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MATHEMATICAL MODELLING AND NUMERICAL SIMULATION OF COULOMB FRICTION

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Abstract. This article proposes the dry friction simulation diagram which highlights the mechanical hysteresis. Two dry friction distinct relations, one non-linear relation and one portion linearized relation are examined and numerically simulated. The numerical simulation highlights how the mechanic hysteresis introduced by dry friction manifests, depending on the value of the travel velocity and on the start /stop process.

Keywords: Coulomb friction, mechanic hysteresis, numerical simulation

1. Introduction

The friction between two bodies in contact becomes visible upon the emergence of a relative velocity between their travel speeds. The expression of this friction force highlights its two components – Coulomb frictional force (dry) and viscous frictional force [3, 5]:

$$F_{fr} = F_{fr,v} + \frac{v}{|v|} F_{fr,U} \tag{1}$$

The travel velocity dependant frictional force v is presented in Figure 1 and can be increased through experiments.

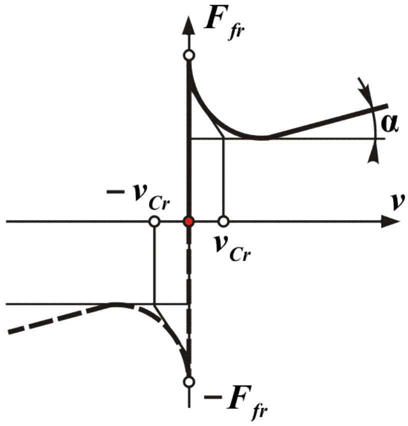


Figure 1. The frictional force

The viscous friction force is pro rata the relative velocity between the surfaces in contact and is determined by the lubricating liquid's viscosity found between these surfaces:

$$F_{fr,v} = c_L\, v \tag{2}$$

where c is the viscous friction coefficient and seldom can be calculated in analytical manner (for instance in case of hydraulic drawers [2]). Usually this coefficient is experimentally determined, representing the tangent of angle α formed by the variation curve of each friction in the field of bigger velocities than the critical velocity. Consequently, the viscous friction force graphics (Figure 2) is represented by a straight line starting from the origin with inclination equal to $c = tg\alpha$.

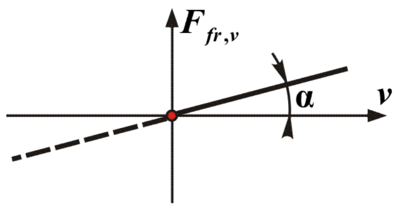


Figure 2. The viscous friction force

The Coulomb friction (dry) force has the known expression:

$$F_{fr,U} = \mu F_N \tag{3}$$

where μ is the dry friction coefficient, depending on the materials of bodies in contact and their roughness, and F_N – the normal force at surfaces in contact. In correspondence with the allure of the graphic of Figure 3, the value of this dry friction coefficient depends on the travel velocity:

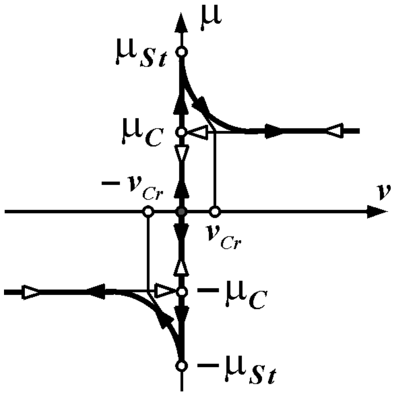


Figure 3. The friction coefficient

- At zero velocity, one can define a static dry friction coefficient μ_{St}, whose value depends on the lubrication conditions of the contact surfaces, depending on the period of stay,
- In the area of subcritical velocities $v < v_{Cr}$, the dry friction coefficient presents exponential variation,
- When exceeding the critical velocity $v > v_{Cr}$, the dry friction coefficient becomes constant being called cinematic friction coefficient μ_C ..

