the geometry and other characteristics of the operating equipment are simplified and theoretical dependencies using empirical coefficients are used. The experimental determination of the digging force leads to considerably more realistic results. It is performed by appropriate laboratory equipment and stands or by specialized, unique for every single case, device in real operating conditions. The working principal of the widely used portable dynamic densitometer [4] does not coincide with the principles of real cutting and digging soil process, which is why it can be used only for qualitative categorization of the soils without defining the values of digging force parameters. The portative manual Christov's device [5] imitates more accurate the real cutting process and makes possible to obtain the cutting force values, but only for the surface of the soil and for lower soil categories. As well, more complicated laboratory stands are known by which is possible to measure more precisely the cutting and digging forces [6], but their running is labour consuming and expensive and their application is very limited.

The control systems of the modern earthmoving machines allow easy measurement, registration, processing and visualization of the forces and displacements in driving hydraulic devices (hydrocylinders and hydromotors), which are a function of the applied to the operating equipment external forces. The unique dependence between the measured forces and the digging force can be used for determination of the digging force parameters – its value and direction.

The purpose of the present work is to propose an easy applied theoretic-experimental approach by which with sufficient for the engineering practice accuracy to determine the digging force parameters in real operating conditions and to allow automatic collection of the statistical information for digging forces and digging processes.

The presented approach will be demonstrated on one of the most typical earth-moving machine – a backhoe hydraulic excavator.

2. Theoretic-experimental approach to computation of digging force

2.1. Selection of dynamometric links from the cinematic chain

Direct measurement of the excavator digging force is extremely difficult and practically unapplied in the real operating conditions of the machine. More appropriate away is to measure forces in different links of the cinematic chain of operating equipment, caused by the digging force. The easiest for realization, theoretically proven and insuring sufficient sensibility and accuracy, is the measurement of pressure (force) and displacements of driving hydrocylinders. For these purposes are used pressure and position sensors additionally mounted or factory built-in as a part of the control system of the modern excavators.

The presence of several driving hydrocylinders at operating equipment (in common case - three) gives an opportunity to realize several different variances of the system for measurement of forces and displacements. The most appropriate variance can be selected by the following two considerations:

a) it is searched the shortest force way from the digging force applied point to the dynamometric hydrocylinder;

b) it must be measured reactive pressure in the dynamometric hydrocylinder and not active.

Considering the above and the fact, that the real digging process is performed by sequences of stick and bucket movements (more than 80% of the working cycle is implemented by the stick), the successful decision is to measure the pressure in the bucket hydrocylinder p_2 or/and the pressure in the stick hydrocylinder p_3 – figure 1. Of course, other decisions are also feasible.

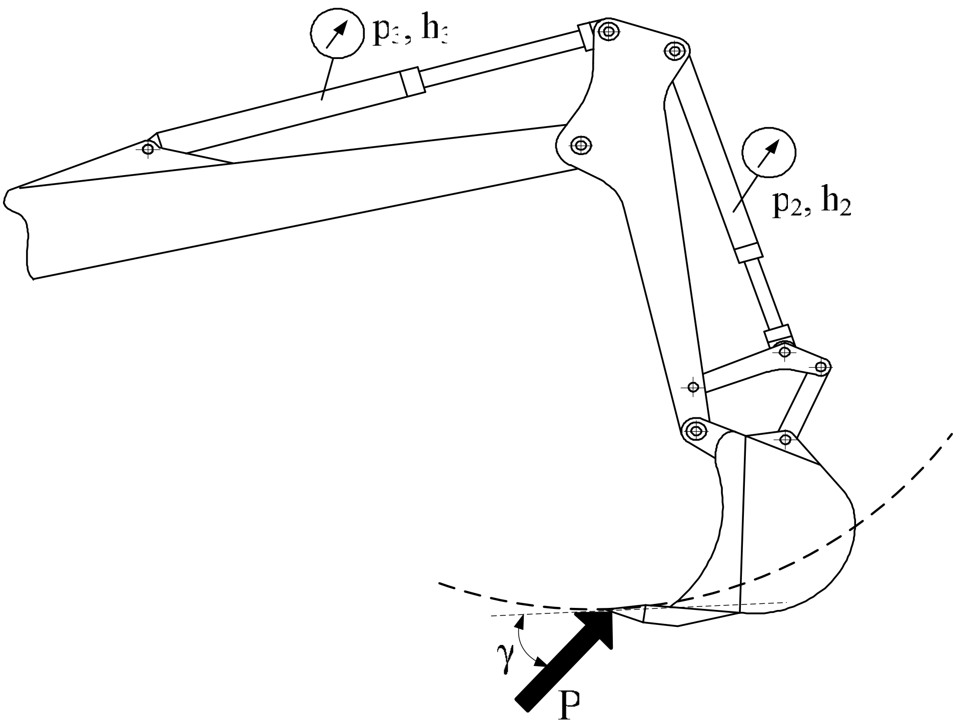


Figure 1. Measurement of the pressure and displacement in bucket and stick hydrocylinders

