THEORETICAL STUDY OF COMPRESSION CHARACTERISTICS OF HIGHLY-VISCOUS PRODUCTS

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Abstract. Simplified rheological model which gives the possibility more certainly describe the deformation behavior of candied fruits and to determine their shear characteristics in production process is proposed.

Keywords: candied fruits, rheological model, shear characteristic, deformation

1. Introduction

The production process of highly viscous products, such as candied fruits, is associated with the mechanical action of the working bodies of vehicles on the product at the different character of interaction with the working surface [1]. One type of such effects is the deformation under uniaxial compression at different rates. Behavior of the product in these conditions and, especially, rheological characteristics, as measured by the deformation, are different from their true values due to relaxation processes accompanying the deformation of the material being tested. For example, when forming candied product, the three-dimensional deformation of the product is observed. These types of deformation characterize the compression characteristics of processes of candied fruits production [2-6].

2. Compression characteristics

Knowing the characteristics of compression is necessary to improve production processes, development of resource-saving processes and equipment. At the same time, their importance for the candied fruit produced by the new technology is not presented in the literature [7-8].

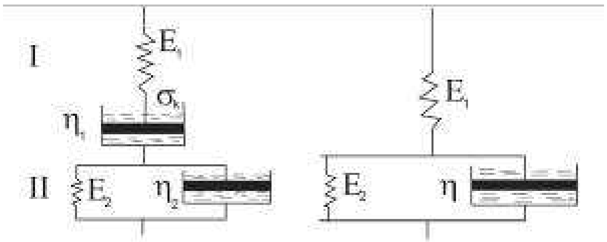
Rheological model of P.A.Rebindera-M.1 is shown in figure 1.

However, suggested (figure 1) is a simplified model M.2, which allows with greater confidence to describe the deformation behavior of the candied fruit masses and to determine their shear characteristics.

Let us consider the deformation behavior of the model M.2 in more detail.

Complete deformation model, in terms of compression (or extension) can be represented as the sum:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 . \qquad (1)$$



a) b)

Figure 1. General rheological model of P.A. Rehbinder M.1 (a); simplified model of M.2 (b) for the case $\sigma < \sigma_k$

The value ε_1 represents the "elastic" part of the deformation, occurring immediately upon application of external load and instantly disappearing after unloading:

$$\varepsilon_1 = \frac{\sigma}{E_1} . \qquad (2)$$

The value of E_1 is an "instantaneous elasticity modulus" or "elastic" modulus.

Component of the deformation ε_2 is "inelastic", but mechanically reversible part of the strain, which appears and disappears with a certain delay with respect to changes in external load. This corresponds to the above-mentioned "elastic spring back" loads. Dependence of ε_2 on the time t can be calculated by solving a differential equation, characterizing the deformation behavior of a model of Kelvin, consisting of elements E_2 and η:

$$\sigma = E_2 \cdot \varepsilon_2 + \eta \cdot \dot{\varepsilon}_2 . \qquad (3)$$

As usual, the dot above ε_2 denotes differentiation with respect to the time. The value E_2 is an "elastic" modulus, the value η has the dimension of the viscosity coefficient.

For a given law of change of the external stress over time σ(t), the solution of equation (3) has the form of:

$$\varepsilon_2(t)=e^{-\frac{E_2}{\eta}t}\left\{const+\int\frac{\sigma(t)}{\eta}e^{\frac{E_2}{\eta}t}dt\right\}. \qquad (4)$$

Moreover, the constant *const* is determined from the initial condition:

$$\varepsilon_2(t=0)=\varepsilon_{2_0}. \qquad (5)$$

By setting the law of change σ(t), ie mode of loading, you can use the formulas (1-5) to calculate the dependence of the total deformation on the time ε(t), describing the deformation behavior of the model. Using the model M.2 can be derived a general equation, which includes the total deformation, ε, but not its terms ε_1 and ε_2. To obtain this equation we should express ε_2 as $\varepsilon_2=\varepsilon-\varepsilon_1$, where $\dot\varepsilon_2=\dot\varepsilon-\dot\varepsilon_1$ and substitute it in (3), so that we obtain:

$$\sigma=E_2\cdot(\varepsilon-\varepsilon_1)+\eta\cdot(\dot\varepsilon-\dot\varepsilon_1). \qquad (6)$$

