International Conference on Economic Engineering and Manufacturing Systems
Braşov, 26 – 27 November 2009

MANUFACTURING SYSTEM MODELING USING PETRI NETS

Daniela COMAN, Adela IONESCU, Mihaela FLORESCU
University of Craiova, Romania

Abstract. This paper presents the modeling and simulation of a manufacturing system using Petri nets. A manufacturing system consists of robots, computer-controlled machines, numerical controlled machines, computers and automated guided vehicle. Timed Petri nets are used to model, simulate and evaluate the manufacturing system. The Petri net model is implemented in Petri Net Toolbox under MATLAB environment. It is achieve the graphic construction of the net. Then, transporting it into a specific mathematical formalism it is made, so that the fulfilled structure to be fully retrieved and used to bring out the internal dynamics of the model. There are determined the global performance indicators in order to evaluate the performance of the manufacturing system.

Keywords: manufacturing system, discrete event, Petri nets

1. Introduction

Today's manufacturing systems are highly complex and many are very costly to build and maintain.

A manufacturing system is a technical system in itself. It consists of robots, computer-controlled machines, numerical controlled machines, computers and automated guided vehicle.

The use of robots in the production segment of manufacturing industries promises a variety of benefits ranging from high utilization to high volume of productivity. Each robotic cell or node will be located along a material handling system such as a conveyor or automatic guided vehicle. The production of each part or work-piece will require a different combination of manufacturing nodes. The movement of parts from one node to another is done through the material handling system. At the end of part processing, the finished parts will be routed to an automatic inspection node, and subsequently unloaded from the Flexible Manufacturing System.

The transport operation plays a very important role within a flexible technological system, especially in the optimization process. We can analyze the concept “transport” without having into account the operations required.

Discrete event simulation has an important role to play in managing these systems.

In this paper, is present an overview of the use of simulation in the design and analysis of manufacturing systems. We intend to present a simulated model according to which we can establish the time variation and the outputs process in a simple production system.

Petri net based distributed system modelling takes place at the state level: it determines the actions that take place in the system, the states that precede these states and the state in which the system will pass after the actions have taken place.

By simulating the state model through the Petri nets, we obtain a description of the system’s behaviour [1, 2].

In section 2, the basic theory of Petri nets it is presented. In section 3, it is illustrated how a manufacturing system is designed using Petri net. In section 4, the Petri net model are implemented in MATLAB environment. Concluding remarks follow in section 5.

2. Petri net structure and graph

Petri nets, developed by Carl Adam Petri in his Ph.D. thesis in 1962, are generally considered as a tool for studying and modeling of systems. A Petri net is foremostly a mathematical description, but it is also a visual or graphical representation of a system. A Petri net can be analysed in order to reveal important information about the structure and behaviour of the modeled system. The information may for instance suggest improvements to the system.

A Petri Net consists of a number of places and transitions with tokens distributed over places. Arcs are used to connect transitions and places. When every input place of a transition contains a token, the transition is enabled and may fire. The result of firing a transition is that a token from every input place is consumed and a token is placed into every output place. The firing of transitions represents causality and inferencing relations. Petri nets have been generalized by allowing multiple token arcs, inhibitor arcs, place capacity, coloured tokens etc. [5].

Formally, the structure of a Petri net, defined by its places, transitions, input function and output function [3, 4, 5].

Definition 1 (Petri Net Structure): A Petri net structure, C, is a four-tuple,

$$C = (P, T, I, O) \quad (1)$$

where: $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places, $n \geq 1$; $T = \{t_1, t_2, \ldots, t_m\}$ is a finite set of transitions, $m \geq 1$; $I : T \rightarrow P^{\infty}$ is the input function, a maping from transitions to bags of places, $O : T \rightarrow P^{\infty}$ is the output function, a mapping from transitions to bags of places. The set of places and the set of transitions are disjoint, i.e., $P \cap T = \emptyset$.

Definition 2 (Petri net graph): A Petri net graph G is a bipartite directed multigraph, $G = (V, A)$, where $V = \{v_1, v_2, \ldots, v_s\}$ is a set of vertices and $A = \{a_1, a_2, \ldots, a_r\}$ is a bag of directed arcs, $a_i = (v_j, v_k)$, with $v_j, v_k \in V$. The set V can be partitioned into two disjoint sets P and T such that $V = P \cup T$, $P \cap T = \emptyset$, and for each directed arc, $a_i \in A$, if $a_i = (v_j, v_k)$, then either $v_j \in P$ and $v_k \in T$ or $v_j \in T$ and $v_k \in P$.