For concrete soil properties, the digging force can be considered as conservative in relation to the trajectory of the bucket tips [5]. The force has approximately a constant angle along the trajectory and variable value. Its application point is accepted concentrated on the teeth tips [1]. In general, if the geometric configuration of the operating equipment is known, the measured hydrocylinders pressures on the following equation could determine the value P of digging force and its angle γ of inclination towards the trajectory:

$$(P,\gamma) = f(p_3, p_2) \qquad (1)$$

For the purpose, it is necessary to develop mechano-mathematical model, by which to define the interdependence between the measured hydrocylinders pressures (respectively forces), current geometric configurations and digging force parameters P and γ. The main elements of the suggested mechano-mathematical model are the algorithms for position and static force analysis of the excavator operating equipment.

2.2. Position analysis of the operating equipment

For the considered mechanical system it is possible to use the well-known classical graph-analytic methods for position analysis [7, 8], as well matrix methods [9-12]. In the present study, transformation matrices are used for the determination of the position of any point from the cinematic chain [9].

The working equipment is treated as planar linkage, which links are driven by hydrocylinders. The stick and the bucket form an open cinematic chain. The driven links (bucket and stick) and the driving hydrocylinders make locally closed loops – two rocker mechanisms and a four bar mechanism.

The cinematic chain is presented in figure 2. It consists of seven rigid bodies, which are interconnected by nine joints. Each body is denoted by consecutive number i in the cinematic chain, and each joint by n, where i = 0, 1, …, r, …, s, …, 6, and n = 1, 2, …, p, …, q, …, 9. The fixed link has a number i = 0.

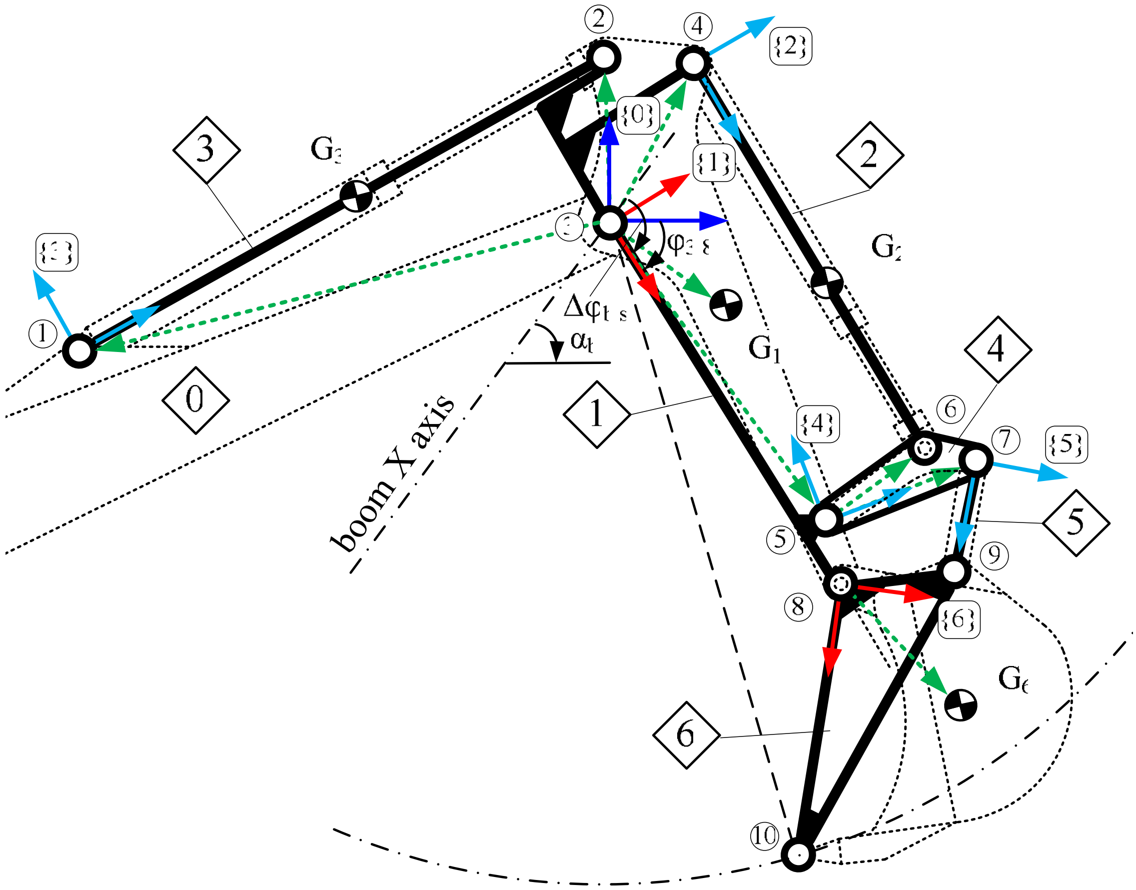


Figure 2. Cinematic chain of the operating equipment
0 - boom; 1 - stick; 2 - bucket hydrocylinder; 3 - stick hydrocylinder; 4 and 5 - links of four bar mechanism; 6 - bucket

