DYNAMIC MODELING OF A VIBRATING SEPARATOR WITH INERTIAL EXCITATION CONSIDERING GYROSCOPIC EFFECTS

Dragomir VRAZHILSKI
Technical University – Sofia, Bulgaria

Abstract. Subject of this work is examination of vibrating separator dynamical behaviour under directed inertial force excitation provided from synchronous spinning of two unbalanced shafts. For that purpose a dynamical model representing the separator as system of three rigid bodies and seven degrees of freedom is created, gyroscopic couples resultant from unbalanced shafts spinning are accounted. With this model are performed numerical experiments for area of small forced displacements around the static equilibrium position. Time and frequency domain characteristics are obtained.

Keywords: spatial oscillations, dynamical modeling, vibrating separator, FFT, gyroscopic effects

1. Introduction

Vibrating separator is intended to sift fine disperse granular materials by non perforated sifting surface. For the purpose of experimental examination of sifting process each point from the sifting surface has to possess equal, rectilinear and steady oscillation trajectories.

As was mentioned in [1] the construction of vibrating separator (VS) (Figure 1) consists from: sifting surface 1, carried from vibrating frame (VF) 2. The VF 2 is elastically coupled to foundation 4 through four identical leaf springs 3. Oscillations excitation is achieved by directed inertial force (F) owned to the opposite and synchronous spinning of two unbalanced shafts (DS) 7. The shafts 7 are symmetrically located about mass center (MC) of separator. They are set in motion by flexible coupling 5 and direct current electrical motor 6, fixed to foundation 4. The opposite and synchronous spinning of shafts 7 is provided from spur gearing situated between them.

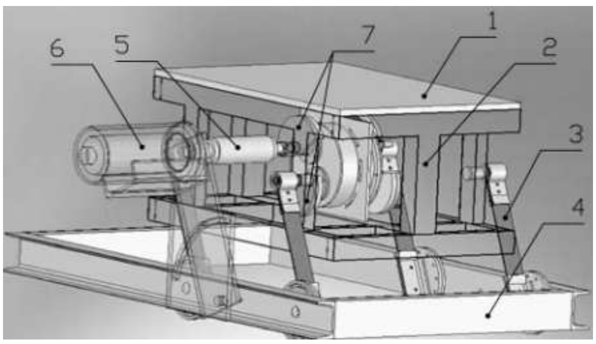

Figure 1. 3D SolidWorks model of Vibrating Separator

Main purposes of this work: to build dynamical model of VS considering DS as a separate body, rotating with respect to the VF. To study the forced spatial vibrations of VS provoked from spinning of DS; to reveal relations between generalized coordinates; to evaluate the sifting surface trajectories.

Similar dynamical model of VS is developed in [1] but the excitation resultant from the DS spinning is decreased to directed sinusoidal force, the gyroscopic moments are neglected. That necessitates creation of more close to the real object dynamical model.

In order to achieve the presented aims is built a dynamical model representing the separator as system of three rigid bodies with seven degrees of freedom (DOF) (Figure 2).

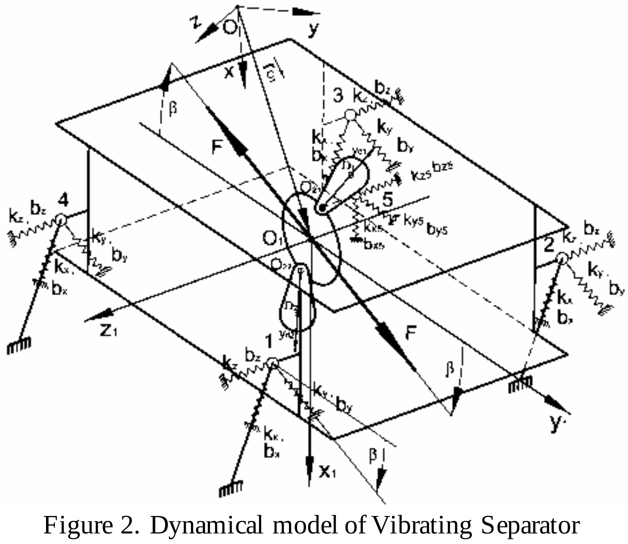

Figure 2. Dynamical model of Vibrating Separator

2. Assumptions and simplifications

The dynamical model is built for vicinity of small displacements around the position of static equilibrium. For this vicinity a linear elastic characteristic of flexible elements and viscous damping is adopted. The mass of treated material is small in comparison to the total vibrating mass, hence being neglected. The foundation and the VF are absolutely rigid. The inertial excitation force F is always inclined on β with respect to the sifting surface. In static position this force is perpendicular to the leaf springs longitudinal axes.

3. Designations and definitions

Defined are the following coordinate systems (CS) (Figures 2 and 3): Oxyz – fixed (reference), O_1xyz – performs pure translation with axes parallel to the axes of Oxyz (point O_1 is positioned at the mass center of the VF), $O_1x_1y_1z_1$ – local (referent), it is invariably connected with VF and its axes are coincident with the principle axes of inertia. This CS performs relative rotation with respect to CS O_1xyz and absolute motion with respect to CS Oxyz. At initial position (static equilibrium) the three coordinate systems are coincident. Also defined are $D_1x_{d1}y_{d1}z_{d1}$ and $D_2x_{d2}y_{d2}z_{d2}$ aligned with principle axes of inertia of the two unbalanced shafts. $O_{21}x_1y_1z_1$ and $O_{22}x_1y_1z_1$ these CS are parallel to $O_1x_1y_1z_1$. The axes $O_{21}z_1$ and $O_{22}z_1$ are rotation axes of unbalanced shafts (Figure 3). At initial position CS $D_1x_{d1}y_{d1}z_{d1}$ and $D_2x_{d2}y_{d2}z_{d2}$ are rotated on angle β about $O_1x_1y_1z_1$. This angle denotes the inclination of inertial excitation force F with respect to the sifting surface.

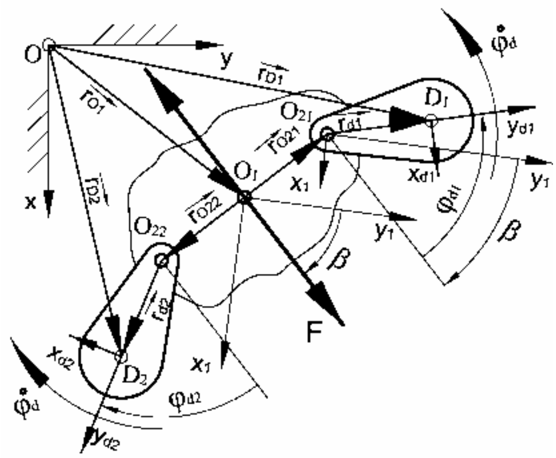

Figure 3. Coordinate systems and position vectors

As generalized coordinates (GC) are adopted:

x, y, z - coordinates of MC (point O_1 or p.O_1) in the fixed CS Oxyz;

ψ, θ, φ - Eulerian angles [2], here they denote rotations about the moving axes, aligned with the VF, the adopted sequence of rotation is x-y-z [1];

φ_d – this GC denotes the rotation angle of the two DS about axes $O_{21}z_1$ and $O_{22}z_1$. Actually they represent two different bodies but because of the spur gearing with only one degree of freedom with respect to the VF. Here φ_d is the rotation angle of $D_1x_{d1}y_{d1}z_{d1}$ with respect to $O_{21}x_1y_1z_1$.

