

## INFLUENCE OF SPOOL SUPPLIED FLOW NON-LINEARITY ON THE BEHAVIOR OF THE HYDRAULIC MOTOR

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**Abstract.** This article studies how non-linearity of the flow supplied by the electrohydraulic servovalve spool influences the dynamic behaviour of the controlled linear hydraulic motor. It approaches the case of hydraulic spool materializing a zero lapped spool, with control edges in circular shape. The analysis is performed comparatively with the linearized flow case, indicating the differences and applications domain.

**Keywords:** zero lapped bridge, non-linear flow

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### 1. Introduction

The spool with four active edges and zero lapped (bridge **A0+A0**) supplied flow are described by the below known expression [3]:

$$Q = B_D Y \sqrt{\Delta P} \quad (1)$$

valid for turbulent flowing in the spool. The following notes are in this relation:  $Y$  – the opening of the hydraulic spool,  $B_D = \alpha_D \pi D_s \sqrt{2/\rho}$  – flow constant of the spool,  $D_s$  – diameter of the hydraulic spool,  $\alpha_D$  – discharge coefficient of the spool is considered constant,  $\Delta P$  – pressure drop on throttling area of the spool.

Expression (1) is non-linear and is the composition between variable  $Y$  and radical of pressure drop  $\Delta P$ . The particular case of spool **A0+A0** controlling a symmetrical hydraulic motor is examined. The input programmed input size is considered the opening  $Y$  of the spool, and as output size the velocity  $v_M$  of the hydraulic motor. The flow supplied to the motor will have a particular form [3, 4]:

$$Q_M = B_D Y \sqrt{P_0 - P_A} = \frac{1}{\sqrt{2}} B_D Y \sqrt{P_0 - P_L} \quad (2)$$

where  $P_L = P_A - P_B$  – motor differential pressures,  $P_A$  – pressure of the motor chamber, and  $P_B$  – pressure of the counter motor chamber.

The linearization of the expression (2) is usually done around the zero points of the hydraulic spool and hydraulic motor. Consequently, the flow linearized expression is obtained:

$$Q_M = K_Q Y \quad (3)$$

within these relations  $K_Q = \frac{1}{\sqrt{2}} B_D \sqrt{P_0}$  represents the flow amplification of the spool.

As a difference from the linearized relation (3), the non-linear expression (2) highlights the negative pressure  $P_A$  feedback on the flow supplied by the spool to the motor. This is expected to influence in a different manner the dynamic behaviour of the motor.

### 2. Mathematical modeling of the symmetrical hydraulic motor

The mathematic model of the linear hydraulic motor will be deducted for highlighting purposes, as interim size, of the motor pressure  $P_A$ . This step will be done by accepting the below working hypothesis [2]:

- Internal and external leakages at the motor are considered null -  $\alpha_M = 0$ ,
- The motor is considered supplied at constant pressure -  $P_0 \approx const.$ ,
- The return pressure is considered atmospheric pressure -  $P_{R0} \approx 0$
- The reference position of the zero lapped spool is its closed position -  $Y = 0$ ,
- the reference position of the motor is the mid position which corresponds to the minimum stiffness of the hydraulic motor;

Based on the above, the mathematical model of the hydraulic motor is described by the continuity equation inside the motor's chamber:

$$Q_M = A_M v_M + C_{HA} \dot{P}_A \quad (4)$$

and by the motor's dynamic equilibrium equation:

$$m_{RM} \dot{v}_M + c_M v_M = A_M P_L - F_{RM} \quad (5)$$

In these equations  $Q_M$  is the flow rate of the hydraulic motor,  $A_M$  – the useful area of the hydraulic motor,  $v_M$  – motor velocity,  $m_{RM}$  – reduced motor mass,  $c_M$  – reduced viscous friction coefficient of the hydraulic motor,  $C_{HA} = V_M / 2E$  – hydraulic capacity of the motor chamber:  $E$  – elasticity module of fluid,  $V_M = 0.25L_M(D_M^2 - d_M^2)$ ,  $D_M$  – piston diameter,  $d_M$  – diameter of the motor strokes,  $L_M$  – maximum displacement of the motor,  $F_{RM}$  – resisting force on the motor.