There is a fixed Cartesian coordinate system {0}, attached to the point 3 with vertical and horizontal axes. To each body is attached local Cartesian coordinate system $\{i\}$, which location and orientation is shown in figure 2. The position of an arbitrary chosen point q from a body in its local coordinate system, attached at the point p, is denoted by vector $\{V_{p.q}^{L}\}$ (see figure 3a):

$$\{V_{p.q}^{L}\} = \{X_q^L \;\; Y_q^L \;\; 1\}^T \qquad (2)$$

Cartesian coordinates of the point q, respectively X_q^L and Y_q^L, are determined by parameters $L_{p.q}$ and $\alpha_{p.q}^{L}$:

$$X_q^L = L_{p.q} \cos \alpha_{p.q}^{L}\,; \;\; Y_q^L = L_{p.q} \sin \alpha_{p.q}^{L} \qquad (3)$$

With $L_{p.q}$ is denoted the distance between points p and q, and with $\alpha_{p.q}^{L}$ - angle between line $\overline{pq}$ and X_i axis of the local coordinate system.

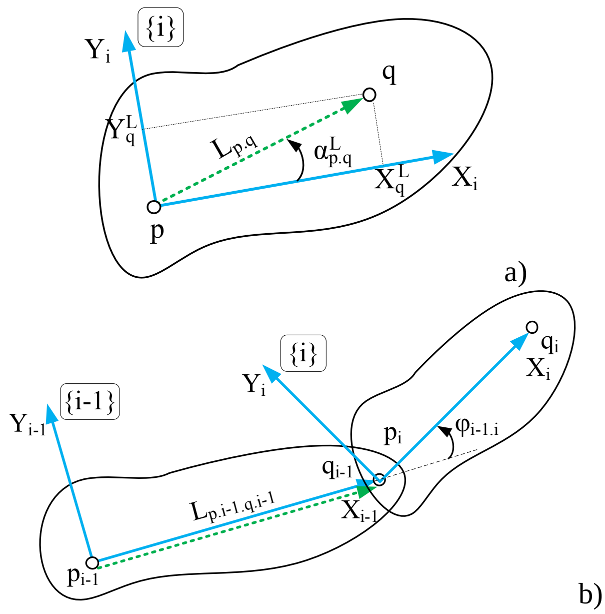


Figure 3: a) position and orientation of the local coordinate system; b) components of the transformation matrix

The stick inclination angle $\varphi_{3.8}$ towards the axis X_0 depends on boom inclination angle α_b towards the same axis X_0 and is determined by the following relation:

$$\varphi_{3.8} = \Delta\varphi_{b.s} - \alpha_b \, , \quad (4)$$

where $\Delta\varphi_{b.s}$ is the current angle between the boom and the stick.

For description of the relative position of two connected by joint bodies, which form an open cinematic chain, following transformation matrices are used (see figure 3b):

$$[T_i^{i-1}] = \begin{bmatrix} \cos\varphi_{i-1.i} & -\sin\varphi_{i-1.i} & L_{p.i-1.q.i-1} \\ \sin\varphi_{i-1.i} & \cos\varphi_{i-1.i} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

where $\varphi_{i-1.i}$ is the angle between links i-1 and i.

For the operating equipment under consideration (figure 2), the open cinematic chain is formed by bodies 0, 1 and 6. Position and orientation of all other bodies depends on them. Transformation matrices between the local coordinate systems of bodies 0, 1 and 6 are:

$$[T_1^0] = \begin{bmatrix} \cos\varphi_{3.8} & -\sin\varphi_{3.8} & 0 \\ \sin\varphi_{3.8} & \cos\varphi_{3.8} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$[T_6^1] = \begin{bmatrix} \cos\varphi_{8.10} & -\sin\varphi_{8.10} & L_{3.8} \\ \sin\varphi_{8.10} & \cos\varphi_{8.10} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (6)$$

The joint coordinates and the body character points at the fixed coordinate system {0} are determined by the equation:

$$\{V_{p.q}^0\} = [T_i^0]\{V_{p.q}^L\} = \{X_q^0 \; Y_q^0 \; 1\}^T \, . \quad (7)$$

For considered mechanical system, the following relations are valid:

- for body 1:

$$\{V_{3.2}^0\} = [T_1^0]\{V_{3.2}^L\}, \quad \{V_{3.4}^0\} = [T_1^0]\{V_{3.4}^L\},$$
$$\{V_{3.5}^0\} = [T_1^0]\{V_{3.5}^L\}, \quad \{V_{3.8}^0\} = [T_1^0]\{V_{3.8}^L\}, \quad (8)$$
$$\{V_{3.G_1}^0\} = [T_1^0]\{V_{3.G_1}^L\};$$

