SPATIAL DYNAMICAL MODELING
OF MULTI POSITIONAL VIBRATION SEPARATOR

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Abstract. Main purpose of this work is to evaluate the spatial dynamical behaviour of vibration separator. In this order is created and examined dynamical model representing the separator as one rigid body with six degrees of freedom, accounted are mass, geometrical, elastic and damping properties. Based on obtained differential equations of motion are performed numerical experiments of free and forced spatial vibrations. The results represented in time and frequency domain are aimed to evaluate the dynamical behaviour of separator, the character of oscillations of points from sifting surface, the relations between generalized coordinates, also to determine the natural frequencies of the system.

Keywords: spatial oscillations, dynamical modeling, vibration separator, FFT

1. Introduction

Vibration separator is intended to sift fine disperse granular materials. For that purpose the sifting surface, have to possess equal and rectilinear oscillation trajectories.

Shortly the construction of vibration separator (VS) (figure 1) consists from: sifting surface 1, carried from vibrating frame 2. The vibrating frame 2 is elastically coupled to foundation 4 through four identical leaf springs 3. Oscillations excitation is achieved by directed inertial disturbance owned to the opposite and synchronous spinning of two disbalanced shafts 7. The shafts 7 are symmetrically located about mass center (MC) of separator. They are set in motion by flexible coupling 5 and direct current electrical motor 6, fixed to foundation 4. The opposite and synchronous spinning of shafts 7 is provided from spur gear situated between them.

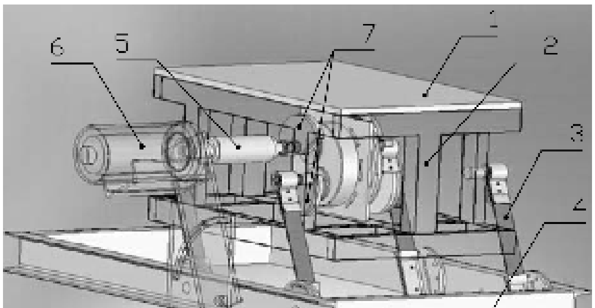

Figure 1. 3D SolidWorks model of vibrating separator

The main purpose of this work is: to study the free and forced spatial vibrations of VS; to determine the natural frequencies of the system; to reveal the relations between generalized coordinates; to evaluate the character of trajectories of points from sifting surface.

Available are many researches in spatial vibrations area of vibration separators, [1, 2] threat the problems of inertial excitation with self synchronized disbalanced shafts, [3] studies the dynamics of separators with electrical excitation. The so far provided dynamical models are oriented toward analytical solution.

In order to achieve the presented aims is built a dynamical model representing the separator as one rigid body with six degrees of freedom (figure 2).

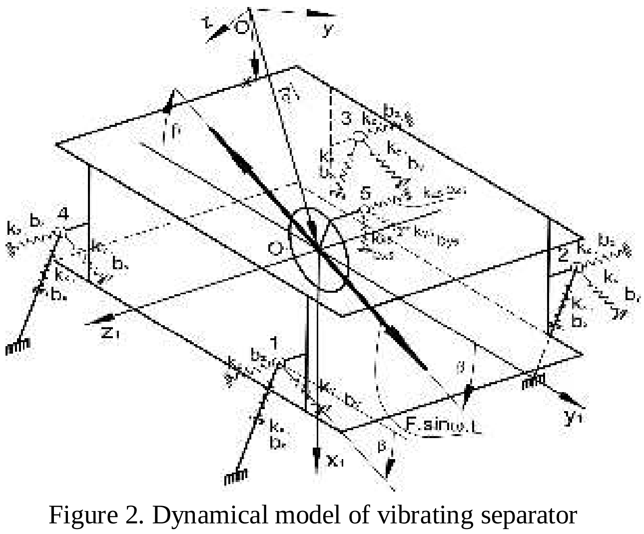

Figure 2. Dynamical model of vibrating separator

2. Assumptions and simplifications

The dynamical model is built for vicinity of small displacements around the static equilibrium position.

For this vicinity is adopted linear elastic characteristic of flexible elements and viscous damping. The mass of treated material is negligibly small with comparison of total vibrating mass and is not taken in to account.

The foundation and the vibrating frame are absolutely rigid. The inertial excitation resultant from spinning of the disbalanced shafts is replaced from directed sinusoidal force, concentrated in MC (point O_1) (figure 2) and inclined on angle β from sifting surface. In static equilibrium position, this force is set to be perpendicular to leaf spring longitudinal axis.

3. Designations and definitions

Defined are the following coordinate systems (CS) (figure 3 and figure 2): Oxyz – fixed (reference), O_1xyz – performs pure translation with axes parallel to the axes of Oxyz (point O_1 is positioned in the mass center of the vibrating frame), $O_1x_1y_1z_1$ – local (referent), it is invariably connected with vibrating frame and its axes are coincident with the principle axes of inertia. This CS performs relative rotation with respect to CS $O_1x_1y_1z_1$ and absolute motion with respect to CS Oxyz.

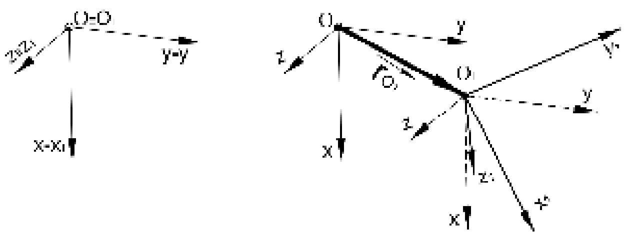

Figure 3. Coordinate systems

In the initial position (static equilibrium) the three coordinate systems are coincident (figure 3, a).