4. Equations of motion

In order to derive the equations of motion the Lagrange's equations are used:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial \dot{q}_i}+\frac{\partial \Pi}{\partial q_i}=Q_i; \quad i=1\div 7 \qquad (1)$$

$q=(x,y,z,\psi,\theta,\varphi,\psi_d)^T$ - GC vector

The superscript T is symbol for transpose.

4.1. Kinetic energy of the system T

$$T=T_{O1}+T_{D1}+T_{D2} \qquad (2)$$

T_{O1} - Kinetic energy of VF

T_{D1}, T_{D2} - Kinetic energy of first respectively second unbalanced shaft.

$$\begin{aligned}
T_{O1}&=\frac{1}{2}M\left(V_{O1x}^2+V_{O1y}^2+V_{O1z}^2\right)+\\
&+\frac{1}{2}\left(Jx_1\cdot\omega_{x1}^2+Jy_1\cdot\omega_{y1}^2+Jz_1\cdot\omega_{z1}^2\right)\\
T_{D1}&=\frac{1}{2}M_d\left(V_{D1x}^2+V_{D1y}^2+V_{D1z}^2\right)+\\
&+\frac{1}{2}\left(Jx_d\cdot\omega_{Dx1}^2+Jy_d\cdot\omega_{Dy1}^2+Jz_d\cdot\omega_{Dz1}^2\right)\\
T_{D2}&=\frac{1}{2}M_d\left(V_{D2x}^2+V_{D2y}^2+V_{D2z}^2\right)+\\
&+\frac{1}{2}\left(Jx_d\cdot\omega_{Dx2}^2+Jy_d\cdot\omega_{Dy2}^2+Jz_d\cdot\omega_{Dz2}^2\right)
\end{aligned} \qquad (3)$$

M and M_d are mass of VF and mass of unbalanced shaft. Jx_1, Jy_1, Jz_1 and Jx_d, Jy_d, Jz_d are moments of inertia of VF and of unbalanced shaft, taken at the center of mass and aligned with the principal axes of inertia. V_{O1}, V_{D1}, V_{D2} are linear velocities of p.O_1, respectively p.D_1 and p.D_2. The subscripts x, y, z denotes their projections on the fixed frame (Oxyz). ω is the full angular velocity of VF, ω_{D1} and ω_{D2} are full angular velocities of the unbalanced shafts. The subscripts x, y, z means that they are projected on the principle axes of inertia.

By [3] for the linear velocity of p.O_1 can be written:

$$\vec{V}_{O1}=\frac{d\vec{r}_{O1}}{dt}=\sum_{i=1}^{7}\frac{d\vec{r}_{O1}}{\partial q_i}\cdot\dot{q}_i=(\dot{x},\dot{y},\dot{z})^T \qquad (4)$$

$\vec{r}_{O1}=(x,y,z)^T$ - Vector describing position of p.O_1 in Oxyz CS - figure 3.

The same expression is used for p.D_1 and p.D_2:

$$\vec{V}_{D1}=\frac{d\vec{r}_{D1}}{dt}=\sum_{i=1}^{7}\frac{d\vec{r}_{D1}}{\partial q_i}\dot{q}_i;\ \vec{V}_{D2}=\sum_{i=1}^{7}\frac{d\vec{r}_{D2}}{\partial q_i}\dot{q}_i \qquad (5)$$

$\vec{r}_{D1}, \vec{r}_{D2}$ - Vectors describing the position of p.D_1 and p.D_2 in Oxyz CS (Figure 3).

$$\begin{aligned} \vec{r}_{D1} &= \vec{r}_{O1} + R\left(\vec{r}_{O21} + R_{\varphi d1} \cdot \vec{r}_{d1}\right) \\ \vec{r}_{D2} &= \vec{r}_{O1} + R\left(\vec{r}_{O22} + R_{\varphi d2} \cdot \vec{r}_{d2}\right) \end{aligned} \tag{6}$$

$\vec{r}_{O21}, \vec{r}_{O22}$ - Vectors describing position of p.O_{21} and p.O_{22} in $O_1x_1y_1z_1$. In this CS they are constants and represent geometrical dimensions.

$$\begin{aligned} \vec{r}_{O21} &= (x_{21}, y_{21}, z_{21},) = const \\ \vec{r}_{O22} &= (x_{22}, y_{22}, z_{22},) = const \end{aligned} \tag{7}$$

$\vec{r}_{d1}, \vec{r}_{d2}$ - Vectors describing position of p.D_1 and p.D_2 in $O_{21}x_{d1}y_{d1}z_{d1}$ and $O_{22}x_{d2}y_{d2}z_{d2}$. These vectors are equal by size. The distance y_d is the eccentricity of unbalanced shafts. For simplicity the current CS are not shown on Figure 3. Their axes are parallel to $D_1x_{d1}y_{d1}z_{d1}$ respectively $D_2x_{d2}y_{d2}z_{d2}$.

$$\vec{r}_{d1} = \vec{r}_{d2} = (0, y_d, 0)^T \tag{8}$$

R - Rotation matrix (direction cosine matrix) [1, 2]. Fully describes the orientation of $O_1x_1y_1z_1$ with respect to Oxyz.

Assuming small oscillations (the Euler angles ψ, θ, φ remains in the interval of ±5°), the sine and cosine functions can be represented only with the first member in Tailor's decomposition [4].

$$R = \begin{pmatrix} 1 & -\varphi & \theta \\ \varphi & 1 & -\psi \\ \theta & \psi & 1 \end{pmatrix} \tag{9}$$

$R_{\varphi d1}, R_{\varphi d2}$ - Rotation matrixes describing the rotation of the two DS around the axes $O_{21}z_1$ and $O_{22}z_1$. These matrixes fully describe the orientation of $D_1x_{d1}y_{d1}z_{d1}$ and $D_2x_{d2}y_{d2}z_{d2}$ with respect to $O_1x_1y_1z_1$. Because the shafts are spinning in opposite directions here is substituted:

$$\varphi_d = \varphi_{d1} = -\varphi_{d2}$$

$$\begin{aligned} R_{\varphi d1} &= \begin{pmatrix} c(\varphi_d + \beta) & -s(\varphi_d + \beta) & 0 \\ s(\varphi_d + \beta) & c(\varphi_d + \beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ R_{\varphi d2} &= \begin{pmatrix} c(-\varphi_d + \beta) & -s(-\varphi_d + \beta) & 0 \\ s(-\varphi_d + \beta) & c(-\varphi_d + \beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \tag{10}$$

The symbols c and s are substitutions of cosine and sine functions.