- for body 6:

$$\{V_{8.9}^0\} = [T_1^0][T_6^1]\{V_{8.9}^L\},$$
$$\{V_{8.10}^0\} = [T_1^0][T_6^1]\{V_{8.10}^L\}, \quad (9)$$
$$\{V_{8.G_6}^0\} = [T_1^0][T_6^1]\{V_{8.G_6}^L\} \, .$$

The inclination angle of each body towards X_0 is determined with the coordinates of two points, which belongs to the body, in the fixed coordinate system {0} by standard function of two arguments atan2(y,x), through which the angle can be calculated in the interval (-π,+ π]. For the studied mechanical system can be written:

$$\varphi_{B1} = \operatorname{atan2}(Y_8, X_8) \, ,$$
$$\varphi_{B5} = \operatorname{atan2}(Y_{10} - Y_8, X_{10} - X_8) \, . \quad (10)$$

The length of the hydrocylinder depends on the current inclination angle of the corresponding body, which forms an open cinematic chain and is calculated by the following equation:

$$L_{Bi} = \sqrt{(X_{pi}^0 - X_{qi}^0)^2 - (Y_{pi}^0 - Y_{qi}^0)^2} \, . \quad (11)$$

According to (11) the lengths of the stick and bucket hydrocylinders are:

$$L_{B3} = \sqrt{(X_2^0 - X_1^0)^2 - (Y_2^0 - Y_1^0)^2} \, ;$$
$$L_{B2} = \sqrt{(X_6^0 - X_4^0)^2 - (Y_6^0 - Y_4^0)^2} \, . \quad (12)$$

The coordinates of hydrocylinders gravity centers in the fixed coordinate system {0} are functions of the position and orientation of the open cinematic chain bodies and can be calculated by equation (13). It is presumed, that the gravity centers are situated it the middle of the hydrocylinders.

$$\begin{Bmatrix} X_{G2}^0 \\ Y_{G2}^0 \end{Bmatrix} = \begin{Bmatrix} X_4^0 \\ Y_4^0 \end{Bmatrix} + \left[R_2^0\right] \begin{Bmatrix} L_{B2}/2 \\ 0 \end{Bmatrix},$$
$$\begin{Bmatrix} X_{G3}^0 \\ Y_{G3}^0 \end{Bmatrix} = \begin{Bmatrix} X_2^0 \\ Y_2^0 \end{Bmatrix} + \left[R_3^0\right] \begin{Bmatrix} L_{B3}/2 \\ 0 \end{Bmatrix}, \quad (13)$$

where $[R_2^0]$ and $[R_3^0]$ are the rotation matrices of the hydrocylinders local coordinate systems in relation to fixed coordinate system:

$$[R_2^0] = \begin{bmatrix} \cos\varphi_{B2} & -\sin\varphi_{B2} \\ \sin\varphi_{B2} & \cos\varphi_{B2} \end{bmatrix};$$
$$[R_3^0] = \begin{bmatrix} \cos\varphi_{B3} & -\sin\varphi_{B3} \\ \sin\varphi_{B3} & \cos\varphi_{B3} \end{bmatrix}. \quad (14)$$

When the lengths of the bodies and the coordinates of the joints 5 and 9 are known, than the current geometric location of the four bar mechanism is defined by the coordinates X_7 and Y_7 of the joint 7 in the fixed coordinate system. X_7 and Y_7 can be calculated from the following system of nonlinear algebraic equations:

$$\left\| \begin{aligned} (X_9^0 - X_7^0)^2 + (Y_9^0 - Y_7^0)^2 = L_{7.9} \\ (X_5^0 - X_7^0)^2 + (Y_5^0 - Y_7^0)^2 = L_{5.7} \end{aligned} \right. \quad (15)$$

Thus, the relations from (7) to (15) define the coordinates of all bodies in the fixed coordinate system, also other geometric parameters – lengths of the hydrocylinders and inclination angles of the bodies.

The performed position analysis is used for definition of the current geometric configuration of the operating equipment and is used in the following-up static force analysis.