A marking μ is an assignment of tokens to the places of a Petri net. A token is a primitive concept for Petri nets, like places and transitions are. Tokens are assigned to, and can be thought to reside in, the places of a Petri net. The number of tokens and the places they reside in may change during the execution of a Petri net. The tokens are used to define the execution of a Petri net.

Definition 3 (Marking): A marking μ of a Petri net $C = (P, T, I, O)$ is a function from the set of places P to the nonnegative integers N, $\mu : P \rightarrow N$. It can also be defined as an n-vector, $n = |P|$, such that $\mu = (\mu(p_1), \mu(p_2), \ldots, \mu(p_i), \ldots, \mu(p_n))$. $\mu(p_i)$ gives the number of tokens in place p_i.

Definition 4 (Marked Petri): A marked Petri net $M = (C, \mu)$ is a Petri net structure $C = (P, T, I, O)$ and a marking μ.

3. Manufacturing system

It is considerate the flexible manufacturing system presented in figure 1. The system consists of two identical machines (M1 and M2) and a robot (R).

Both machines are loaded by the robot and are automatically unloaded.

On M1 machine, there are processed the parts of type 1 from L1 lot, for which the loaded period with R is uniformly distributed between 2 and 4 minutes; and the processing period (including the release of M1 and R) is uniformly distributed between 8 and 12 minutes. Similarly, on M2 machine, there are processed the parts of type 2 from L2 lot, for which the loaded period with R is uniformly distributed between 3 and 7 minutes; and the processing period (including the release of M1 and R) is uniformly distributed between 17 and 25 minutes. It is considered that there are sufficiently large numbers of both parts and that the robot will proceed to load a new non-machined part as soon as it is free.

Regarding the robot, there is no mechanism for the preferential choice of the type of parts, which are loaded (from L1 or L2 lot), the two types of parts having equal probability of serving.

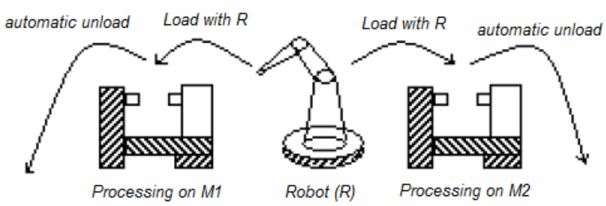

Figure 1. Structure of the manufacturing system

The following places and transitions are defined for the manufacturing system modeling in Petri-net.

Structure: (P, T, I, O).
Places: $P = \{M1, R, M2, L1, P1, L2, P2\}$.
Transitions: $T = \{t1, t2, t3, t4, t5, t6\}$.
Input function: $I(t1) = \{M1, R\}$, $I(t2) = \{L1\}$, $I(t3) = \{P1\}$, $I(t4) = \{M2, R\}$, $I(t5) = \{L2\}$, $I(t6) = \{P2\}$.
Output function: $O(t1) = \{L1\}$, $O(t2) = \{P1\}$, $O(t3) = \{M1, R\}$, $O(t4) = \{L2\}$, $O(t5) = \{P2\}$, $O(t6) = \{M2, R\}$.

The following transitions are defined to model the manufacturing system:

t1 and t4 - modeling the choice of the part to be loaded on one machine;

t2 and t3 - modeling the loading operations of one part from L1 lot respectively the processing operations of this part on M1 machine;

t5 and t6 - modeling the loading operations of one part from L2 lot respectively the processing operations of this part on M2 machine;

The Petri net model is presented in figure 2.

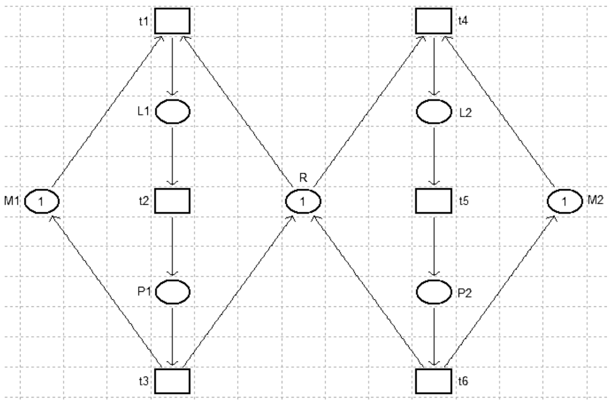

Figure 2. Petri net model for the manufacturing system

The input matrix A^-, and the output matrix A^+ is computed directly from the graphical model, as follows:

$$A^{-}=\begin{bmatrix}1&1&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&1&1&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix} \quad (2)$$

$$A^{+}=\begin{bmatrix}0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\1&1&0&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\\0&1&1&0&0&0&0\end{bmatrix} \quad (3)$$

