THEORETICAL CONSIDERATIONS CONCERNING THE DETERMINATION OF VALUE FOR THE CRITICAL SPEED OF THE BALL SCREWS FROM NUMERICAL AXES STRUCTURE

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Abstract. A servo performance limit of the numerical axes from the numerically controlled machine tool structure is the first natural frequency of their mechanical structure, because this limits the maximum amplification on the system control loops. For this reason, it is very important the appraisal for the critical speed of the ball screws from the numerical axes structure made by a more exactly calculation procedure.

Generally, the ball screw manufacturers offer in the instruction manuals, charts for the ball screw critical speed appraisal dependent on dimensions and type of their pillows. This method for the appraisal of the critical speed value is not a very accurate one, so that, while the feed velocities achieved by the numerical axes of the numerically controlled machine tools are increased, a more exactly calculation method is necessary for the optimization of the amplification parameters for the control loops.

This paper aims to elaborate a calculation method for the critical speed of the ball screws from the numerical axes structure, as critical bending speed, which corresponds to one of the natural resonant frequencies, the first natural resonant frequency of the assembly consisted of the ball screw and its pillows being considered as significant frequency in this case.

Keywords: ballscrew drives, critical speed

1. Introduction

The ball screw of the numerical axes structure can be easy modelled from standpoint of the dynamic behaviour, as a rotor [3]. According to [1], the critical speed of a rotor is the typical speed that sparks off the resonance of an excited system. Also, it is defined in [1], the critical speed – the speed when the bending of a rotor is maximum, so that the relative deflection is more important than the displacement of the support journals and the rigid-rotor-mode critical speed – the speed when the displacement of the support journals is maximum, so that this displacement is more important than the bending of the rotor.

For the achievement of the main objective proposed by this paper, respectively for the development of a calculation method for the critical speeds of the ball screws, the ball screw – support bearing system will be dynamically modelled in the variant of the flexible rotor fixed on stiff supports, because the ball screw deformation is obviously much more than the journal displacement and this deformation causes dramatic effects upon the dynamic behaviour of the numerical axis in assembly.

The determinations of the all inherit frequencies and the relative vibration modes are necessary as the free vibrations of a system to be solved. For many times in practice, it is necessary only some inherit frequencies to be known, sometimes only one [2]. This case is analyzed in this paper, where the first natural frequency of the mechanical structure limits the amplification on the control loops of a numerical axis servo-system. It will be considered that this first natural frequency corresponds to the critical speed of the ball screw.

2. Mathematical model

The classic method for the solution of a vibration problem consists of the writing of one or more motion equations, by applying of the Newton second law [4]. This method will be utilized, using the modeling of the ball screw- support bearing system like that shown in the figure 1, a.

In order that the motion equation to be found, it will be started from the known relation [2]:

$$EI\frac{d^2y}{dx^2} = M \tag{1}$$

where: E – modulus of longitudinal elasticity; I – minimum moment of inertia of threaded shaft; M – bending moment from current section.

This equation gives the curvature of a beam (that models the ball screw) dependent on the bending moment in a certain section of the beam, by assumption that the material is a homogeneous, isotropic one and it is submited to the Hooke law and by considering that the beam is a straight one and with a constant cross section on the whole length. The equation is valid only for small deformations and for beams having their length of big dimensions related to their cross dimensions (this being the case of the ball screw from the numerical axes structure), when the effects of the displacement due to the shearing are negligible. The effects of the section sliding and rotation are negligible.