As generalized coordinates (GC) are adopted x, y, z coordinates of MC (point O_1) in the fixed CS Oxyz and ψ, θ, φ angles where the last ones represent the x-y-z Euler angles [4], figure 4, here they define rotations about the moving axes, aligned with the body.

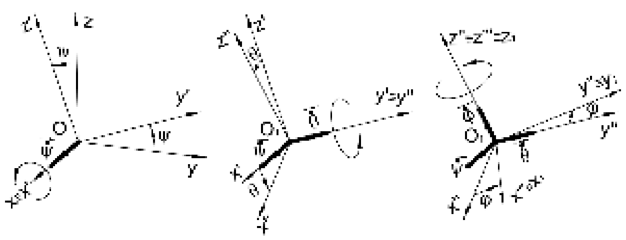

Figure 4. Euler angles and rotation sequence

4. Equations of motion

In order to derive the equations of motion the Lagrange's equations will be used equation (1):

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial \Pi}{\partial q_i}+\frac{\partial \Phi}{\partial \dot{q}_i}=Q_i\,; \quad i=1\div 6 \qquad (1)$$

$q=(x,y,z,\psi,\theta,\varphi)^T$ - GC vector.

4.1. Kinetic energy of the system *T*

By the Koenig's theorem for the full kinetic energy of the system can be written:

$$T=\frac{1}{2}\cdot M\cdot\left[\dot{x}^2+\dot{y}^2+\dot{z}^2\right]+ +\frac{1}{2}\cdot\left[Jx_1\cdot\omega_{x_1}^2+Jy_1\cdot\omega_{y_1}^2+Jz_1\cdot\omega_{z_1}^2\right] \qquad (2)$$

Here M is mass of the separator, Jx_1, Jy_1, Jz_1 are moments of inertia, taken at the center of mass and aligned with the principal axes of inertia, ω_{x_1}, ω_{y_1}, ω_{z_1} are projections of the full angular velocity ω (equation (3), [5]) on these axes (figure 4).

$$\omega=\begin{pmatrix}\omega_{x_1}\\ \omega_{y_1}\\ \omega_{z_1}\end{pmatrix}=R_{III}^T\cdot R_{II}^T\cdot\begin{pmatrix}\dot{\psi}\\ 0\\ 0\end{pmatrix}+R_{III}^T\cdot\begin{pmatrix}0\\ \dot{\theta}\\ 0\end{pmatrix}+\begin{pmatrix}0\\ 0\\ \dot{\varphi}\end{pmatrix}= =\begin{pmatrix}\dot{\psi}\cdot c\theta\cdot c\varphi+\dot{\theta}\cdot s\varphi\\ \dot{\theta}\cdot c\varphi-\dot{\psi}\cdot c\theta\cdot s\varphi\\ \dot{\varphi}+\dot{\psi}\cdot s\theta\end{pmatrix} \qquad (3)$$

R_I, R_{II}, R_{III} (equation (4)) are rotation matrixes (direction cosine matrix), [4], which represent the rotation around the moving axes (figure 4). The superscript *T* is symbol for transpose, *c* and *s* are substitutions of cosine and sine functions.

$$R_I=\begin{pmatrix}1 & 0 & 0\\ 0 & c\psi & -s\psi\\ 0 & s\psi & c\psi\end{pmatrix};\quad R_{II}=\begin{pmatrix}c\theta & 0 & s\theta\\ 0 & 1 & 0\\ -s\theta & 0 & c\theta\end{pmatrix};\quad R_{III}=\begin{pmatrix}c\varphi & -s\varphi & 0\\ s\varphi & c\varphi & 0\\ 0 & 0 & 1\end{pmatrix} \qquad (4)$$

In order to fully describe the orientation of frame $O_1x_1y_1z_1$ with respect to Oxyz the rotation matrix *R* (equation (5), [4]) is used.

$$R=R_I\cdot R_{II}\cdot R_{III}$$

$$R=\begin{pmatrix}c\theta\cdot c\varphi & -c\theta\cdot s\varphi & s\theta\\ s\psi\cdot s\theta\cdot c\varphi+c\psi\cdot s\varphi & c\psi\cdot c\varphi-s\psi\cdot s\theta\cdot s\varphi & -s\psi\cdot c\theta\\ s\psi\cdot s\varphi-c\psi\cdot s\theta\cdot c\varphi & c\psi\cdot s\theta\cdot s\varphi+s\psi\cdot c\varphi & c\psi\cdot c\theta\end{pmatrix} \qquad (5)$$

Assuming small oscillations (the Euler angles ψ, θ, φ remains in the interval of ±5°), the sine and cosine functions can be represented only with the first member in Tailor's decomposition [6]. Thus the rotation matrix R obtains the form equation (6):

$$R=\begin{pmatrix}1 & -\varphi & \theta\\ \varphi & 1 & -\psi\\ -\theta & \psi & 1\end{pmatrix} \qquad (6)$$

In these conditions and according to [3, 7] the full angular velocity ω acquires the following simplified form:

$$\omega=\begin{pmatrix}\omega_{x_1}\\ \omega_{y_1}\\ \omega_{z_1}\end{pmatrix}=\begin{pmatrix}\dot\psi+\dot\theta\cdot\varphi\\ \dot\theta-\dot\psi\cdot\varphi\\ \dot\varphi+\dot\psi\cdot\theta\end{pmatrix} \qquad (7)$$

Finally, after substituting equation (7) in equation (2) for the full kinetic energy of the system can be written:

$$T=\frac{1}{2}\cdot M\cdot\left[\dot x^2+\dot y^2+\dot z^2\right]+ \\ +\frac{1}{2}\cdot\left[Jx_1\cdot\left(\dot\psi+\dot\theta\cdot\varphi\right)^2+Jy_1\cdot\left(\dot\theta-\dot\psi\cdot\varphi\right)^2+Jz_1\cdot\left(\dot\varphi+\dot\psi\cdot\theta\right)^2\right] \qquad (8)$$

4.2. Potential energy of the system Π

The potential energy of the system is formed as a result from deformation of the flexible elements.