After substitutions and simplifications is achieved:

$$\begin{aligned} \vec{r}_{D1} &= \begin{pmatrix} x + x_{21} - \varphi . y_{21} + \theta . z_{21} - \begin{pmatrix} s(\varphi_d + \beta) + \\ + \varphi \cdot c(\varphi_d + \beta) \end{pmatrix} y_d \\ y + \varphi . x_{21} + y_{21} - \psi . z_{21} + \begin{pmatrix} c(\varphi_d + \beta) - \\ - \varphi \cdot s(\varphi_d + \beta) \end{pmatrix} y_d \\ z - \theta . x_{21} + \psi . y_{21} + z_{21} + \begin{pmatrix} \theta \cdot s(\varphi_d + \beta) + \\ + \psi \cdot c(\varphi_d + \beta) \end{pmatrix} y_d \end{pmatrix} \\ \vec{r}_{D2} &= \begin{pmatrix} x + x_{22} - \varphi \cdot y_{22} + \theta \cdot z_{22} - \begin{pmatrix} s(\beta - \varphi_d) + \\ + \varphi \cdot c(\beta - \varphi_d) \end{pmatrix} y_d \\ y + \varphi \cdot x_{22} + y_{22} - \psi \cdot z_{22} + \begin{pmatrix} c(\beta - \varphi_d) - \\ - \varphi \cdot s(\beta - \varphi_d) \end{pmatrix} y_d \\ z - \theta \cdot x_{22} + \psi \cdot y_{22} + z_{22} + \begin{pmatrix} \theta \cdot s(\beta - \varphi_d) + \\ + \psi \cdot c(\beta - \varphi_d) \end{pmatrix} y_d \end{pmatrix} \end{aligned} \tag{11}$$

Consequently the linear velocities of p.D_1 and p.D_2 are obtained in function of GC:

$$\vec{V}_{D1} = \begin{pmatrix} \dot{x} - \dot{\varphi} \cdot y_{21} + \dot{\theta} \cdot z_{21} - \begin{pmatrix} \dot{\varphi}_d \cdot c(\varphi_d + \beta) + \\ + \dot{\varphi} \cdot c(\varphi_d + \beta) - \\ - \varphi \cdot \dot{\varphi}_d \cdot s(\varphi_d + \beta) \end{pmatrix} y_d \\ \dot{y} + \dot{\varphi} \cdot x_{21} - \dot{\psi} \cdot z_{21} - \begin{pmatrix} \dot{\varphi} \cdot s(\varphi_d + \beta) + \\ + \dot{\varphi}_d \cdot s(\varphi_d + \beta) \\ + \dot{\varphi} \cdot \dot{\varphi}_d \cdot c(\varphi_d + \beta) \end{pmatrix} y_d \\ \dot{z} - \dot{\theta} \cdot x_{21} + \dot{\psi} \cdot y_{21} + \begin{pmatrix} \dot{\theta} \cdot s(\varphi_d + \beta) + \\ + \theta \cdot \dot{\varphi}_d \cdot c(\varphi_d + \beta) + \\ + \dot{\psi} \cdot c(\varphi_d + \beta) - \\ - \psi \cdot \dot{\varphi}_d \cdot s(\varphi_d + \beta) \end{pmatrix} y_d \end{pmatrix} \tag{12}$$

Acquisition of expression for $\vec{V}_{D2}$ is analogical.

Accordingly to [4, 5] the full angular velocity of VF ω is:

$$\omega = \begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \end{pmatrix} = \begin{pmatrix} \dot{\psi} \cdot c\theta \cdot c\varphi + \dot{\theta} \cdot s\varphi \\ \dot{\theta} \cdot c\varphi - \dot{\psi} \cdot c\theta \cdot s\varphi \\ \dot{\varphi} + \dot{\psi} \cdot s\theta \end{pmatrix} \tag{13}$$

But if the simplified form of rotation matrix R Eq. (9) is adopted, the expression for the angular velocity ω (according to [4]) is allowable to be decreased to the following simplified form:

$$\omega=\begin{pmatrix}\omega_{x1}\\ \omega_{y1}\\ \omega_{z1}\end{pmatrix}=\begin{pmatrix}\dot\psi\\ \dot\theta\\ \dot\varphi\end{pmatrix} \qquad (14)$$

The full angular velocities of unbalanced shafts, projected on their principle axes of inertia are Eqs. (15).

$$\vec{\omega}_{D1}=R^T_{\varphi d1}\,\vec{\omega}+\vec{\omega}_{d1}=\begin{pmatrix}\dot\psi\cdot c(\varphi_d+\beta)+\dot\theta\cdot s(\varphi_d+\beta)\\ -\dot\psi\cdot s(\varphi_d+\beta)+\dot\theta\cdot c(\varphi_d+\beta)\\ \dot\varphi+\dot\varphi_d\end{pmatrix} \qquad (15a)$$

$$\vec{\omega}_{D2}=R^T_{\varphi d2}\,\vec{\omega}+\vec{\omega}_{d2}=\begin{pmatrix}\dot\psi\cdot c(\beta-\varphi_d)+\dot\theta\cdot s(\beta-\varphi_d)\\ -\dot\psi\cdot s(\beta-\varphi_d)+\dot\theta\cdot c(\beta-\varphi_d)\\ \dot\varphi-\dot\varphi_d\end{pmatrix} \qquad (15b)$$

$\vec{\omega}_{d1},\vec{\omega}_{d2}$ - Relative angular velocities of DS with respect to VF.

$$\vec{\omega}_{d1}=\begin{pmatrix}0, & 0, & \dot\varphi_d\end{pmatrix}^T;\ \vec{\omega}_{d2}=\begin{pmatrix}0, & 0, & -\dot\varphi_d\end{pmatrix}^T \qquad (16)$$

Finally after substitutions for the full kinetic energy of the system is written Eq. (17).