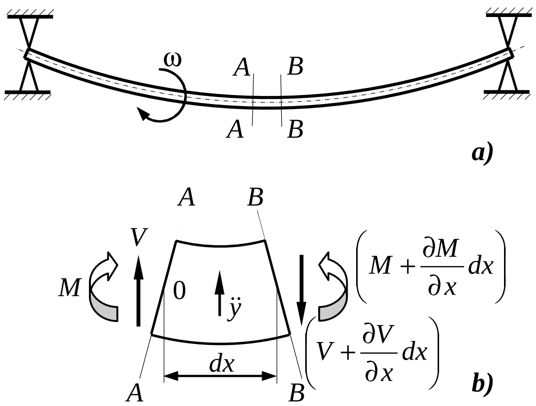

Figure 1. Dynamic model of the ball screw

For the case of the ball screw cross vibration, the motion equation is deduced by consideration of the forces that act on the small infinite element from its length (figure 1, b), limited by two plans A-A and B-B, that are normally on the longitudinal threaded shaft axis. In each section, the total vertical shearing force is consisted of two parts: the force generated by the static load including the threaded shaft weight and the force generated by vibration [5, 6]. Part of the shearing force generated by the static load balances exactly this load and therefore when the motion equation is deduced, these forces can be neglected, if the all displacements will be taken from the balance position of the ball screw under the load. Sum of the rest vertical forces that act on the considered element must be equal with the product of the element mass by the acceleration on cross direction $\ddot{y}=\partial^2 y/\partial t^2$

$$V+\frac{\partial V}{\partial x}dx-V=\frac{\partial V}{\partial x}dx=-\frac{A\gamma}{g}\frac{\partial^2 y}{\partial t^2}dx \qquad (2)$$

or:

$$\frac{\partial V}{\partial x}=-\frac{\gamma A}{g}\frac{\partial^2 y}{\partial t^2} \qquad (3)$$

If the moments are calculated related to the point O of the element from figure 1, then:

$$Vdx=\frac{\partial M}{\partial x}dx \text{ şi } V=\frac{\partial M}{\partial x} \qquad (4)$$

The other terms contain higher-order derivatives and can be neglected. By replacing this equation (3), it is obtained:

$$-\frac{\partial^2 M}{\partial x^2}=\frac{A\gamma}{g}\frac{\partial^2 y}{\partial t^2} \qquad (5)$$

where: A is sectional area of threaded shaft, γ - specific gravity, and g - acceleration of gravity. By substituting in the equation (1) results:

$$-\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)=\frac{\gamma A}{g}\frac{\partial^2 y}{\partial t^2} \qquad (6)$$

If EI is constant, The solution of this equation has the form:

$$y=X(x)\left[\cos(\omega_n t+\theta)\right] \qquad (7)$$

where X is a x function only. By writing:

$$k^4=\frac{\omega_n^2\gamma A}{EIg} \qquad (8)$$

and dividing (7) by $\left[\cos(\omega_n t+\theta)\right]$, it is obtained:

$$\frac{d^2 X}{dx^4}=k^4 X \qquad (9)$$

where X is a function having the fourth derivative equal with a constant multiplied by function itself.

The equation solution is given by a sum of functions (linearly independent ones) that checks the equation:

$$X=A_1\sin kx+A_2\cos kx+A_3\text{sh}\,kx+A_4\text{ch}\,kx \qquad (10)$$

The solution can be also expressed by terms consisted of exponential functions, but the trigonometric and hyperbolic functions are usually more easy to be used.

For the ball screws having different bearing conditions, the constants A_1, A_2, A_3 and A_4 are settled from the limit conditions. For the solution finding, it is suitable as the equation to be written in the below form, where two from constants are null, for each of the usual limit conditions:

$$X=A(\cos kx+\text{ch}\,kx)+B(\cos kx-\text{ch}\,kx)+ \\ +C(\sin kx+\text{sh}\,kx)+D(\sin kx-\text{sh}\,kx) \qquad (11)$$

In the application of the limit conditions the following relations are used, where are firstly shown the successive derivative related to x:

- deflection is proportional to X and is null on a stiff support;
- rotation is proportional to X' and is null at a built-in end;
- bending moment is proportional to X'' and is null at a free or jointed end;
- shearing force is proportional to X'' and is null at a free end.