In this model, every flexible element (leaf springs and flexible coupling) is substituted from set of three mutually perpendicular linear springs and viscous dampers (figure 5), angular springs and dampers are neglected.

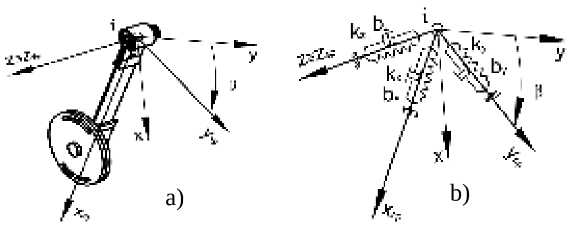

Figure 5. Flexible elements substitution and CS

The connection points (1 ÷ 4) (figure 2), (table 1) between the leaf springs and the vibrating frame are symmetrical about the MC (point O_1) and lies on the $O_1y_1z_1$ plane, but the connection point 5 (figures 2 and 7) of the flexible coupling is aside from point O_1. That's why in this model the mass center and the center of flexibility are not coincident.

Table 1.Coordinates of connection points

i	x_i	y_i	z_i
1	0	y_1	z_1
2	0	y_1	$-z1$
3	0	$-y_1$	$-z1$
4	0	$-y_1$	z_1
5	x_5	y_5	z_5

The stiffness and damping coefficients are assigned with k_{xi}, k_{yi}, k_{zi} and respectively b_{xi}, b_{yi}, b_{zi}.

The coefficients of the leaf springs are:

$$k_{x_1}=k_{x_2}=k_{x_3}=k_{x_4}=k_x$$
$$k_{y_1}=k_{y_2}=k_{y_3}=k_{y_4}=k_y$$
$$k_{z_1}=k_{z_2}=k_{z_3}=k_{z_4}=k_z$$

$$b_{x_1}=b_{x_2}=b_{x_3}=b_{x_4}=b_x$$
$$b_{y_1}=b_{y_2}=b_{y_3}=b_{y_4}=b_y$$
$$b_{z_1}=b_{z_2}=b_{z_3}=b_{z_4}=b_z$$

The coefficients of the flexible coupling are:

$$k_{x_5},k_{y_5},k_{z_5};\quad b_{x_5},b_{y_5},b_{z_5}.$$

Accordingly to [6] the deformation Δ (figure 6) of a spring can be represented as translation of its connection point. In order to represent this translation in the fixed CS Oxyz is used equation (9).

$$\vec\Delta_i=\vec r_{O1}+R\cdot\vec r_i-\vec r_i,\qquad i=1\div 5$$

$$\begin{pmatrix}u_i\\ v_i\\ w_i\end{pmatrix}=\begin{pmatrix}x\\ y\\ z\end{pmatrix}+\begin{pmatrix}1 & -\varphi & \theta\\ \varphi & 1 & -\psi\\ -\theta & \psi & 1\end{pmatrix}\cdot\begin{pmatrix}x_i\\ y_i\\ z_i\end{pmatrix}-\begin{pmatrix}x_i\\ y_i\\ z_i\end{pmatrix}= \\ =\begin{pmatrix}x-\varphi\cdot y_i+\theta\cdot z_i\\ y+\varphi\cdot x_i-\psi\cdot z_i\\ z-\theta\cdot x_i+\psi\cdot y_i\end{pmatrix} \qquad (9)$$

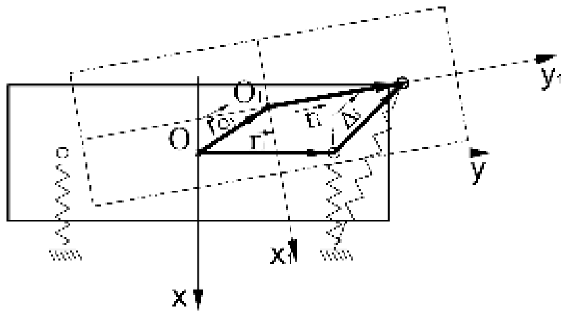

Figure 6. Springs deformation

Here i is a connection point number, u_i, v_i, w_i are projections of the deformation vector $\vec\Delta_i$ in CS Oxyz, $\vec r_{O1}$ - vector describing the position of p.O1 in CS Oxyz, $\vec r_i$ - vector describing the connection point position in CS $O_1x_1y_1z_1$, this vector is time independent.

The stiffness and damping coefficients of the leaf springs are known in CS $ix_{sp}y_{sp}z_{sp}$ (figures 5 and 7). This CS is coincident with the principle axes of inertia of the spring cross section. That's why for convenience the deformation is represented in CS $Ox_{sp}y_{sp}z_{sp}$ which is rotated on β about Oz axis (equation (10)).