$$\begin{aligned}
T_{O1}=&\frac{1}{2}M\left(\dot x^2+\dot y^2+\dot z^2\right)+\frac{1}{2}\left(Jx_1\cdot\dot\psi^2+Jy_1\cdot\dot\theta^2+Jz_1\cdot\dot\varphi^2\right)+\frac{1}{2}M_d\left(\left(\dot x-\dot\varphi\cdot y_{21}+\dot\theta\cdot z_{21}-\begin{pmatrix}\dot\varphi_d\cdot c(\varphi_d+\beta)+\\ +\dot\varphi\, c(\varphi_d+\beta)-\\ -\varphi\cdot\dot\varphi_d\cdot s(\varphi_d+\beta)\end{pmatrix}y_d\right)^2+\right.\\
&+\left(\dot y+\dot\varphi\cdot x_{21}-\dot\psi\cdot z_{21}-\begin{pmatrix}\dot\varphi\cdot s(\varphi_d+\beta)+\\ +\dot\varphi_d\cdot s(\varphi_d+\beta)+\\ +\varphi\cdot\dot\varphi_d\cdot c(\varphi_d+\beta)\end{pmatrix}y_d\right)^2\left.+\left(\dot z-\dot\theta\cdot x_{21}+\dot\psi\cdot y_{21}+\begin{pmatrix}\dot\theta\, s(\varphi_d+\beta)+\theta\cdot\dot\varphi_d\cdot c(\varphi_d+\beta)+\\ +\dot\psi\cdot c(\varphi_d+\beta)-\psi\cdot\dot\varphi_d\cdot s(\varphi_d+\beta)\end{pmatrix}y_d\right)^2\right)+\\
&+\frac{1}{2}\left(Jx_d\left(\dot\psi.c(\varphi_d+\beta)+\dot\theta.s(\varphi_d+\beta)\right)^2+Jy_d\left(-\dot\psi\cdot s(\varphi_d+\beta)+\dot\theta\cdot c(\varphi_d+\beta)\right)^2+Jz_d\left(\dot\varphi+\dot\varphi_d\right)^2\right)+\\
&+\frac{1}{2}M_d\left(\left(\dot x-\dot\varphi\cdot y_{22}+\dot\theta\cdot z_{22}-\begin{pmatrix}-\dot\varphi_d\cdot c(\beta-\varphi_d)+\\ +\dot\varphi\, c(\beta-\varphi_d)+\\ +\varphi\cdot\dot\varphi_d\cdot s(\beta-\varphi_d)\end{pmatrix}y_d\right)^2+\left(\dot y+\dot\varphi\cdot x_{22}-\dot\psi\cdot z_{22}+\begin{pmatrix}-\dot\varphi\cdot s(\beta-\varphi_d)+\\ +\dot\varphi_d\cdot s(\beta-\varphi_d)+\\ +\varphi\cdot\dot\varphi_d\cdot c(\beta-\varphi_d)\end{pmatrix}y_d\right)^2+\right.\\
&\left.+\left(\dot z-\dot\theta\cdot x_{22}+\dot\psi\cdot y_{22}+\left(\dot\theta\cdot s(\beta-\varphi_d)+\theta\cdot\dot\varphi_d\cdot c(\beta-\varphi_d)+\dot\psi\cdot c(\beta-\varphi_d)-\psi\cdot\dot\varphi_d\cdot s(\beta-\varphi_d)\right)y_d\right)^2\right)+\\
&+\frac{1}{2}\left(Jx_d\left(\dot\psi\cdot c(\beta-\varphi_d)+\dot\theta\cdot s(\beta-\varphi_d)\right)^2+Jy_d\left(-\dot\psi\cdot s(\beta-\varphi_d)+\dot\theta\cdot c(\beta-\varphi_d)\right)^2+Jz_d\left(\dot\varphi-\dot\varphi_d\right)^2\right)
\end{aligned} \qquad (17)$$

4.2. Potential energy of the system Π

The potential energy of the system is formed from deformation of the flexible elements and from the weight of DS and its relative vertical displacement about p.O_{21} and p.O_{22}.

$$\Pi=\Pi_1+\Pi_{D1}+\Pi_{D2} \qquad (18)$$

Π_1 - Potential energy from deformation of all flexible elements. In this model every flexible element (leaf springs and flexible coupling) is substituted from set of three mutually perpendicular linear springs and viscous dampers, angular springs and dampers are neglected.

Π_1 is absolutely equal with the full potential energy obtained in [1] and the derivation won't be repeated here.

Π_{D1},Π_{D2} - Potential energy from unbalanced shafts. Here g is earth acceleration, r_{d1x}, r_{d2x} are projections of vectors $\vec{r}_{d1},\vec{r}_{d2}$ on the axes O_{21}x and

$O_{22}x$. Finally for the potential energy of the system is obtained:

$$\begin{aligned}
\Pi_{D1} &= -M_d \cdot g \cdot r_{d1x} = M_d \cdot g \cdot [s(\varphi_d + \beta) + \varphi \cdot c(\varphi_d + \beta)] \cdot y_d \\
\Pi_{D2} &= -M_d \cdot g \cdot r_{d2x} = M_d \cdot g \cdot [s(\beta - \varphi_d) + \varphi \cdot c(\beta - \varphi_d)] \cdot y_d \\
\Pi_{D1} + \Pi_{D2} &= 2M_d \cdot g \cdot y_d \cdot c\varphi_d \cdot (s\beta + \varphi \cdot c\beta)
\end{aligned} \tag{20}$$

$$\begin{aligned}
\Pi = &\frac{1}{2}\left(k_x \sum_{i=1}^{4} \left(u_i^{sp}\right)^2 + k_y \sum_{i=1}^{4} \left(v_i^{sp}\right)^2 + k_z \sum_{i=1}^{4} \left(w_i^{sp}\right)^2 \right) + \frac{1}{2}\left(k_{x5} \cdot u_5^2 + k_{y5} \cdot v_5^2 + k_{z5} \cdot w_5^2\right) + \Pi_{D1} + \Pi_{D2} = \\
&= 2\left(k_x \cdot c^2\beta + k_y \cdot s^2\beta\right)x^2 + 4\left(k_x - k_y\right)x \cdot y \cdot c\beta \cdot s\beta + 2\left(k_x \cdot s^2\beta + k_y \cdot c^2\beta\right)y^2 + 2k_z z^2 + \\
&+ 2k_x \cdot z_1^2 \cdot \theta^2 \cdot c^2\beta + 2k_y \cdot z_1^2 \cdot \theta^2 \cdot s^2\beta + 2k_x \cdot y_1^2 \cdot \varphi^2 \cdot c^2\beta + 2k_y \cdot y_1^2 \cdot \theta^2 \cdot s^2\beta - 4k_x \cdot z_1^2 \cdot \theta \cdot \psi \cdot c\beta \cdot s\beta + \\
&+ 4k_y \cdot z_1^2 \cdot \theta \cdot \psi \cdot c\beta \cdot s\beta + 2k_z \cdot y_1^2 \cdot \psi^2 + 2k_y \cdot z_1^2 \cdot \psi^2 \cdot c^2\beta + 2k_x \cdot z_1^2 \cdot \psi^2 \cdot s^2\beta + \\
&+ \frac{1}{2}\left(k_{x5}(x - \varphi \cdot y_5 + \theta \cdot z_5)^2 + k_{y5}(y + \varphi \cdot x_5 - \psi \cdot z_5)^2 + k_{z5}(z - \theta \cdot x_5 + \psi \cdot y_5)^2\right) + \\
&+ 2M_d \cdot g \cdot y_d \cdot c\varphi_d \cdot (s\beta + \varphi \cdot c\beta)
\end{aligned} \tag{21}$$