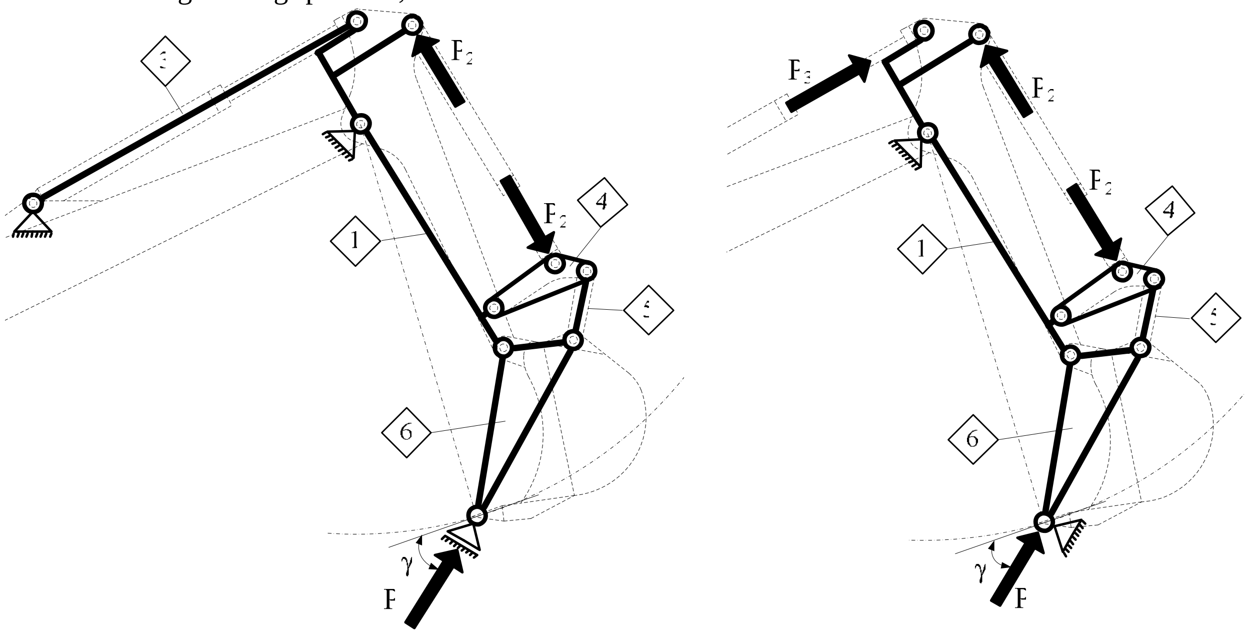
2.3. Static force analysis of the mechanical system

The goal of the performed static force analysis is to set up the relation between measured hydrocylinder forces and the digging force parameters – its value and direction. Also could be determined the joints reactions in different geometric configurations.

For the engineering practice, there are two essential cases for indirect determination of the digging force:

a) Determination of the digging force value if its direction is known. It is performed by measurement of the force P_2 in the bucket hydrocylinder (figure 4a). This case is applicable when the properties of the working environment are known;

b) Determination of the digging force value and its direction by simultaneous measurement of the stick and bucket hydrocylinder forces P_3 and P_2 (figure 4b). This approach is used when it is necessary to perform full identification of the digging force – its value P and inclination angle γ towards the trajectory tangent line.



a) b)

Figure 4. Structural schemes of the mechanical system

In order to apply the conditions for static equilibrium of the bodies (figure 4) it is necessary the number of the degrees of freedom h to be equal to zero for both cases. The Chebishev's formula (16) applied to the first (17) and the second (18) case has following form:

$$h = 3n - 2p_5 - p_4 = 0, \qquad (16)$$

$$h_I = 3.5 - 2.7 - 1 = 0, \qquad (17)$$

$$h_{II} = 3.4 - 2.6 - 0 = 0, \qquad (18)$$

where n is the number of links, p_5 and p_4 are the numbers of cinematic joints from fifth and fourth class.

The applied external forces are: gravity forces; experimentally measured force P_2 – for the first case (figure 4a) or measured forces P_2 and P_3 –for the second case (figure 4b).

The direction of the digging force in both cases is considered as conservative in relation to trajectory. The force makes approximately a constant angle with the trajectory tangential line.

The normal P_n and tangential P_t components of the digging force P are defined in the coordinate system $\{P\}$. This coordinate system is attached to the tip of bucket tooth, its x and y axes are normal and tangential respectively to the trajectory (figure 5a). The following relations are valid:

$$\gamma = \operatorname{atan}(P_n / P_t) = \operatorname{atan} f, \qquad (19)$$

$$\alpha = \operatorname{atan}(P_x / P_y) = \operatorname{atan} k, \qquad (20)$$

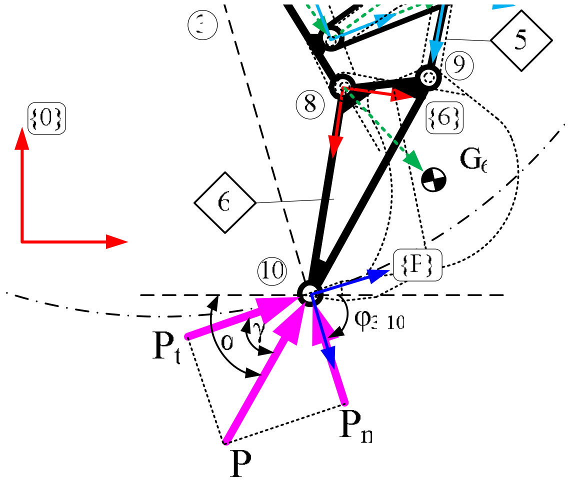
$$k = \frac{\cos\phi_{3.10} f + \sin\phi_{3.10}}{\sin\phi_{3.10} f - \cos\phi_{3.10}}, \qquad (21)$$

$$f = \frac{\cos\phi_{3.10}k + \sin\phi_{3.10}}{\sin\phi_{3.10}k - \cos\phi_{3.10}}, \qquad (22)$$