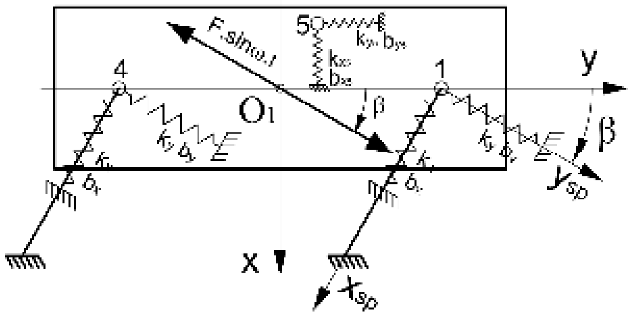

Figure 7. Dynamical model in view from z axis

$$\vec{\Delta}i_{sp} = R_\beta^T \cdot \vec{\Delta}_i\,, \quad i = 1 \div 4$$

$$\begin{pmatrix} u_i^{sp} \\ v_i^{sp} \\ w_i^{sp} \end{pmatrix} = \begin{pmatrix} c\beta & s\beta & 0 \\ -s\beta & c\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} = \qquad (10)$$

$$= \begin{pmatrix} c\beta\cdot(x-\varphi\cdot y_i+\theta\cdot z_i)+s\beta\cdot(y+\varphi\cdot x_i-\psi\cdot z_i) \\ -s\beta\cdot(x-\varphi\cdot y_i+\theta\cdot z_i)+c\beta\cdot(y+\varphi\cdot x_i-\psi\cdot z_i) \\ z-\theta\cdot x_i+\psi\cdot y_i \end{pmatrix}$$

$$R_\beta = \begin{pmatrix} c\beta & -s\beta & 0 \\ s\beta & c\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (11)$$

Finally, for the potential energy of the system can be written:

$$\begin{aligned}
\Pi = &\frac{1}{2}\cdot\left(k_x\cdot\sum_{i=1}^{4}\left(u_i^{sp}\right)^2+k_y\cdot\sum_{i=1}^{4}\left(v_i^{sp}\right)^2+k_z\cdot\sum_{i=1}^{4}\left(w_i^{sp}\right)^2\right)+\\
&+\frac{1}{2}\cdot\left(k_{x_5}\cdot u_5^2+k_{y_5}\cdot v_5^2+k_{z_5}\cdot w_5^2\right)=\\
&=2\cdot\left(k_x\cdot c^2\beta+k_y\cdot s^2\beta\right)\cdot x^2+4\cdot\left(k_x-k_y\right)\cdot x\cdot y\cdot c\beta\cdot s\beta+\\
&+2\cdot\left(k_x\cdot s^2\beta+k_y\cdot c^2\beta\right)\cdot y^2+2\cdot k_z\cdot z^2+\\
&+2\cdot k_x\cdot z_1^2\cdot\theta^2\cdot c^2\beta+2\cdot k_y\cdot z_1^2\cdot\theta^2\cdot s^2\beta+\\
&+2\cdot k_x\cdot y_1^2\cdot\varphi^2\cdot c^2\beta+2\cdot k_y\cdot y_1^2\cdot\varphi^2\cdot s^2\beta-\\
&-4\cdot k_x\cdot z_1^2\cdot\theta\cdot\psi\cdot c\beta\cdot s\beta+4\cdot k_y\cdot z_1^2\cdot\theta\cdot\psi\cdot c\beta\cdot s\beta+\\
&+2\cdot k_z\cdot y_1^2\cdot\psi^2+2\cdot k_y\cdot z_1^2\cdot\psi^2\cdot c^2\beta+\\
&+2\cdot k_x\cdot z_1^2\cdot\psi^2\cdot s^2\beta+\\
&+\frac{1}{2}\cdot\begin{pmatrix} k_{x_5}\cdot(x-\varphi\cdot y_5+\theta\cdot z_5)^2+k_{y_5}\cdot(y+\varphi\cdot x_5-\psi\cdot z_5)^2+ \\ +k_{z_5}\cdot(z-\theta\cdot x_5+\psi\cdot y_5)^2 \end{pmatrix}
\end{aligned} \qquad (12)$$

4.3. Dissipative energy of the system Φ

According to [3, 6] the velocities of the connection points can be driven from deformation after differentiation about time. So for the full dissipative energy of the system can be written:

$$\begin{aligned}
\Phi = &\frac{1}{2}\left(b_x\cdot\sum_{i=1}^{4}\left(\frac{du_i'}{dt}\right)^2+b_y\cdot\sum_{i=1}^{4}\left(\frac{dv_i'}{dt}\right)^2+b_z\cdot\sum_{i=1}^{4}\left(\frac{dw_i'}{dt}\right)^2\right)+\\
&+\frac{1}{2}\left(b_{x_5}\cdot\left(\frac{du_{i5}}{dt}\right)^2+b_{y_5}\cdot\left(\frac{dv_{i5}}{dt}\right)^2+b_{z_5}\cdot\left(\frac{dw_{i5}}{dt}\right)^2\right)=\\
&=2\cdot\left(b_x\cdot c^2\beta+b_y\cdot s^2\beta\right)\cdot x+4\cdot\left(b_x-b_y\right)\cdot x\cdot y\cdot c\beta\cdot s\beta+\\
&+2\cdot\left(b_x\cdot s^2\beta+b_y\cdot c^2\beta\right)\cdot\dot{y}^2+2\cdot b_z\cdot\dot{z}^2+2\cdot b_x\cdot z_1^2\cdot\dot{\theta}^2\cdot c^2\beta+\\
&+2\cdot b_y\cdot z_1^2\cdot\dot{\theta}^2\cdot s^2\beta+2\cdot b_x\cdot y_1^2\cdot\dot{\varphi}^2\cdot c^2\beta+\\
&+2\cdot b_y\cdot y_1^2\cdot\dot{\varphi}^2\cdot s^2\beta-4\cdot b_x\cdot z_1^2\cdot\dot{\theta}\cdot\dot{\psi}\cdot c\beta\cdot s\beta+\\
&+4\cdot b_y\cdot z_1^2\cdot\dot{\theta}\cdot\dot{\psi}\cdot c\beta\cdot s\beta+2\cdot b_z\cdot y_1^2\cdot\dot{\psi}^2+\\
&+2\cdot b_y\cdot z_1^2\cdot\dot{\psi}^2\cdot c^{\beta}+2\cdot b_x\cdot z_1^2\cdot\dot{\psi}^2\cdot s^2\beta+\\
&+\frac{1}{2}\begin{pmatrix} b_{x_5}\cdot(\dot{x}-\dot{\varphi}\cdot y_5+\dot{\theta}\cdot z_5)^2+b_{y_5}\cdot(\dot{y}+\dot{\varphi}\cdot x_5-\dot{\psi}\cdot x_5)^2+ \\ +b_{z_5}\cdot(\dot{z}-\dot{\theta}\cdot x_5+\dot{\psi}\cdot y_5)^2 \end{pmatrix}
\end{aligned} \qquad (13)$$