For the usual limit conditions, two of constants are null, so being two equations with two constants. These can be combined and it is obtained an equation where only the frequency is unknown. By using the frequency, one of constants can be expressed dependent on the other one.

3. Determination of the critical speed dependent on the features of the ball screw bearings

In the numerical axes configuration, the combinations of the bearing types where the ball screw is supported are shown in figure 2.

By using the mathematical model developed in the previous section, the inherit frequencies of the ball screw will be firstly determined for the configuration a) from figure 2 of the numerical axis.

The limit conditions are:

$$\text{- at } x=0,\ X=0 \text{ and } X'=0 \quad (12)$$

$$\text{- at } x=l_b,\ X''=0 \text{ and } X'''=0 \quad (13)$$

From these conditions successively results:

$$A=0;\ C=0;$$

$$0=B(-\cos kl_b-\operatorname{ch}kl_b)+D(-\sin kl_b-\operatorname{sh}kl_b) \quad (14)$$
$$0=B(\sin kl_b-\operatorname{sh}kl_b)+D(-\cos kl_b-\operatorname{ch}kl_b)$$

By solving the two equations (143) related to D/B and by equalling the results, the following equation is obtained:

$$\frac{D}{B}=-\frac{\cos kl_b+\operatorname{ch}kl_b}{\sin kl_b+\operatorname{sh}kl_b}=\frac{\sin kl_b-\operatorname{sh}kl_b}{\cos kl_b+\operatorname{ch}kl_b} \quad (15)$$

The equation (15) is reduced at the form:

$$\cos kl_b\cdot\operatorname{ch}kl_b=-1 \quad (16)$$

where: k has the significance of the expression from the equation (8), and l_b the mounting length, (fig. 2).

Only the first solution of the equation (16) is interesting for this paper objective, this being typical to the first vibration mode of the system. For this solution finding, proper mathematical software can be used, for example Matlab, the following solution being obtained [7, 8]

$$k\cdot l_b=1.87510406\,871196 \quad (17)$$

The value of expression $k\cdot l_b$ will be a typical one for each from those 4 situations described in fig.2. This means that the relation (17) so can be written:

$$k\cdot l_b=\lambda,\ \text{or}\ k=\frac{\lambda}{l_b} \quad (18)$$

where λ represents the value of the product $k\cdot l_b$.

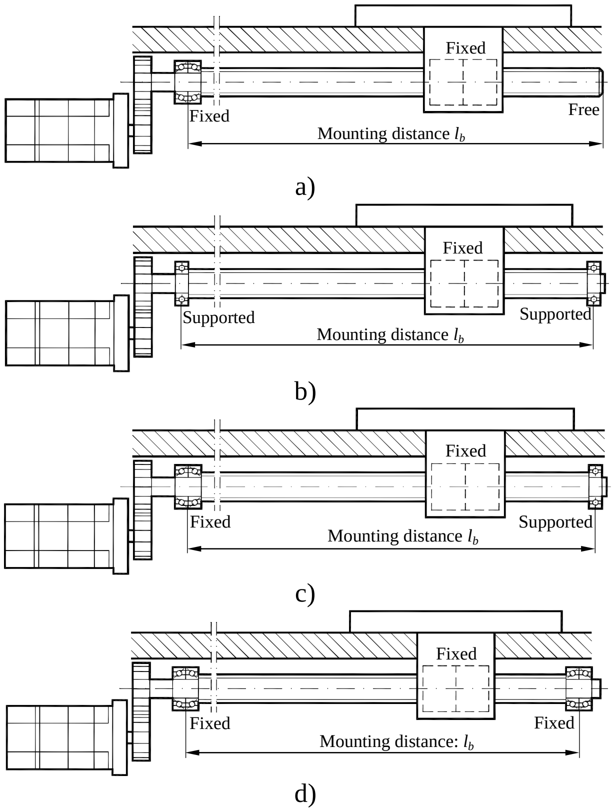

Figure 2. Combinations of the bearing types for the ball screws used in the numerical axes

By using the limit conditions suitable to the bearing mode of the ball screw for the situations b), c) and d) from figure 2, the values of λ, from table 1, are obtained – by a procedure like that presented for the case a).