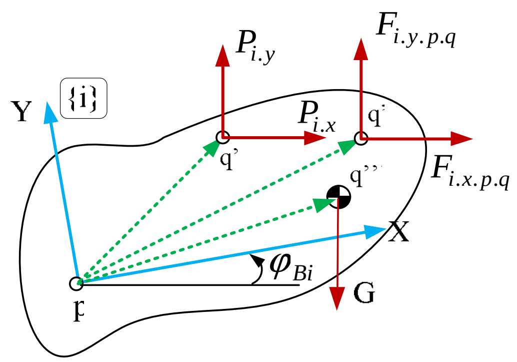
$$\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} = [R_P^0] \begin{Bmatrix} -P_n \\ P_t \end{Bmatrix}, \qquad (23)$$

$$[R_P^0] = \begin{bmatrix} \cos\phi_{3.10} & -\sin\phi_{3.10} \\ \sin\phi_{3.10} & \cos\phi_{3.10} \end{bmatrix}, \qquad (24)$$

$$P_t = \sqrt{\frac{P^2}{1+f^2}}. \qquad (25)$$



a)



b)

Figure 5: a) digging force and its components; b) free body diagram of the body *i* and applied to it internal and external forces

The conditions for static equilibrium of the bodies are obtained by removing the joints and application of internal and external forces on the bodies (figure 5b).

As known, the necessary and sufficient conditi-on for equilibrium of body *i* with applied planar forces is:

$$\sum \begin{Bmatrix} \{F_{i.p.q}\} \\ M_{F.i.p} \end{Bmatrix} + \sum \begin{Bmatrix} \{P_i\} \\ M_{P.i.p} \end{Bmatrix} + \begin{Bmatrix} \{G_i\} \\ M_{G.i.p} \end{Bmatrix} = 0, \qquad (26)$$

where $\{F_{i.p.q}\} = \{F_{i.x.p.q} \quad F_{i.y.p.q}\}^T$, $\{P_i\} = \{P_{i.x} \quad P_{i.y}\}^T$ and $\{G_i\} = \{0 \quad -G_{i.y}\}^T$ are the vectors of internal forces $F_{i.p.q}$, external forces P_i and gravity forces G_i in the fixed coordinate system; $M_{F.i.p} = \{F_{i.p.q}\}^T [A_i] \{V_{p.q}^L\}$- the moments of internal forces; $M_{P.i.p} = \{P_i\}^T [A_i] \{V_{p.q}^L\}$- the moments of external forces; $M_{G.i.p} = \{G_i\}^T [A_i] \{V_{p.q}^L\}$ - the moments of gravity forces in the local coordinate systems.

The matrix $[A_i]$ has the following form:

$$[A_i] = \begin{bmatrix} -\sin\varphi_{Bi} & -\cos\varphi_{Bi} \\ \cos\varphi_{Bi} & -\sin\varphi_{Bi} \end{bmatrix}. \qquad (27)$$

In the figure 6 are shown free body diagrams for the first considered case.

For the second case, the body 3 does not participate in the conditions of static equilibrium, and in joint 2 of the body 1 unknown reactions are replaced with projections P_{3x} and P_{3y} of the measured force P_3. A half of hydrocylinder gravity force G_3 also should be added in vertical direction.

The conditions of static equilibrium are composed by application of equation (26) to each body for both cases. The solution of the obtained system of linear algebraic equations ((15) in the first case and (12) in the second case) gives the reactions in all joints and the searched components of the digging force *P*.

3. Numerical example

The developed theoretic-experimental approach is used for determination of the digging force in operating equipment of backhoe hydraulic excavator BEN 195. The digging process is realized with stick hydrocylinder by constant angle between the stick and the bucket. The hydrocylinder forces P_2 and P_3 are measured and presented (figure 7) as a function of the body 3 stroke (stick hydrocylinder, figure 6).

An interactive Mathcad program is developed for computation of digging force parameters based on the developed original algorithms for position and static force analysis. The digging force parame-ters are identified in real operating conditions. In the figure 8 are presented the digging force *P*, its components P_x and P_y, and also the ratio *f* between P_x and P_y. The cinematic scheme of the working equipment at the particular geometric configuration is visualized by means of MathCAD (figure 9).