Here i is a connection point number, $u_i^{sp}, v_i^{sp}, w_i^{sp}$ are projections of the deformation vector $\vec{\Delta}_i$ in CS $Ox_{sp}y_{sp}z_{sp}$ which is rotated on β about Oz axis, u_5, v_5, w_5 are deformations of flexible coupling in CS Oxyz.

The stiffness and damping coefficients of the leaf springs are:

$k_{x1} = k_{x2} = k_{x3} = k_{x4} = k_x \quad b_{x1} = b_{x2} = b_{x3} = b_{x4} = b_x$

$k_{y1} = k_{y2} = k_{y3} = k_{y4} = k_y \quad b_{y1} = b_{y2} = b_{y3} = b_{y4} = b_y$

$k_{z1} = k_{z2} = k_{z3} = k_{z4} = k_z \quad b_{z1} = b_{z2} = b_{z3} = b_{z4} = b_z$

The coefficients of the flexible coupling are: k_{x5}, k_{y5}, k_{z5}; b_{x5}, b_{y5}, b_{z5}

4.3. Dissipative energy of the system Φ

According to [4] the velocities of the connection points can be driven from deformation after differentiation about time. So for the full dissipative energy of the system can be written:

$$\begin{aligned}
\Phi = &\frac{1}{2}\left(b_x \sum_{i=1}^{4} \left(\frac{du_i'}{dt}\right)^2 + b_y \sum_{i=1}^{4} \left(\frac{dv_i'}{dt}\right)^2 + b_z \sum_{i=1}^{4} \left(\frac{dw_i'}{dt}\right)^2 \right) + \frac{1}{2}\left(b_{x5}\left(\frac{du_5'}{dt}\right)^2 + b_{y5}\left(\frac{dv_5'}{dt}\right)^2 + b_{z5}\left(\frac{dw_5'}{dt}\right)^2 \right) = \\
&= 2\left(b_x \cdot c^2\beta + b_y \cdot s^2\beta\right)\dot{x}^2 + 4\left(b_x - b_y\right)\dot{x} \cdot \dot{y} \cdot c\beta \cdot s\beta + 2\left(b_x \cdot s^2\beta + b_y \cdot c^2\beta\right)\dot{y}^2 + 2b_z \cdot \dot{z}^2 + \\
&+ 2b_x \cdot z_1^2 \cdot \dot{\theta}^2 \cdot c^2\beta + 2b_y \cdot z_1^2 \cdot \dot{\theta}^2 \cdot s^2\beta + 2b_x \cdot y_1^2 \cdot \dot{\varphi}^2 \cdot c^2\beta + 2b_y \cdot y_1^2 \cdot \dot{\varphi}^2 \cdot s^2\beta + \\
&+ 4\left(b_y - b_x\right)\dot{\theta} \cdot \dot{\psi} \cdot c\beta \cdot s\beta + 2b_z \cdot y_1^2 \cdot \dot{\psi}^2 + 2b_y \cdot z_1^2 \cdot \dot{\psi}^2 \cdot c^2\beta + 2b_x \cdot z_1^2 \cdot \dot{\psi}^2 \cdot s^2\beta + \\
&+ \frac{1}{2}\left(b_{x5}\left(\dot{x} - \dot{\varphi} \cdot y_5 + \dot{\theta} \cdot z_5\right)^2 + b_{y5}\left(\dot{y} - \dot{\varphi} \cdot x_5 + \dot{\psi} \cdot z_5\right)^2 + b_{z5}\left(\dot{z} - \dot{\theta} \cdot x_5 + \dot{\psi} \cdot z_5\right)^2 \right)
\end{aligned} \tag{22}$$

4.4. Excitation Q

External excitation is available only by GC φ_d. This is the torque T provided from the DC electrical motor with approximately linear torque–speed characteristic.

$$T = T \cdot s - \frac{T \cdot s}{\omega} \cdot \dot{\varphi}_d \tag{23}$$

Ts is stall torque, ω is required working angular velocity.

After substituting Eqs. (17), (21), (22) and (23) in (1), the system differential Eqs. (24) that describes this dynamical model is obtained.

Because of quantitative reasons here in Eqs. (24) are shown equations only for GC x, y, z and φ_d.

All symbolical operations are performed in MATHEMATICA, after that the dynamical model is transferred to MATLAB and than numerical solution is obtained.

$$
\begin{aligned}
&(M+2M_d)\ddot{x}+(z_{21}+z_{22})M_d\cdot\ddot{\theta}-(y_{21}+y_{22}+2\cdot c\beta\cdot c\varphi_d\cdot y_d)M_d\cdot\ddot{\varphi}+2(s\beta+\varphi\cdot s\beta)\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\ddot{\varphi}+\\
&\quad+2(s\beta+\varphi\cdot c\beta)\cdot c\varphi_d\cdot M_d\cdot y_d\cdot\dot{\varphi}_d^{\,2}+4\cdot c\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\dot{\varphi}\cdot\dot{\varphi}_d+\left(b_{x5}+4b_x\cdot c^2\beta+4b_y\cdot s^2\beta\right)\dot{x}+\\
&\quad+4\left(b_x-b_y\right)\cdot c\beta\cdot s\beta\cdot\dot{y}+b_{x5}\cdot z_5\cdot\dot{\theta}-b_{x5}\cdot y_5\cdot\dot{\varphi}+\left(4k_x\cdot c^2\beta+4k_y\cdot s^2\beta+k_{x5}\right)x+\\
&\quad+\left(4k_x-4k_y\right)\cdot c\beta\cdot s\beta\cdot y+k_{x5}\cdot z_5\cdot\theta-k_{x5}\cdot y_5\cdot\varphi=0
\end{aligned}
$$