In compliance with the relations between the pressures of the motor chambers, valid for a symmetrical hydraulic motor [3]:

$$\begin{cases} P_A + P_B = P_0 \\ P_A - P_B = P_L \end{cases} \quad (6)$$

the dynamics of the hydraulic motor in time domain is described by the following system of equations:

$$\begin{cases} C_{HA} \dot{P}_A = Q_M - A_M v_M \\ m_{RM} \dot{v}_M + c_M v_M = A_M (2P_A - P_0) - F_{RM} \end{cases} \quad (7)$$

and in complex, by the below system of equations:

$$\begin{cases} P_A(s) = \frac{1}{T_1 s + 1} [Q_M(s) - K_1 v_M(s)] \\ v_M(s) = \frac{K_2}{T_2 s + 1} \{K_1 [2P_A(s) - P_0(s)] - F_{RM}(s)\} \end{cases} \quad (8)$$

where:

$$K_1 = A_M ; K_2 = 1/c_M ; T_1 = C_{HA} ; T_2 = m_{RM}/c_M .$$

Based on this system of equations (8) and on the relation (2), the block – diagram of Figure 1 was indicated, and underlining the non-linear flow resulted from the displacement  $y = Y$  of one hydraulic control spool.

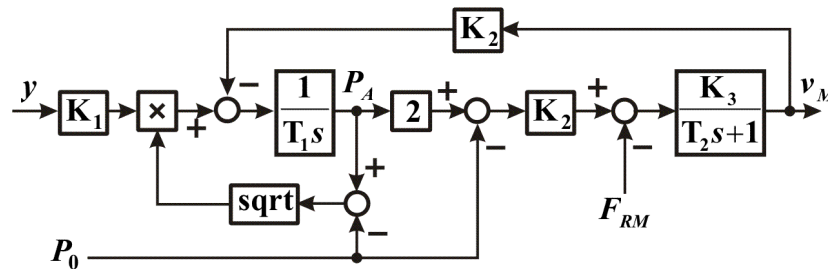


Figure 1. Block-diagram of the system

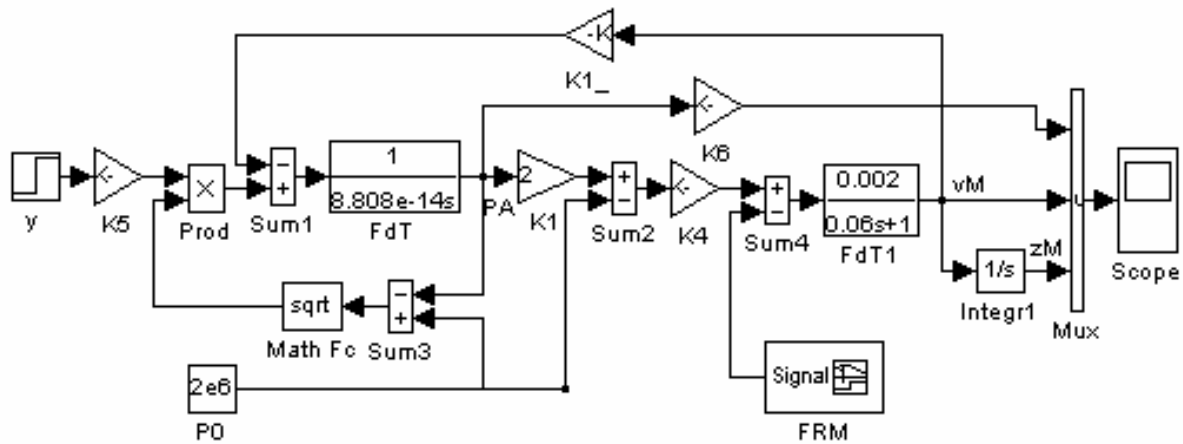


Figure 2. Simulation diagram in Simulink-Matlab

### 3. Numerical simulation of the hydraulic motor controlled by non-linear flow spool

Based on the block-diagram at figure 1, was formed the simulation diagram (figure 2), using Simulink of Matlab [6] programming environment. The numerical simulation of the hydraulic motor was made for the following numerical data of the hydraulic spool and hydraulic motor [1, 5]:

$$D_s = 8[mm], \quad \alpha_D = 0.64, \quad \rho = 920[kg/m^3],$$

$$P_0 = 20 [bar], \quad D_M = 62 [mm], \quad d_M = 40 [mm],$$

$$m_{RM} = 10[Kg], \quad c_M = 200[N/ms]$$

$$E = 0.4 \cdot 10^9 [N/m^2], \quad F_{RM} = 300 + 200 \sin(100\pi) [N],$$

$$F_{RM} = [N], \quad y = 0,2 [mm].$$

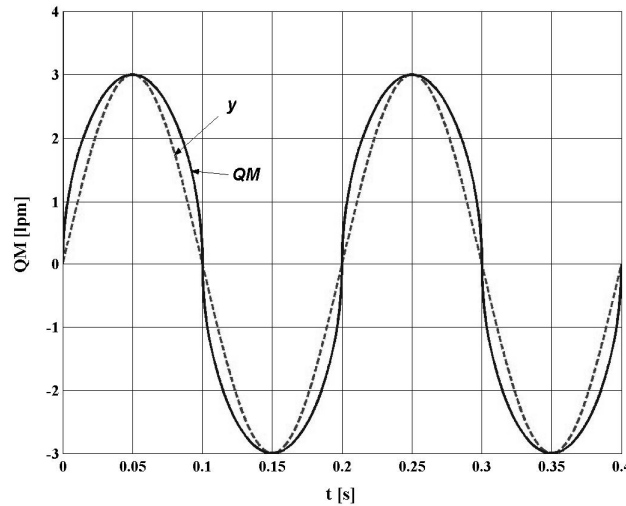


Figure 3. Response in frequency of the hydraulic spool

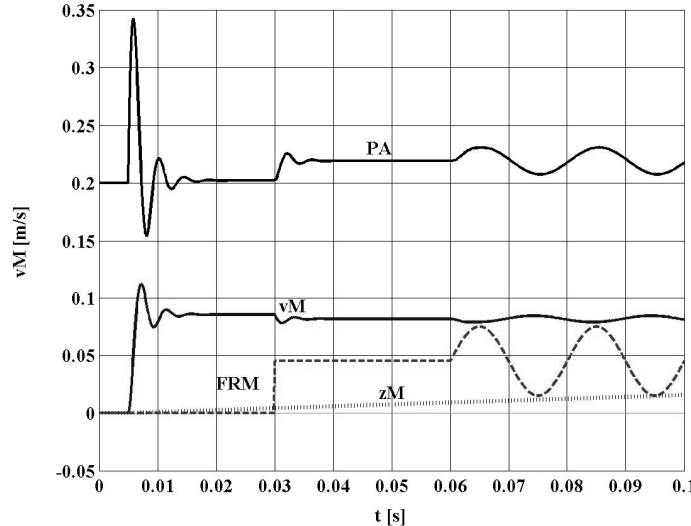


Figure 4. Feedback of the hydraulic motor

Figure 3 displays the response in frequency of the hydraulic spool, taking as input size the displacement  $y$  of the hydraulic spool and output size, the velocity of the hydraulic motor being considered constant. If the input size has sinusoidal shape (dotted line of the diagram) one can note that non-linearity as radical shape leads to obtaining a periodical output wave of a shape with deviations

from the sinusoidal form (continuous line diagram).

Figure 4 displays the feedback of the hydraulic motor, taking as programmed input size the displacement  $y$  of the spool supplying the motor's non-linear flow and as input size the resisting force  $F_{RM}$  (dotted line graphics), and as output size the hydraulic motor velocity  $v_M$

(continuous line). The same graphics displays the pressure variation of the hydraulic motor chamber  $P_A$  (dot line). The four operating stages are underlined:

- Coupling the hydraulic supply equipment is characterized by the presence of pressure  $P_0$ , when the spool and hydraulic motor are at zero,
- Spool's opening is done in the absence of the force resistance to the motor, which leads to the occurrence of velocity at the motor,
- Emergence at time  $t_1 = 0.03$  seconds of a step of the force resistant  $F_{RM}$  to the motor, leading to the increase of the motor pressure and decrease of the motor's travel velocity,
- Subsequent emergence of an additional variation of  $F_{RM}$ , of sinusoidal shape, leading to periodical oscillations of pressure  $P_A$  and velocity  $v_M$ , with deviations from the sinusoidal form specific to linear systems.

Overshooting output sizes can be compared to the overshoot occurred in case of linearization of the hydraulic spool supplied flow.

The most interesting observation related to the motor's feedback is that, unlike the motor controlled by one spool supplying a linearized flow, the hydraulic motor transient regimes last much shorter (less than one second) and the frequency of oscillations within these is also much lower. This is determined by the negative feedback of the pressure  $P_A$  on the flow supplied by the spool to the motor, and implicitly on its travel velocity.

#### 4. Conclusions

Considering the non-linearity of the flow supplied by spool to the hydraulic motor, results in a response much different from linearized flow case. This is manifested by shortening the transient regime within the indicial response and by output deviations from sinusoidal share, in the case of frequency response.

In conclusion, the linearization of the hydraulic spool flow around zero position, leads to linear models valid only for extremely small variations around this point. In order to obtain mathematical models displaying much smaller deviations from actual status, it must necessarily be considered the non-linear character of the hydraulic spool flow.

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