The incidence matrix A of a Petri net will be:

$$A=\begin{bmatrix}-1&-1&0&1&0&0&0\\0&0&0&-1&1&0&0\\1&1&0&0&-1&0&0\\0&-1&-1&0&0&1&0\\0&0&0&0&0&-1&1\\0&1&1&0&0&0&-1\end{bmatrix} \quad (4)$$

The incidence matrix has rank 4, so that the Petri net model shown in Fig. 2 will have:

m - rank (A) = 3 $\Rightarrow$ at most 3 P - invariants are linearly independent,

n − rank(A) = 2 $\Rightarrow$ at most 2 T - invariants are liniarly independent.

4. Simulation studies

The simulation studies are carried out for the Petri net giving statistics on events and resources for model [3, 6].

The simulation of the proposed manufacturing system using timed Petri nets provides the possibility to view the manufacturing process in time.

After achieving the graphic construction of the net, transporting it into a specific mathematical formalism has been made, so that the fulfilled structure to be fully retrieved and used to bring out the internal dynamics of the model.

The results from the mathematical method of checking through the invariants method associated transitions and the corresponding positions after calculating the incidence matrix of the net have been validated through the simulations using Petri Net Toolbox in Matlab environment. It was validated in this way that the net topology, the evolution of (their dynamics), as well as structural and behavioural properties (figure 3).

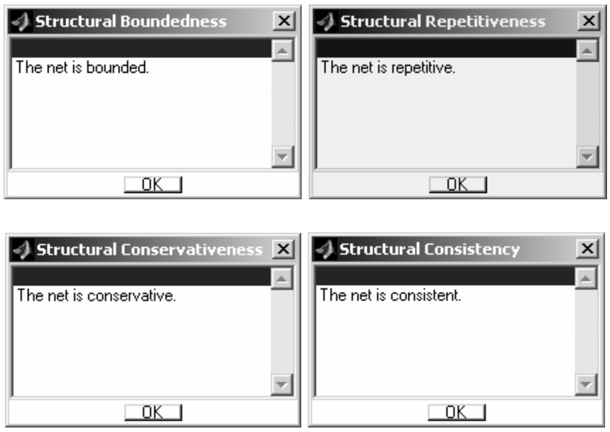

Figure 3. Study of the structural properties

By the simulation, in the Petri Net Toolbox, of the system functioning for a period of 2000 hours have been obtained global indicators (related to model transitions and positions).

The following two tables (Table 1- Global Statistics Places and Table 2 - Global Statistics Transitions) present the complete lists of global indices associated with the places and the transitions considered in the architecture of Petri net that modeling the manufacturing system.

Table 1. Global Statistics Places

Place Name	Arrival Sum	Arrival Rate	Arrival Dist.	Throughput Sum	Throughput Rate	Throughput Dist.	Waiting Time	Queue Length
M1	3079	0.025655	38.9785	3079	0.025655	38.9785	26.0385	0.66802
R	6159	0.051319	19.4861	6159	0.051319	19.4861	0	0
M2	3080	0.025664	38.9658	3080	0.025664	38.9658	12.9358	0.33198
L1	3079	0.025655	38.9785	3079	0.025655	38.9785	3.02	0.077479
P1	3079	0.025655	38.9785	3079	0.025655	38.9785	9.92	0.2545
L2	3080	0.025664	38.9658	3080	0.025664	38.9658	5.02	0.12883
P2	3080	0.025664	38.9658	3080	0.025664	38.9658	21.01	0.53919

Table 2. Global Statistics Transitions

Transition Name	Service Sum	Service Rate	Service Dist.	Service Time	Utilization
t1	3079	0.025655	38.9785	0	0.00032471
t2	3079	0.025655	38.9785	3.02	0.077778
t3	3079	0.025655	38.9785	9.92	0.25472
t4	3080	0.025664	38.9658	0	0.00021689
t5	3080	0.025664	38.9658	5.02	0.12901
t6	3080	0.025664	38.9658	21.01	0.53919

Based on these indices, are calculated, following the performance of manufacturing: number of parts processed entirely on M1 machine: 3.079; number of parts processed entirely on M2 machine: 3.080; average time required to process a part from L1 lot: 13 minutes; average time required to process a part from L2 lot: 26 minutes; utilization of M1 machine: 0.33; utilization of M2 machine: 0.66; and utilization of robot: 1.