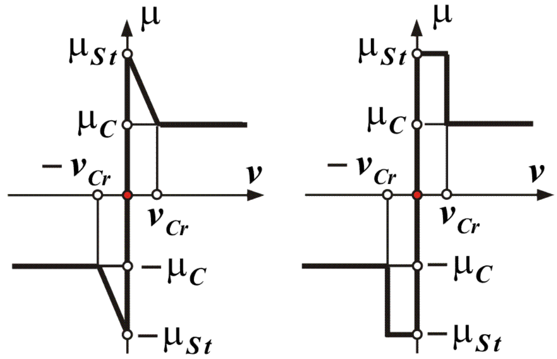
The graphic of Figure 3 [1] highlights the variation of the dry friction coefficient and implicitly of the dry friction force for the two distinct travel processes – **start of** movement (continuous line graphics and black marking arrows) and movement **stop** (or line graphics and white marking arrows).

The allure of dry friction graphics and the one above presented highlight the following non-linearities of the dry friction force:

- Mechanic hysteresis (different characteristics upon the START /STOP of the movement),
- Variable gain (variable friction coefficient per various segments, including different values during the stay period – depending on the inactivity period),
- Relay type feature.

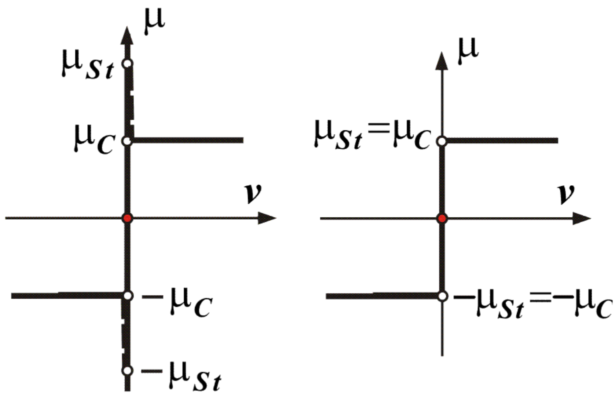
Depending on how approximation in the field $v < v_{Cr}$ is made and taking into consideration of the static friction coefficient μ_{St}, the dry friction graphics can be approximated in several ways:

- Considering μ_{St} and its approximation through an oblique line of the domain $v < v_{Cr}$ (Figure 4a),
- Considering μ_{St} and approximating it through a horizontal line of the domain $v < v_{Cr}$ (Figure 4b),



a) ***b***)

Figure 4. Approximation of dry friction force



a) ***b***)

Figure 5. Approximation of the friction force

- Taking into account μ_{St} only at the zero velocity point (Figure 5a),
- Considering $\mu_{St} = \mu_C$ and approximating it through bi-stable relay type (Figure 5b).

2. Mathematical modelling and numerical simulation of dry friction

Furthermore, the mathematical modelling of the dry friction force will be made considering $\mu_{St} = const$. With this, the dry friction force is described by the relation:

$$F_{fU} = F_{fUC} + \Delta F_{SC} \cdot \exp\left(-\frac{v}{v_{Cr}}\right) \tag{4}$$

or by the portion linearized relation:

$$\begin{cases} F_{fU} = F_{fUC} - \dfrac{\Delta F_{SC}}{v_{Cr}} \cdot v \quad pt.\, v < v_{Cr} \\ F_{fU} = F_{fUC} \quad pt.\, v \geq v_{Cr} \end{cases} \tag{5}$$

where F_{fUC} is the cinematic dry friction force, F_{fUS} – static dry friction force, and $\Delta F_{SC} = F_{fUS} - F_{fUC}$.

Using Simulink within Matlab [4] programming environment, the numerical simulation of dry friction was made, based on the simulation diagram at Figure 6. This was made starting from the relation (4), with the following numerical data: $F_{fUS} = 25\,[N], F_{fUC} = 20\,[N],\ v_{Cr} = 5\,[mm/s]$.