4.4. Excitation Q

The excitation vector *Q* is built from the inertial force projections over the GC. This inertial force is result from spinning of the unbalanced shafts with angular velocity ω, it lies in $O_1x_1y_1$ plane, also is constant by direction in CS $O_1x_1y_1z_1$ and is applied at point O_1. The direction of this force is displaced on angle β (β = const) from O_1y_1 axis (figures 2 and 7). In this model is assumed that the spinning of the unbalanced shafts causes no moments about the MC point O_1. So for the inertial force ($P(t)$) and its projections in CS $O_1x_1y_1z_1$ can be written:

$$P(t) = \begin{pmatrix} F\cdot\sin(\omega\cdot t)\cdot\sin\beta \\ F\cdot\sin(\omega\cdot t)\cdot\cos\beta \\ 0 \end{pmatrix}; \quad F = m_d\cdot e\cdot\omega^2 \qquad (14)$$

Here m_d is unbalanced mass of the rotating shafts, *e* is eccentricity, ω is angular velocity.

Finally, in order to represent the excitation vector *Q* is used equation (15):

$$\begin{pmatrix} Q_x \\ Q_y \\ Q_z \end{pmatrix} = \begin{pmatrix} 1 & -\varphi & \theta \\ \varphi & 1 & -\psi \\ -\theta & \psi & 1 \end{pmatrix} \cdot \begin{pmatrix} F\cdot\sin(\omega\cdot t)\cdot\sin\beta \\ F\cdot\sin(\omega\cdot t)\cdot\cos\beta \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} Q_\psi \\ Q_\theta \\ Q_\varphi \end{pmatrix} = 0 \qquad (15)$$

After substituting equations (8), (12), (13) and (15) in (1), and using Mathematica programming environment, the system differential equations (16) that describe this dynamical model is obtained:

$$
\begin{aligned}
&M\cdot\ddot{x}+b_{x_5}\cdot\dot{x}+4\cdot\dot{x}\cdot\left(b_x\cdot c^2\beta+b_y\cdot s^2\beta\right)+4\cdot\dot{y}\cdot\left(b_x-b_y\right)\cdot c\beta\cdot s\beta+b_{x_5}\cdot\dot{\theta}\cdot z_5-b_{x_5}\cdot\dot{\varphi}\cdot y_5+\\
&\quad+4\cdot x\cdot\left(k_x\cdot c^2\beta+k_y\cdot s^2\beta\right)+k_{x_5}\cdot x+4\cdot y\cdot\left(k_x-k_y\right)\cdot c\beta\cdot s\beta+k_{x_5}\cdot\theta\cdot z_5-k_{x_5}\cdot\varphi\cdot y_5=Q_x\\
&M\cdot\ddot{y}+4\cdot\dot{x}\cdot\left(b_x-b_y\right)\cdot c\beta\cdot s\beta+4\cdot\dot{y}\cdot\left(b_x\cdot s^2\beta+b_y\cdot c^2\beta\right)+b_{y_5}\cdot\dot{y}-b_{y_5}\cdot\dot{\psi}\cdot z_5+b_{y_5}\cdot\dot{\varphi}\cdot x_5+\\
&\quad+4\cdot x\cdot\left(k_x-k_y\right)\cdot c\beta\cdot s\beta+4\cdot y\cdot\left(k_y\cdot c^2\beta+k_x\cdot s^2\beta\right)+k_{y_5}\cdot y-k_{y_5}\cdot\psi\cdot z_5+k_{y_5}\cdot\varphi\cdot x_5=Q_y\\
&M\cdot\ddot{z}+\left(4\cdot b_z+b_{z_5}\right)\cdot\dot{z}+b_{z_5}\cdot\dot{\psi}\cdot y_5-b_{z_5}\cdot\dot{\theta}\cdot x_5+\left(4\cdot k_z+k_{z_5}\right)\cdot z+k_{z_5}\cdot\psi\cdot y_5-k_{z_5}\cdot\theta\cdot x_5=Q_z\\
&\left(Jx_1+Jy_1\cdot\varphi^2+Jz_1\cdot\theta^2\right)\cdot\ddot{\psi}+\left(Jx_1-Jy_1\right)\cdot\ddot{\theta}\cdot\varphi+Jz_1\cdot\ddot{\varphi}\cdot\theta+2\cdot Jy\cdot\dot{\psi}\cdot\dot{\varphi}\cdot\varphi+\left(Jx_1-Jy_1+Jz_1\right)\cdot\dot{\theta}\cdot\dot{\varphi}+\\
&\quad+2\cdot Jz_1\cdot\dot{\psi}\cdot\dot{\theta}\cdot\theta+4\cdot b_z\cdot\dot{\psi}\cdot y_1^2+b_{z_5}\cdot\dot{\psi}\cdot y_5^2+b_{y_5}\cdot\dot{\psi}\cdot z_5^2-b_{y_5}\cdot\dot{y}\cdot z_5+b_{z_5}\cdot\dot{z}\cdot y_5+\\
&\quad+4\cdot\dot{\psi}\cdot z_1^2\cdot\left(b_x\cdot s^2\beta+b_y\cdot c^2\beta\right)+4\cdot\dot{\theta}\cdot z_1^2\cdot\left(-b_x+b_y\right)\cdot c\beta\cdot s\beta-b_{z_5}\cdot\dot{\theta}\cdot x_5\cdot y_5-b_{y_5}\cdot\dot{\varphi}\cdot x_5\cdot z_5-\\
&\quad-k_{y_5}\cdot y\cdot z_5+k_{z_5}\cdot z\cdot y_5+4\cdot\psi\cdot z_1^2\cdot\left(k_x\cdot s^2\beta+k_y\cdot c^2\beta\right)+4\cdot k_z\cdot\psi\cdot y_1^2+k_{y_5}\cdot\psi\cdot z_5^2+k_{z_5}\cdot\psi\cdot y_5^2+\\
&\quad+4\cdot\theta\cdot z_1^2\cdot\left(-k_x+k_y\right)\cdot c\beta\cdot s\beta-k_{z_5}\cdot\theta\cdot x_5\cdot y_5-k_{y_5}\cdot\varphi\cdot x_5\cdot z_5=0\\
&Jy_1\cdot\ddot{\theta}+Jx_1\cdot\ddot{\theta}\cdot\varphi^2+\left(Jx_1-Jy_1\right)\cdot\ddot{\psi}\cdot\varphi+\left(Jx_1-Jy_1-Jz_1\right)\cdot\dot{\psi}\cdot\dot{\varphi}-Jz_1\cdot\dot{\psi}^2\cdot\theta+2\cdot Jx_1\cdot\dot{\theta}\cdot\dot{\varphi}\cdot\varphi+\\
&\quad+b_{x_5}\cdot\dot{x}\cdot z_5-b_{z_5}\cdot\dot{z}\cdot x_5+4\cdot\dot{\psi}\cdot z_1^2\cdot\left(-b_x+b_y\right)\cdot c\beta\cdot s\beta-b_{z_5}\cdot\dot{\psi}\cdot x_5\cdot y_5+4\cdot\dot{\theta}\cdot z_1^2\cdot\left(b_x\cdot c^2\beta+b_y\cdot s^2\beta\right)+\\
&\quad+b_{x_5}\cdot\dot{\theta}\cdot z_5^2-b_{x_5}\cdot\dot{\varphi}\cdot y_5\cdot z_5+b_{z_5}\cdot\dot{\theta}\cdot x_5^2+k_{x_5}\cdot x\cdot z_5-k_{z_5}\cdot z\cdot x_5+4\cdot\psi\cdot z_1^2\cdot\left(-k_x+k_y\right)\cdot c\beta\cdot s\beta-\\
&\quad-k_{z_5}\cdot\psi\cdot x_5\cdot y_5+4\cdot\theta\cdot z_1^2\cdot\left(k_x\cdot c^2\beta+k_y\cdot s^2\beta\right)+k_{x_5}\cdot\theta\cdot z_5^2+k_{z_5}\cdot\theta\cdot x_5^2-k_{x_5}\cdot\varphi\cdot y_5\cdot z_5=0\\
&Jz_1\cdot\ddot{\varphi}+Jz_1\cdot\ddot{\psi}\cdot\theta+\left(-Jx_1+Jy_1+Jz_1\right)\cdot\dot{\psi}\cdot\dot{\theta}-Jy_1\cdot\dot{\psi}^2\cdot\varphi-Jx_1\cdot\dot{\theta}^2\cdot\varphi+4\cdot\dot{\varphi}\cdot y_1^2\cdot\left(b_x\cdot c^2\beta+b_y\cdot s^2\beta\right)-\\
&\quad-b_{x_5}\cdot\dot{x}\cdot y_5+b_{y_5}\cdot\dot{y}\cdot x_5^2-b_{y_5}\cdot\dot{\psi}\cdot x_5\cdot z_5-b_{x_5}\cdot\dot{\theta}\cdot y_5\cdot z_5+b_{x_5}\cdot\dot{\varphi}\cdot y_5^2+b_{y_5}\cdot\dot{\varphi}\cdot x_5^2-k_{x_5}\cdot x\cdot y_5+\\
&\quad+k_{y_5}\cdot y\cdot x_5-k_{y_5}\cdot\psi\cdot x_5\cdot z_5-k_{x_5}\cdot\theta\cdot y_5\cdot z_5+4\cdot\varphi\cdot y_1^2\cdot\left(k_x\cdot c^2\beta+k_y\cdot s^2\beta\right)+k_{x_5}\cdot\varphi\cdot y_5^2+k_{y_5}\cdot\varphi\cdot x_5^2=0
\end{aligned}
\tag{15}
$$

The complex structure and the non linear relations in the system differential equations (16) make the searching of analytical solution too complex task. So the programming environment of MATLAB is used to obtain numerical solution.

5. Physical parameters of the separator

The mass and geometrical parameters are driven after modeling in SolidWorks.

M = 67.1 [kg]	k_x = 882143.2 [N/m]
m_d = 9.1375 [kg]	k_y = 4801.0 [N/m]
e = 0 ÷ 30 [mm]	k_z = 189460.3 [N/m]
Jx_1 = 2.97 [kg·m^2]	k_{x5} = 9996.7 [N/m]
Jy_1 = 1.77 [kg·m^2]	k_{y5} = 9996.7 [N/m]
Jz_1 = 2.89 [kg·m^2]	k_{z5} = 54699.9 [N/m]
b_x = 273.5 [N·s/m]	x_1 = 0
b_y = 63.5 [N·s/m]	y_1 = 235 [mm]
b_z = 77.0 [N·s/m]	z_1 = 290 [mm]
b_{x5} = 126.4 [N·s/m]	$x_5 = -r_c\cdot\cos\beta$
b_{y5} = 126.4 [N·s/m]	$y_5 = r_c\cdot\cos\beta$
b_{z5} = 87.4 [N·s/m]	z_5 = – 320 [mm]
β = 0º ÷ 90º	r_c = 50 [mm]

The stiffness and damping coefficients are experimentally measured from the real flexible elements (leaf springs and flexible coupling).