Also, the special options of Petri Net Toolbox, which confers a high capacity of analysis, has made possible a synthesis of this Petri net model which allows exploring the dependences of global performance indicators associated with the net positions/transitions on two “Design Parameters” (being considered transitions t2 and t5) for the various parameters of the simulation (Figure 4 - Dependency of the Service Sum index associated t3 transition, Figure 5 - Dependency of the Service Sum index associated t5 transition, Figure 6 – Dependency of the Queue length index associated M1 position, Figure 7 - Dependency of the Queue length index associated M2 position, Figure 8 - Dependency of the Queue length index associated L1 position, Figure 9 - Dependency of the Queue length index associated L2 position, Figure 10 - Dependency of the Waiting Time index associated M1 position, Figure 11 - Dependency of the Waiting Time index associated M2 position, Figure 12 - Dependency of the Utilization index associated t6 transition, Figure 13 - Dependency of the Throughput Dist. index associated M1 position).

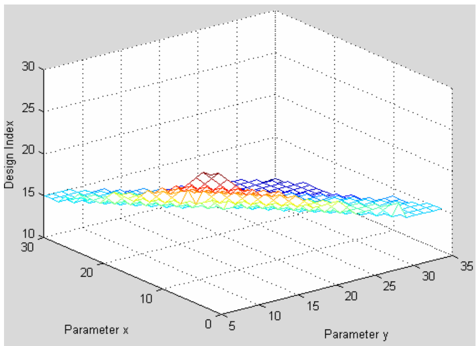

Figure 4. Dependency of the Service Sum index associated t3 transition

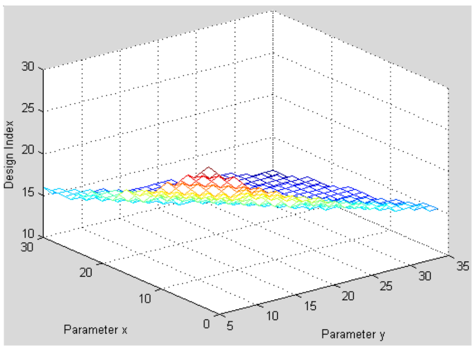

Figure 5. Dependency of the Service Sum index associated t5 transition

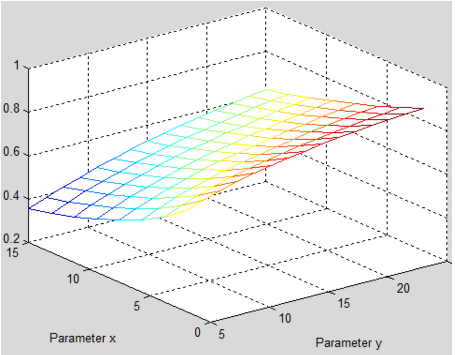

Figure 6. Dependency of the Queue length index associated M1 position

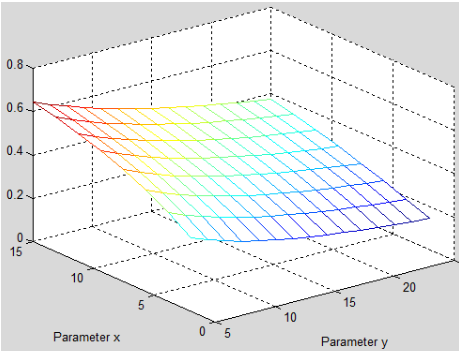

Figure 7. Dependency of the Queue length index associated M2 position

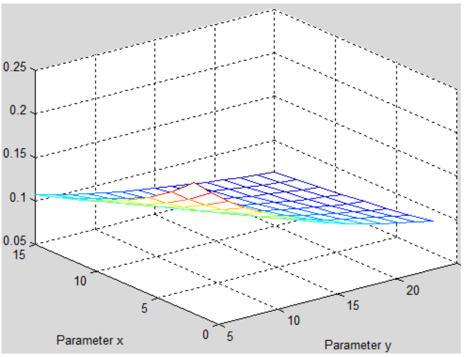

Figure 8. Dependency of the Queue length index associated L1 position

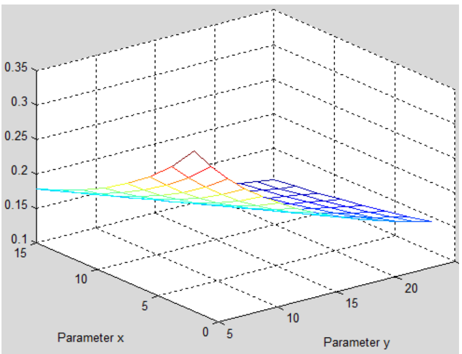

Figure 9. Dependency of the Queue length index associated L2 position

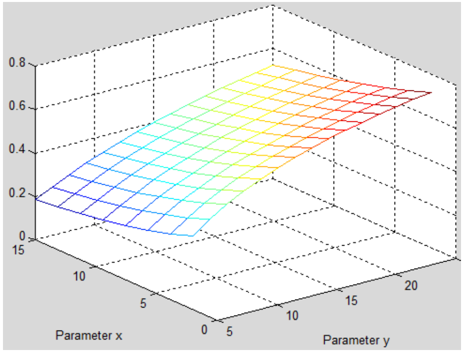

Figure 10. Dependency of the Waiting Time index associated M1 position

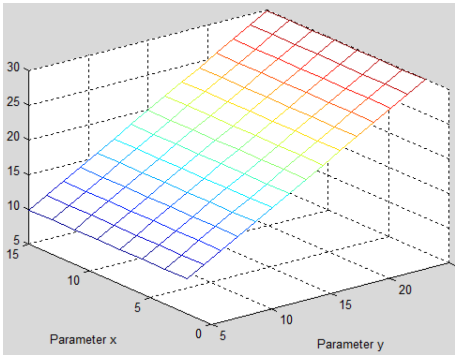

Figure 11. Dependency of the Waiting Time index associated M2 position

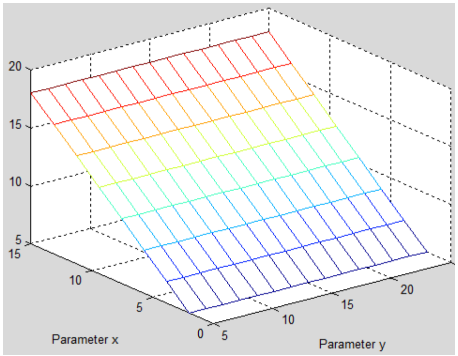