$$
\begin{aligned}
&(M+2M_d)\ddot{y}-(z_{21}+z_{22})M_d\cdot\ddot{\psi}-(x_{21}+x_{22}-2\cdot s\beta\cdot c\varphi_d\cdot y_d)M_d\cdot\ddot{\varphi}+2(\varphi\cdot s\beta-c\beta)\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\ddot{\varphi}_d+\\
&\quad+2(\varphi\cdot s\beta-c\beta)\cdot c\varphi_d\cdot M_d\cdot y_d\cdot\dot{\varphi}_d^{\,2}+4\cdot s\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\dot{\varphi}\cdot\dot{\varphi}_d+\left(b_{y5}+4b_y\cdot c^2\beta+4b_x\cdot s^2\beta\right)\dot{y}+\\
&\quad+4\left(b_x-b_y\right)\cdot c\beta\cdot s\beta\cdot\dot{x}-b_{y5}\cdot z_5\cdot\dot{\psi}+b_{y5}\cdot x_5\cdot\dot{\varphi}+\left(4k_x\cdot s^2\beta+4k_y\cdot c^2\beta+k_{x5}\right)y+\left(4k_x-4k_y\right)\cdot c\beta\cdot s\beta\cdot x-\\
&\quad-k_{y5}\cdot z_5\cdot\psi+k_{y5}\cdot x_5\cdot\varphi=0
\end{aligned}
$$

$$
\begin{aligned}
&(M+2M_d)\ddot{z}+(y_{21}+y_{22}+2\cdot c\beta\cdot c\varphi_d\cdot y_d)M_d\cdot\ddot{\psi}+(-x_{21}-x_{22}+2\cdot s\beta\cdot c\varphi_d\cdot y_d)M_d\cdot\ddot{\theta}-\\
&\quad-2(\psi\cdot c\beta+\theta\cdot s\beta)\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\ddot{\varphi}_d-2(\psi\cdot c\beta+\theta\cdot s\beta)\cdot c\varphi_d\cdot M_d\cdot y_d\cdot\dot{\varphi}_d^{\,2}-\\
&\quad-4(\dot{\psi}\cdot c\beta+\dot{\theta}\cdot s\beta)\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\dot{\varphi}_d+4(b_z-b_{z5})\cdot\dot{z}+b_{z5}\cdot y_5\cdot\dot{\psi}-b_{z5}\cdot x_5\cdot\dot{\theta}+\\
&\quad+(4k_z+4k_{z5})\cdot z+k_{z5}\cdot y_5\cdot\psi-k_{z5}\cdot x_5\cdot\theta=0
\end{aligned}
$$

$$
\begin{aligned}
&2\cdot s\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\ddot{x}+2\cdot c\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\varphi\cdot\ddot{x}-2\cdot c\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\ddot{y}+2\cdot s\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\varphi\cdot\ddot{y}-\\
&\quad-2\cdot s\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\theta\cdot\ddot{z}-2\cdot c\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\psi\cdot\ddot{z}+s(\varphi_d-\beta)\cdot M_d\cdot z_{21}\cdot y_d\cdot\varphi\cdot\ddot{\psi}-\\
&\quad-c(\varphi_d+\beta)\cdot M_d\cdot z_{22}\cdot y_d\cdot\varphi\cdot\ddot{\psi}-(c\beta\cdot c\varphi_d-s\beta\cdot s\varphi_d)\cdot M_d\cdot z_{21}\cdot y_d\cdot\ddot{\theta}+(c\beta\cdot c\varphi_d+s\beta\cdot s\varphi_d)\cdot\\
&\quad\cdot M_d\cdot z_{22}\cdot y_d\cdot\ddot{\theta}+(s\beta\cdot c\varphi_d+c\beta\cdot s\varphi_d)\cdot M_d\cdot x_{21}\cdot y_d\cdot\psi\cdot\ddot{\theta}-(s\beta\cdot c\varphi_d-c\beta\cdot s\varphi_d)\cdot M_d\cdot x_{22}\cdot y_d\cdot\psi\cdot\ddot{\theta}-\\
&\quad-4\cdot c\beta\cdot s\beta\cdot c\varphi_d\cdot s\varphi_d\cdot M_d\cdot y_d^2\cdot\psi\cdot\ddot{\theta}-(c\beta\cdot c\varphi_d+s\beta\cdot s\varphi_d)\cdot M_d\cdot x_{21}\cdot y_d\cdot\theta\cdot\ddot{\theta}+(c\beta\cdot c\varphi_d+s\beta\cdot s\varphi_d)\cdot\\
&\quad\cdot M_d\cdot x_{22}\cdot y_d\cdot\theta\cdot\ddot{\theta}+2\left(c^2\beta-s^2\beta\right)\cdot c\varphi_d\cdot s\varphi_d\cdot M_d\cdot y_d^2\cdot\theta\cdot\ddot{\theta}+s(\varphi_d+\beta)\cdot M_d\cdot z_{21}\cdot y_d\cdot\varphi\cdot\ddot{\theta}+s(\varphi_d+\beta)\cdot\\
&\quad\cdot M_d\cdot z_{22}\cdot y_d\cdot\varphi\cdot\ddot{\theta}-(s\beta\cdot c\varphi_d+c\beta\cdot s\varphi_d)\cdot M_d\cdot x_{21}\cdot y_d\cdot\ddot{\varphi}+(s\beta\cdot c\varphi_d-c\beta\cdot s\varphi_d)\cdot M_d\cdot x_{22}\cdot y_d\cdot\ddot{\varphi}+\\
&\quad+(c\beta\cdot c\varphi_d-s\beta\cdot s\varphi_d)\cdot M_d\cdot y_{21}\cdot y_d\cdot\ddot{\varphi}-(c\beta\cdot c\varphi_d+s\beta\cdot s\varphi_d)\cdot M_d\cdot y_{22}\cdot y_d\cdot\ddot{\varphi}-c(\varphi_d+\beta)\cdot M_d\cdot x_{21}\cdot y_d\cdot\varphi\cdot\ddot{\varphi}+\\
&\quad+c(\varphi_d+\beta)\cdot M_d\cdot x_{22}\cdot y_d\cdot\varphi\cdot\ddot{\varphi}-s(\varphi_d+\beta)\cdot M_d\cdot y_{21}\cdot y_d\cdot\varphi\cdot\ddot{\varphi}-s(\varphi_d-\beta)\cdot M_d\cdot y_{22}\cdot y_d\cdot\varphi\cdot\ddot{\varphi}+\\
&\quad+2Jz_d\cdot\ddot{\varphi}_d+2M_d\cdot y_d^2\cdot\ddot{\varphi}_d+2\left(s^2\beta\cdot c^2\varphi_d+c^2\beta\cdot s^2\varphi_d\right)\cdot M_d\cdot y_d^2\cdot\psi^2\cdot\ddot{\varphi}_d+\\
&\quad+2\left(c^2\beta\cdot c^2\varphi_d+s^2\beta\cdot s^2\varphi_d\right)\cdot M_d\cdot y_d^2\cdot\theta^2\cdot\ddot{\varphi}_d+2M_d\cdot y_d^2\cdot\varphi^2\cdot\ddot{\varphi}_d-\\
&\quad-4\left(c^2\varphi_d-s^2\varphi_d\right)\cdot c\beta\cdot s\beta\cdot M_d\cdot y_d^2\cdot\psi\cdot\theta\cdot\ddot{\varphi}_d+s(\varphi_d+\beta)\cdot M_d\cdot z_{21}\cdot y_d\cdot\ddot{\varphi}_d-s(\varphi_d-\beta)\cdot M_d\cdot z_{22}\cdot y_d\cdot\ddot{\varphi}_d-\\
&\quad-s(\varphi_d+\beta)\cdot M_d\cdot y_{21}\cdot y_d\cdot\psi\cdot\ddot{\varphi}_d-s(\varphi_d-\beta)\cdot M_d\cdot y_{22}\cdot y_d\cdot\psi\cdot\ddot{\varphi}_d+2\left(s^2\beta-c^2\beta\right)\cdot c\varphi_d\cdot s\varphi_d\cdot M_d\cdot y_d^2\cdot\psi\cdot\ddot{\varphi}_d+\\
&\quad+c(\varphi_d+\beta)\cdot M_d\cdot y_{21}\cdot y_d\cdot\theta\cdot\ddot{\varphi}_d-c(\varphi_d-\beta)\cdot M_d\cdot y_{22}\cdot y_d\cdot\theta\cdot\ddot{\varphi}_d-4\cdot c\beta\cdot s\beta\cdot c\varphi_d\cdot s\varphi_d\cdot M_d\cdot y_d^2\cdot\theta\cdot\ddot{\varphi}_d+\\
&\quad+2\left(c^2\beta-s^2\beta\right)\cdot c\varphi_d\cdot s\varphi_d\cdot Jx_d\cdot\dot{\psi}^2-2\left(c^2\beta-s^2\beta\right)\cdot c\varphi_d\cdot s\varphi_d\cdot Jy_d\cdot\dot{\psi}^2-2(Jx_d-Jy_d)\cdot c\varphi_d\cdot s\varphi_d\cdot c(2\beta)\cdot\dot{\theta}^2+\\
&\quad+2\left(s^2\beta-c^2\beta\right)\cdot c\varphi_d\cdot s\varphi_d\cdot M_d\cdot y_d^2\cdot\theta^2\cdot\dot{\varphi}_d^2+8\cdot c\beta\cdot s\beta\cdot c\varphi_d\cdot s\varphi_d\cdot M_d\cdot y_d^2\cdot\psi\cdot\theta\cdot\dot{\varphi}_d^2+
\end{aligned}
\tag{24}
$$