Table 1. Values of the coefficient λ for the situations from figure 2.

Case (figure 2)	λ
a): Fixed – Free	1.87510406871196
b): Supported – Supported	3.14159265358979
c): Fixed – Supported	3.92660231204791
d) Fixed – Fixed	4.73004074486270

By replacing the k expression from the relation (18) in equation (8), it is obtained:

$$\omega_n=\frac{\lambda^2}{l_b^2}\sqrt{\frac{EIg}{\gamma A}} \quad (19)$$

which represents the general expression of the critical velocity corresponding to the first vibration mode of the ball screw from a numerical axis structure. In practice, the value of the critical speed is more useful:

$$n_n = \frac{30 \cdot \lambda^2}{\pi \cdot l_b^2} \sqrt{\frac{EIg}{\gamma A}} \tag{20}$$

4. Calculations and simulations

This is interesting for the analysis of the dependence between the critical speed value of the ball screws and their mounting length, respectively the bearing mode, the chart of dependence $n_n = f(l_b)$.

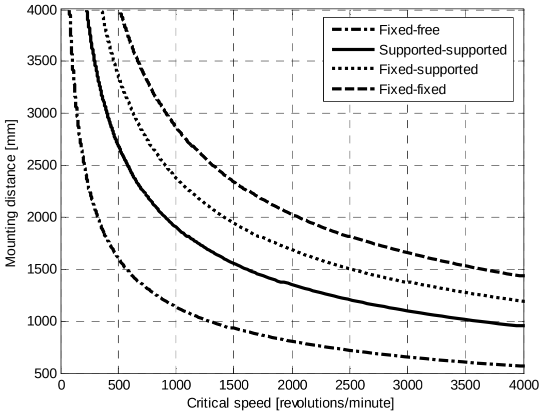

Figure 3. Critical speed dependence on l_b and λ (threaded shaft diameter: 30.1 mm)

In the figures 3 and 4, are presented these dependences for the ball screws having the root diameter of 30.1 mm (figure 3), respectively 44.1 mm (figure 4) with mounting distances between 500 and 4000 mm.

5. Conclusions

This paper aimed to develop a more precise calculation methodology for the critical speed of the ball screws from the numerical axes structure, as necessary parameter in the optimization of its servo-system. For this, starting from the known dynamic model that of a flexible rotor fixed on stiff supports, a calculation relation for the critical speed was deduced and by means of this, the dependences between the critical speed value and the mounting length, respectively the bearing type which supports the ball screw in the numerical axis structure, were plotted.

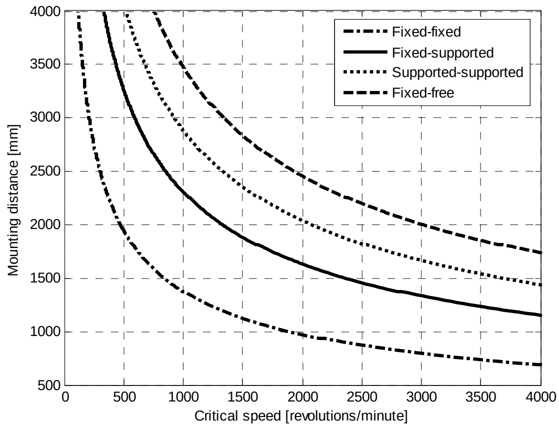

Figure 4. Critical speed dependence on l_b and λ (threaded shaft diameter: 44.1 mm)

The calculation relation is useful in the preliminary design stage for the dimensioning of the numerical axis mechanical structure. Surely, a more exact value for the critical speed of the ball screw, which depends on the real behaviour of the numerical axis whole mechanical structure assembly, can be obtained by experimental measurements on the physical model of the numerically controlled machine tool.

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