6. Results

The results from numerical solution are time domain representation of acceleration, velocity and displacement of the separator's body over the GC. Also after applying the Fast Fourier transformation (FFt) are obtained and frequency domain characteristics.

All results are given in graphical form. Because of the considerable number of graphics which are provided from this model here are presented only graphics for displacement and its single sided amplitude spectrum. Also the angle β is set to β = 0°. In this position the vector of excitation is coincident with axis O_1y_1.

6.1. Free vibrations

The free vibrations solution is achieved after applying initial displacement 50 mm on GC y. Time domain results are shown on figure 8.

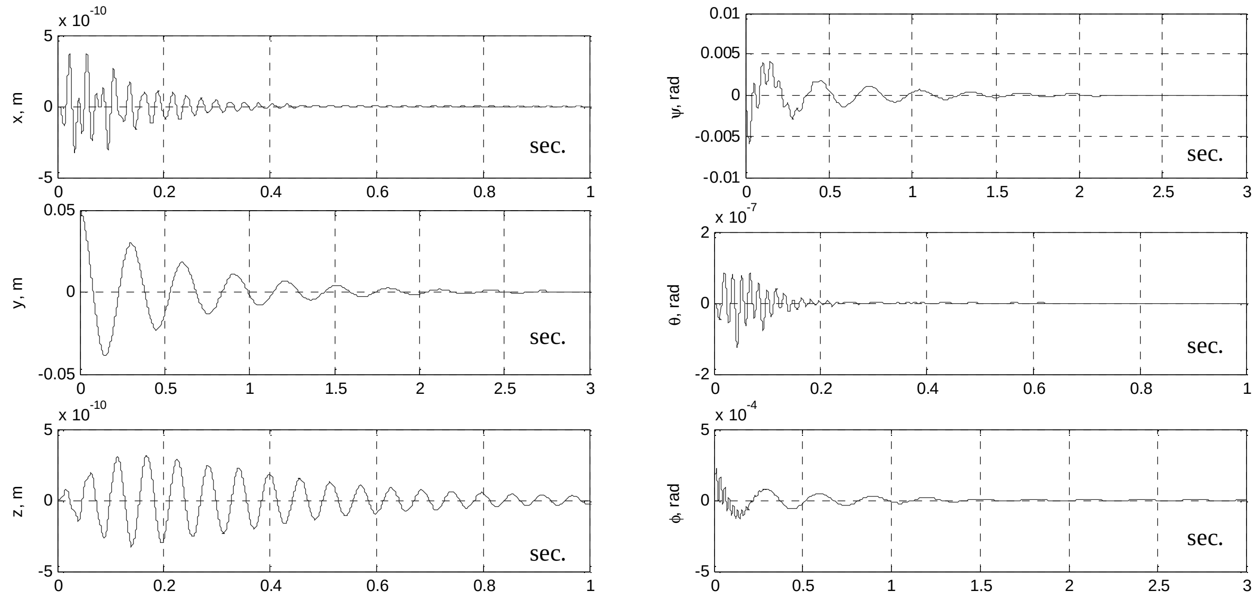

Figure 8. Free vibrations in time domain

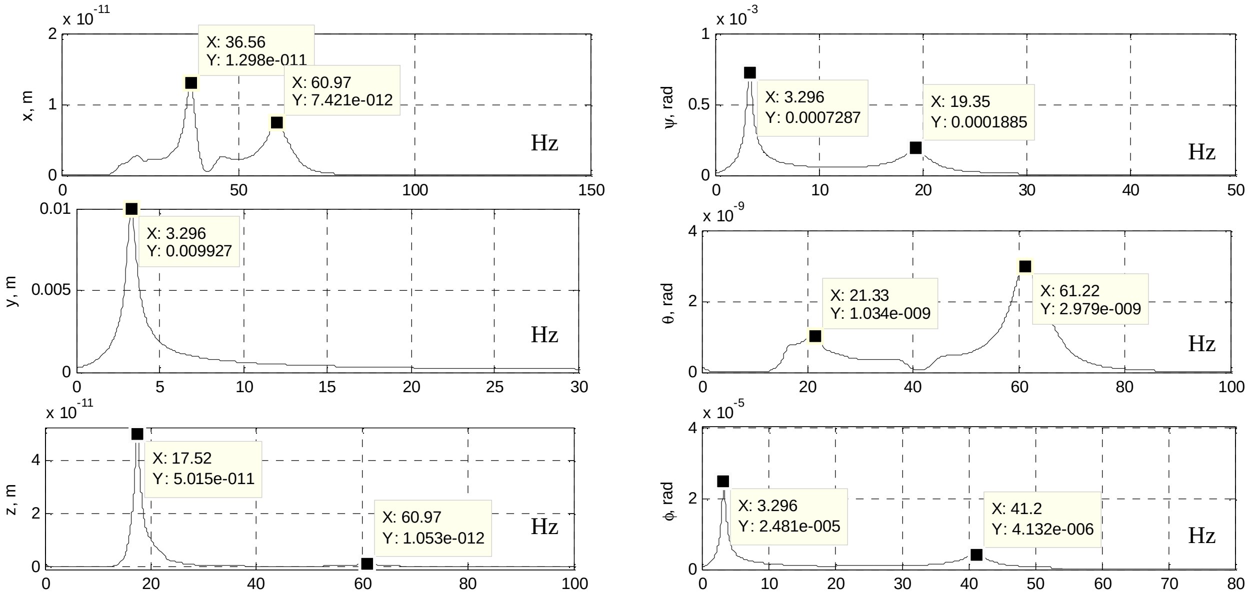

Figure 9. Free vibrations in frequency domain
(Single sided amplitude spectrum)

On figure 9 which presents single sided amplitude spectrum are denoted natural frequencies of the system.