$$+2\left(c^2\beta - s^2\beta\right)\cdot c\varphi_d\cdot s\varphi_d\cdot M_d\cdot y_d^2\cdot\psi^2\cdot\dot{\varphi}_d^2 + 8\cdot c\beta\cdot s\beta\cdot c\varphi_d\cdot s\varphi_d\cdot\left(Jx_d - Jy_d\right)\cdot\dot{\psi}\cdot\dot{\theta} +$$
$$+4\left(s^2\varphi_d - c^2\varphi_d\right)\cdot c\beta\cdot s\beta\cdot M_d\cdot y_d^2\cdot\theta\cdot\dot{\psi}\cdot\dot{\varphi}_d + 4\left(s^2\beta\cdot c^2\varphi_d + c^2\beta\cdot s^2\varphi_d\right)\cdot M_d\cdot y_d^2\cdot\psi\cdot\dot{\psi}\cdot\dot{\varphi}_d -$$
$$-4\cdot c\beta\cdot s\beta\cdot c(2\varphi_d)\cdot M_d\cdot y_d^2\cdot\psi\cdot\dot{\theta}\cdot\dot{\varphi}_d + 4\left(s^2\beta\cdot s^2\varphi_d + c^2\beta\cdot c^2\varphi_d\right)\cdot M_d\cdot y_d^2\cdot\theta\cdot\dot{\theta}\cdot\dot{\varphi}_d + 4M_d\cdot y_d^2\cdot\varphi\cdot\dot{\varphi}\cdot\dot{\varphi}_d -$$
$$-2\cdot c\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\varphi\cdot g - 2\cdot s\beta\cdot s\varphi_d\cdot M_d\cdot y_d\cdot\varphi\cdot g = T\cdot s - \frac{T\cdot s}{\omega}\cdot\dot{\varphi}_d$$

5. Physical parameters of the separator

The mass and geometrical parameters are obtained after modeling in SolidWorks.

$M = 52.37$ kg	$M_d = 7.37$ kg
$Jx_1 = 2.88$ kg·m^2	$Jx_d \approx 0.07$ kg·m^2
$Jy_1 = 1.63$ kg·m^2	$Jy_d \approx 0.07$ kg·m^2
$Jz_1 = 2.81$ kg·m^2	$Jz_d \approx 0.015$ kg·m^2
$y_d = 0 \div 19$ mm	$x_1 = 0$ mm
$y_1 = 235$ mm	$z_1 = 290$ mm
$b_x = 273.5$ N·s/m	$k_x = 882143.2$ N/m
$b_y = 26.5$ N·s/m	$k_y = 4801.0$ N/m
$b_z = 77.0$ N·s/m	$k_z = 18946.3$ N/m
$b_{x5} = b_{y5} = 126.4$ N·s/m	$k_{x5} = k_{y5} = 9996.7$ N/m
$b_{z5} = 87.4$ N·s/m	$k_{z5} = 54699.9$ N/m
$x_5 = -r_c\cdot\cos\beta$, mm	$y_5 = r_c\cdot\sin\beta$, mm
$z_5 = -320$ mm	$r_c = 50$ mm $\beta = 0 \div 90°$

6. Results

The results from numerical solution of system Eq. (24) are presented in graphical form. Because of the considerable number of graphics provided from this model here are presented only graphics for displacement (linear and angular) of VF in time domain (Figure 4). After implementation of Fast Fourier Transformation (FFT) the single sided amplitude spectrum is obtained (Figure 5).

The results (Figures 4 and 5) are achieved under the following initial conditions:

$$y_d = 5.7,\ \text{mm};\quad \omega = 104.72,\ \text{rad/s};\quad \beta = 0^o$$

$$\vec{r}_{o21} = \left(-50,\ 0,\ 0\right)^T,\ \text{mm};\quad \vec{r}_{o22} = \left(50,\ 0,\ 0\right)^T,\ \text{mm}$$

7. Conclusions

Three mass and seven degrees of freedom nonlinear dynamical model with account of gyroscopic effects resultant from unbalanced shafts spinning is created.