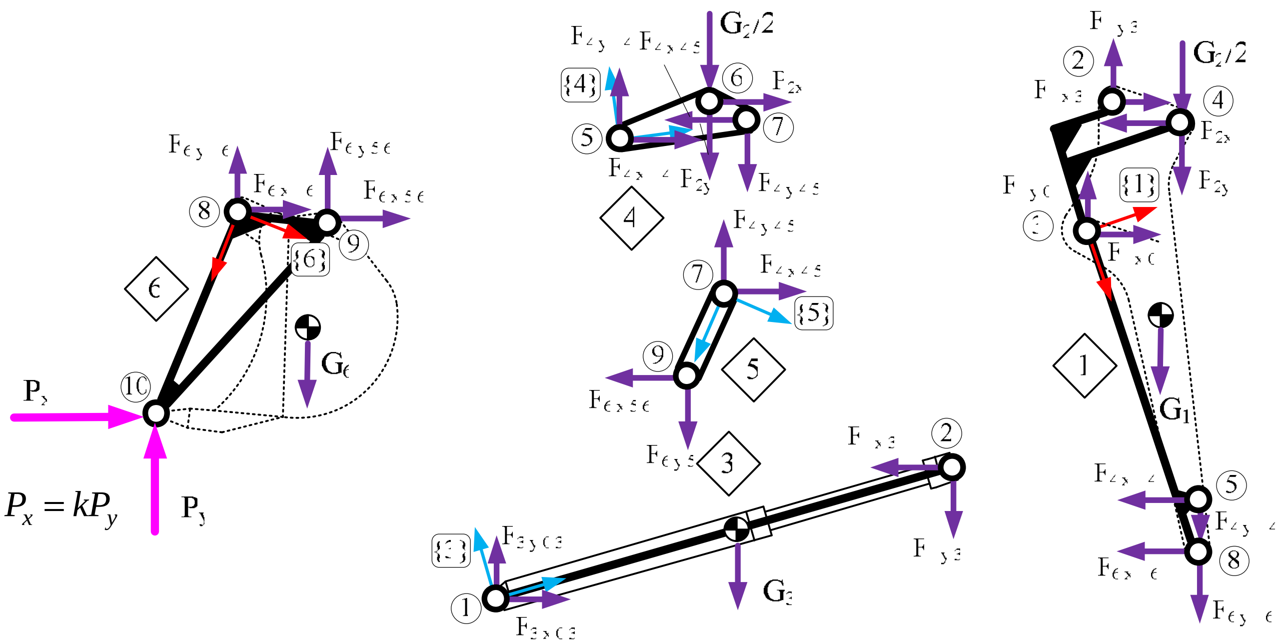


Figure 6. Free body diagrams for the first considered case

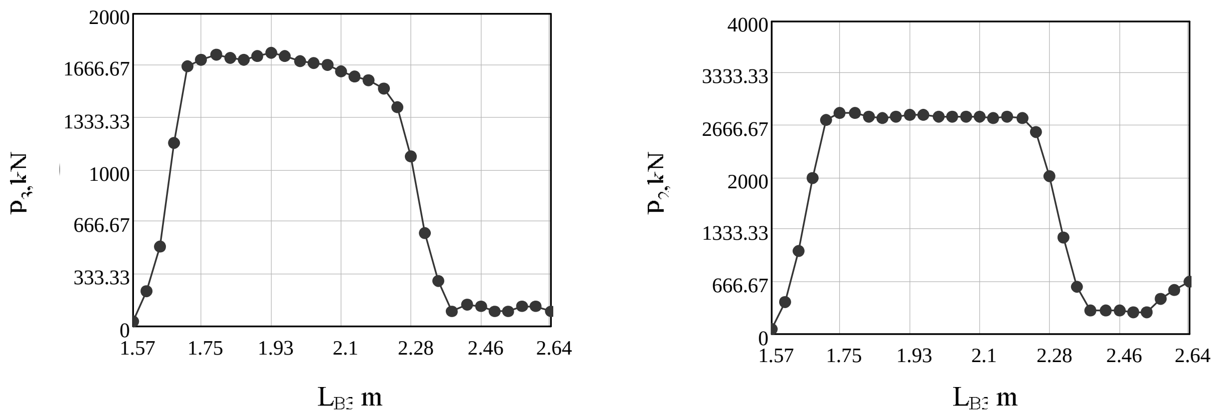


Figure 7. Experimentally measured forces P_3 and P_2

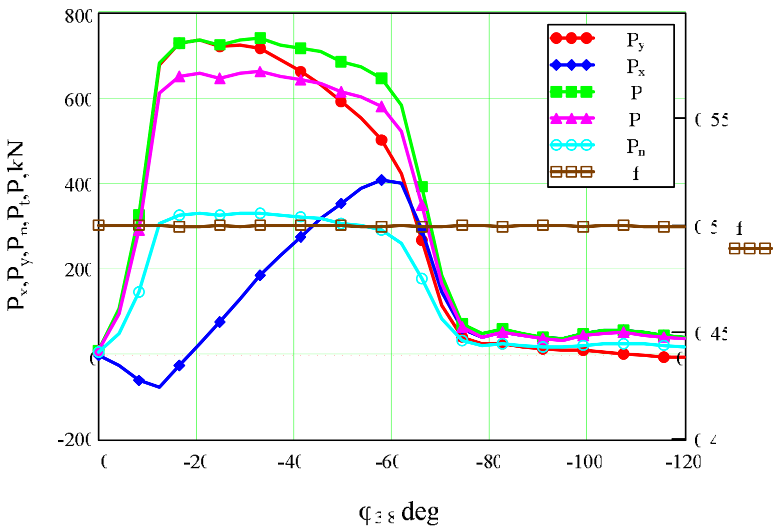


Figure 8. Graphical visualization of the cinematic scheme

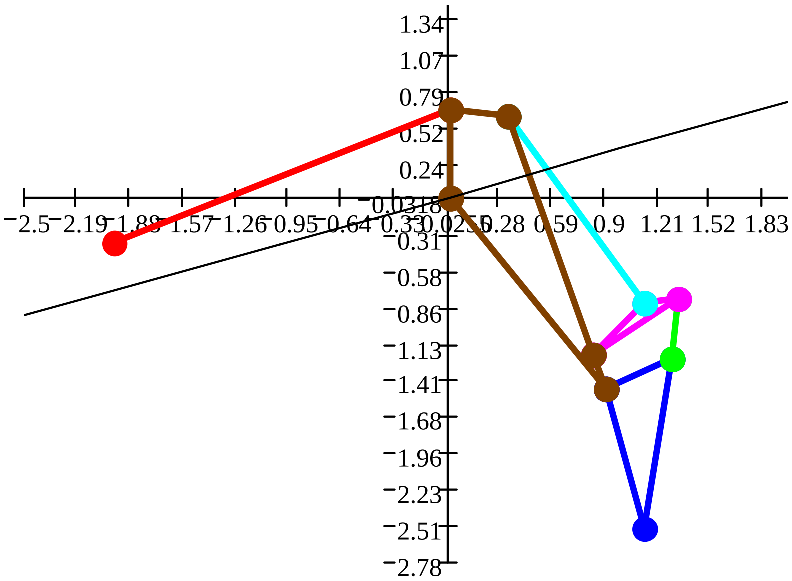


Figure 9. Digging force parameters

4. Conclusions

4.1. An easy to apply theoretic-experimental approach to computation of digging force in arbitrary chosen points from implemented tool trajectories of earthmoving machines is proposed.

4.2. The approach allows automatic collection of statistical information for digging forces and digging processes in real operating conditions.

4.3. The original algorithms for position and static force analysis of the backhoe hydraulic excavator are developed. These algorithms can be used independently.

4.4. An interactive MathCAD computer program that realizes the suggested algorithms is developed;

4.5. The developed theoretic-experimental approach is used for determination of the digging force parameters for hydraulic excavator BEN 195, operating in real conditions. The necessary pressures (forces) and strokes of bucket and stick hydrocylinders are measured.

References

1. Volkov, D., Krikun, V., Totolin, P., Gaewskaia, K., Nikulin, P.: *Excavating machines* (*Mashini dlia zemlianih rabot*). Moskva, Mashinostroenie, 1992 (in Russian)
2. He, J., Miedema, S., Vlasblom, W.: *FEM Analyses of Cutting of Anisotropic Densely Compacted and Saturated Sand*. WEDAXXV & TAMU37, New Orleans, USA, June 2005
3. Zhao, Y.: *The FEM calculation of pore water pressure in sand cutting process by SEPRAN*. MSc assignment, Delft University of Technology, Chair of Dredging Technology, Delft, Holland, 2000