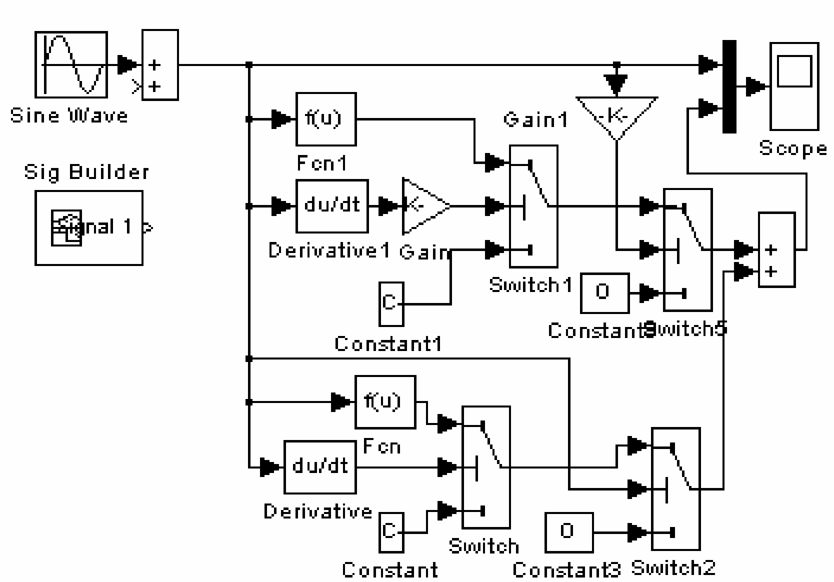
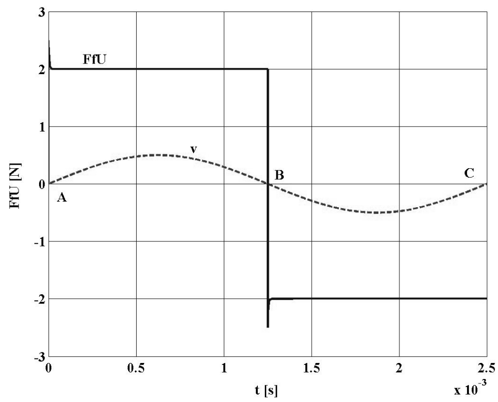
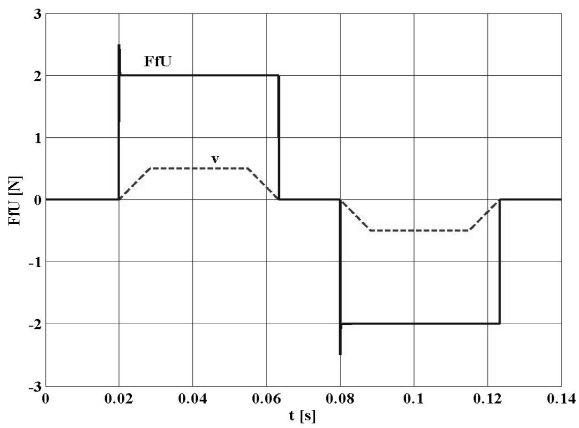


Figure 6. The simulation diagram



a)



b)

Figure 7. Numerical simulation of dry friction

Figure 7a displays the dry friction graphics, obtained at sinusoidal variation of the travel velocity. One can note how dry friction hysteresis' manifestation is highlighted. Static friction coefficient μ_{St} and exponential variation in the domain $v < v_{Cr}$ is manifested at starting movement only – on the right side of **A** and **B**. In exchange, when movement is stopped – on the left side of **B** and **C**, right at the entry in domain $v < v_{Cr}$, the friction coefficient remains equal to μ_C.

Figure 7b discloses the evolution of dry friction in case the travel velocity has zero velocity areas alternating with trapezoidal signals. The same way of manifestation of mechanical hysteresis phenomena associated to dry friction is noticed.

3. Final observations

Numerical simulation of dry friction described by the approximate relation (5) is similarly made and will lead to similar results. Though this leads to a certain approximation, the use of linearity on portions of dry friction leads to obtaining linear systems, will all derived advantages.

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