The equations of motion describing the dynamical model are solved numerically in the programming environment of MATLAB. The results are represented graphically in time and frequency domain.

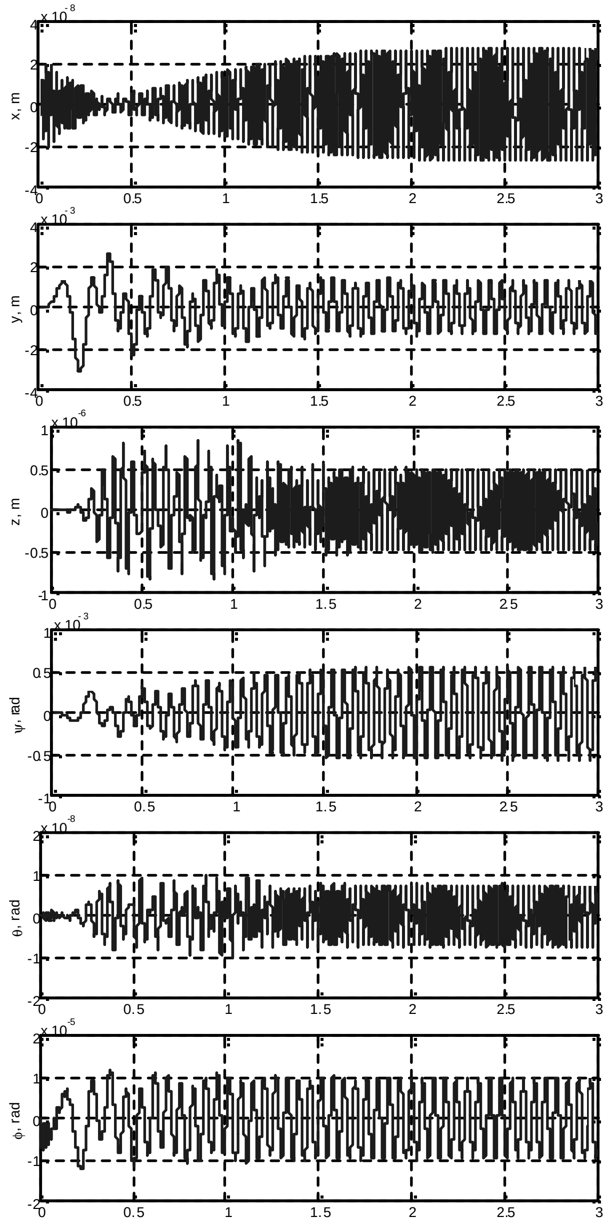

Figure 4. Forced vibrations in time domain

These results are in high degree similar with the results obtained in [1]. On that basis can be confirmed the conclusion that for the presented construction of VS the relations between GC are negligibly weak, the forced oscillations trajectory of sifting surface is steady and rectilinear.

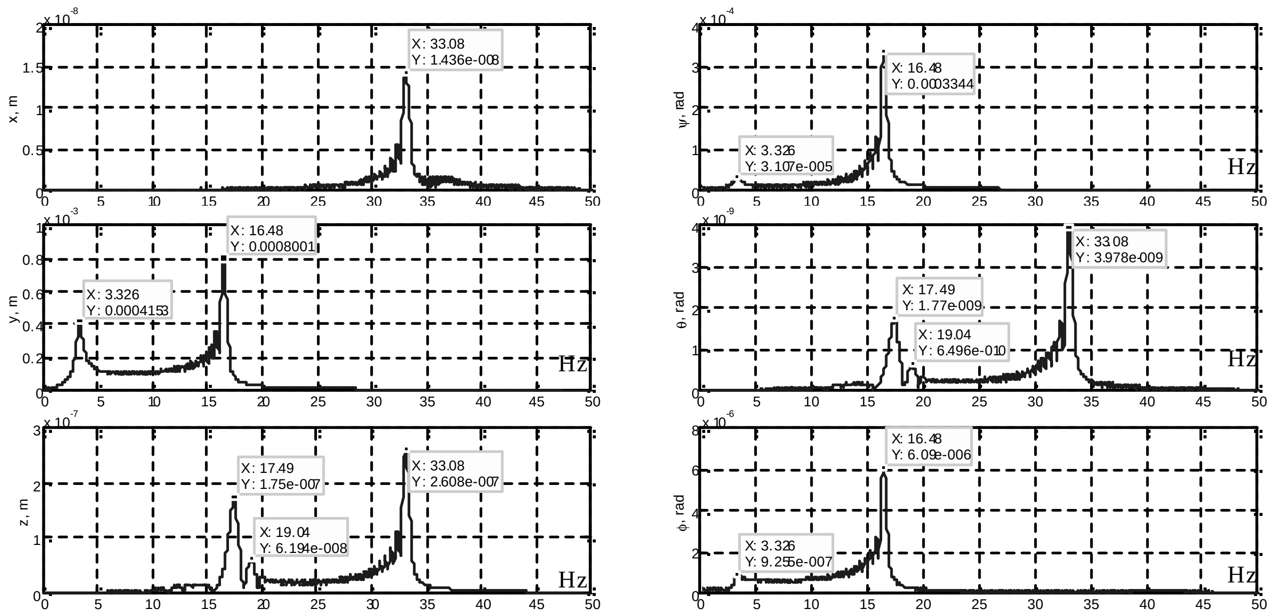

Figure 5. Forced vibrations in frequency domain
(Single sided amplitude spectrum)

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References

1. Vrazhilski, Dr.: *Spatial Dynamical Modelingof Multi Positional Vibrating separator*. RECENT, Vol. 9, no. 2(23), July 2008, p. 72-78, ISSN 1582-0246, Brasov, Romania
2. Craig, J.J.: *Introduction To Robotics: Mechanics and Control. 2nd ed*. Addison-Wesley publishing company, ISBN 0-201-09528-9, USA, 1989
3. Buchvarov, S., Tcherneva, Z.: *Vibration, Vibroprotection and Noiseprotection of Machines*. St. Kliment Ohridski University Press, ISBN954-07-1200-9, Bulgaria, Sofia, 1998 (in Bulgarian)
4. Ganiev, R.F., Kononenko, W.O.: *Vibration of hard solids* (*Kolebania tverdih tel*). Nauka, UDC 539.30/32, Moskow, Russia, 1976, (in Russian)
5. de Silva, C.W: *Vibration: Fundamentals and Practice*. CRC Press, ISBN 0-8493-1808-4, USA, 2000

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