Figure 12. Dependency of the Utilization index associated t6 transition

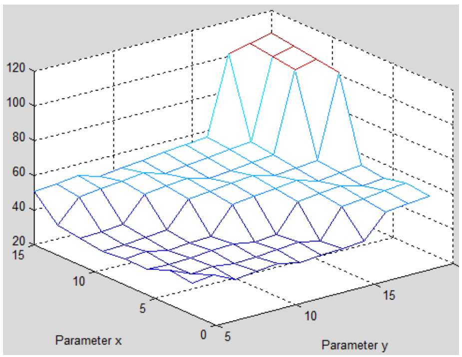

Figure 13. Dependency of the Throughput Dist. index associated M1 position

5. Conclusion

In this paper, a Petri net model is used for designing a manufacturing system. These Petri net models are implemented in MATLAB environment. The simulations studies was validated that the net topology, the evolution of (their dynamics), as well as structural and behavioural properties and was provided the global performance indices associated with the places and the transitions and also the whole set of global indices associated with all the nodes of the net.

The simulation of the proposed manufacturing system using timed Petri nets provides the possibility to view the manufacturing process in time. We have obtained graphic representations regarding the evolution of average manufacturing time durations and the evolution of the average equipment use duration.

References

1. Fowler, J., Schomig, A.: *Applied system simulation: methodologies and applications*, Kluer Academic Publishers, 2003, ISNB 1-4020-7603-7
2. Lee, E.J., Dangoumau, N., Toguyeni, A.: *A Petri net based approach to design controllers for Flexible Manufacturing Systems.* Proceeding of 17th IMACS World Congress Scientific Computation, Applied Mathematics and Simulation-IMACS 2005, France, July 2005
3. Pastravanu, O., Matcovschi, M., Mahulea, C.: *Aplicații ale rețelelor Petri în analiza sistemelor discrete (Aplications of Petri Nets in discreet systems analysis).* “Gh. Asachi” University Publishing House, Iasi, Romania, 2002 (in Romanian)
4. Pastravanu, O.: *Analiza sistemelor discrete. Metode calitative bazate pe formalismul rețelelor Petri (Discreet systems analysis. Qualitative techniques based on Petri net formalism).* MatrixRom Publishing House, Bucharest, 1997, ISBN 973-9254-61-6 (in Romanian)
5. Peterson, J. L.: *Petri net theory and modelling of systems*. Prentice-Hall Inc., ISBN: 0136619835, 1981
6. ***: http://www.ac.tuiasi.ro/pntool/. Accessed: 2009-04-12