4. Zelenin, A., Balovnev, V., Kerov, I.: *Excavating machines* (*Mashini dlia zemlianih rabot*). Moskva, Mashinostroenie, 1975 (in Russian)
5. Danchev, D., Hristov, D.: *Fundamentals of road and construction machines* (*Osnovi na patni i stroitelni mashini*). Tehnika, Sofia, Bulgaria, 1990 (in Bulgarian)
6. Balovnev, V.: *Modeling of rocks cutting* (*Fizicheskoe modelirovanie rezania gruntov*). Moskva, Mashinostroenie, 1969 (in Russian)
7. Hsin-Sheng, L., Shinn-Liang, C., Kuo-Huang, L.: *A study of the design, manufacture and remote control of a pneumatic excavator*. The International Journal of Mechanical Engineering Education, ISSN 0306-4190, Volume 32, Issue 4, October 2004, p. 345-362
8. Minchev, N., Zhivkov, V., Enchev, K., Stoianov, P.: *Theory of machines and mechanisms* (*Teoria na mashinite i mehanizmite*). Tehnika, Sofia, Bulgaria, 1991 (in Bulgarian)
9. Craig, J.: *Introduction to robotics. Mechanics and control*. Addison-Wesley Publishing Company, 1989
10. Koivo, A. *Kinematics of excavators (backhoes) for transferring surface material*. Journal of Aerospace Engineering, 7(1), 1994, p. 17–32
11. Hofstra, C., Hemmen, A., Miedema, S., Hulsteyn, J.: *Describing the position of backhoe dredges*. Texas A&M 32nd Annual Dredging Seminar. Warwick, Rhode Island, June 25-28, 2000
12. Janssen, B., Nievelstein, M.: *Multi-loop Linkage Dynamics via Geometric Methods: A Case Study on a RH200 Hydraulic Excavator*. 2005 Internship report, Eindhoven University of Technology Department of Mechanical Engineering Dynamics and Control Group. Available at: http://library.tue.nl/csp/dare/LinkToRepository.csp?recordnumber=612753. Accessed: 2008.03.04

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