6.2. Forced vibrations

Forced vibrations (figures 10 and 11) are received under the following conditions:

m_d = 9.14 [kg]; e = 9.2 [mm]; ω = 104.72 [rad/s].

7. Conclusions

One mass and six degrees of freedom nonlinear dynamical model is created and examined.

The natural frequencies and the frequency response of the system under forced excitation are studied. The nonlinear but weak relations between GC y and the other GC are obvious.

From figure 11 is possible to evaluate that considerable forced oscillations are available only by GC y. This allows the conclusion that under forced vibrations the trajectory of the sifting surface is steady and rectilinear.

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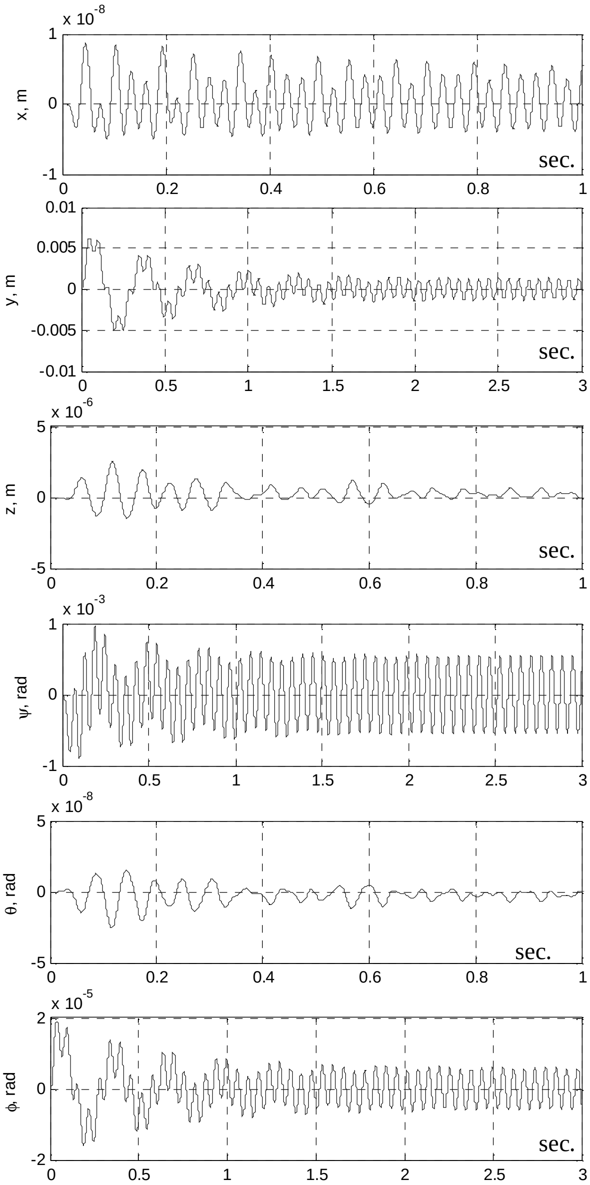

Figure 10. Forced vibrations in time domain

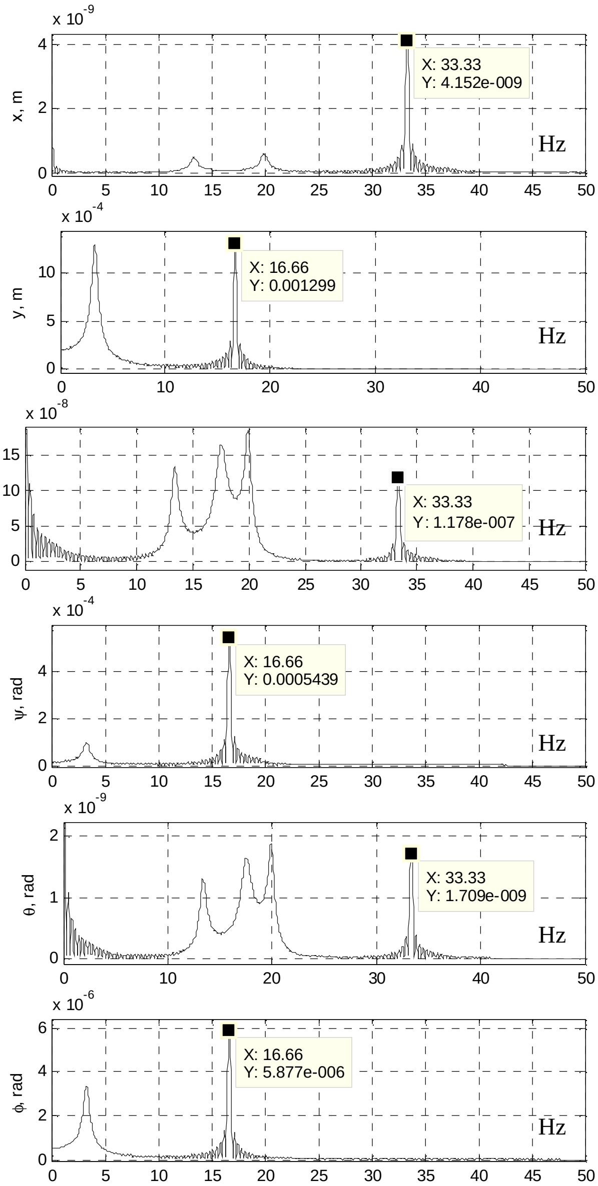

Figure 11. Forced vibrations in frequency domain (Single sided amplitude spectrum)

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