According to (2) there is:

$$\dot\varepsilon_1=\frac{\dot\sigma}{E_1}. \qquad (7)$$

From (2), (6) (7) we obtain:

$$\sigma=E_2\cdot\left(\varepsilon-\frac{\sigma}{E_1}\right)+\eta\cdot\left(\dot\varepsilon-\frac{\dot\sigma}{E_1}\right). \qquad (8)$$

After some transformations we bring this equation to a more symmetrical form:

$$\sigma+\tau_1\cdot\dot\sigma=E\cdot(\varepsilon+\tau_2\dot\varepsilon) \qquad (9)$$

where

$$\tau_1=\frac{\eta}{E_1+E_2};\quad \tau_2=\frac{\eta}{E_2};\quad E=\frac{E_1E_2}{E_1+E_2}. \qquad (10)$$

Equation (9) has a very large community, because, in its most general form, expresses the relationship between stress, strain and their first derivatives in time. It is used in many fields of science for a quantitative description of the various relaxation phenomena (in this case instead of σ and ε in the equation of the type (9) are present the other values: *D* and *E*, *B* and *H*, etc.). In particular, equation (9) is widely used to describe the relaxation phenomena in solids, the internal friction in metals, etc. In these cases, it is usually called the equation of the "standard linear body," or "Voigt body."

The solution of equation (9), with the given law σ(t), allows to calculate the dependence ε(t), not dividing ε on the elements ε_1 and ε_2. However, when analyzing the physical meaning of the obtained solutions and the visual representation of the deformation behavior of the model M.2, such division is necessary.

Let us now consider the deformation behavior of the model M.2 in different modes of loading. The simplest possible mode is instantaneous application of external load, which in the future remains constant: σ = const = σ_0. According to (2), immediately there appears an "elastic" part of the deformation:

$$\varepsilon_1=\frac{\sigma_0}{E_1}. \qquad (11)$$

Equation (3) becomes

$$\dot\varepsilon_2+\frac{E_2}{\eta}\varepsilon=\frac{\sigma_0}{\eta}. \qquad (12)$$

Its solution with the initial condition of $\varepsilon_2(t=0)=0$ is:

$$\varepsilon_2=\frac{\sigma_0}{E_2}\left(1-e^{-\frac{E_2}{\eta}t}\right). \qquad (13)$$

With $t\to\infty$ quantity ε_2 tends to the value:

$$\varepsilon_2(t\to\infty)=\varepsilon_{2\infty}=\frac{\sigma_0}{E_2}. \qquad (14)$$

The expression for the total deformation ε is:

$$\varepsilon_2(t\to\infty)=\varepsilon_{2\infty}=\frac{\sigma_0}{E_2}. \qquad (15)$$

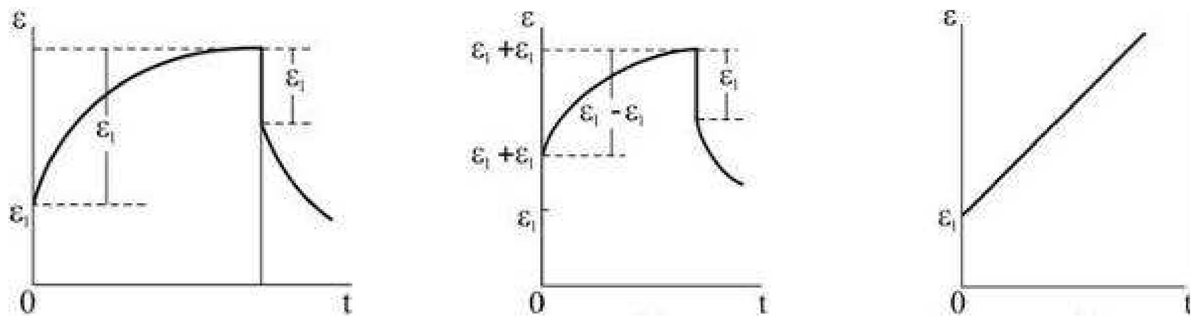
The general form of the dependence ε(t), under these conditions, is shown in figure 2 (left part).

During the instantaneous complete removal of the external load the value ε_1 instantly disappears, and ε_2 decreases gradually over time. Solving equation (12), with the initial condition of $\varepsilon_2(t=0)=\varepsilon_{2\infty}=\sigma_0/E_2$ (ie, with the "relaxation" ε_2 from the previously achieved asymptotic value $\varepsilon_{2\infty}$), we find the law of decrease ε_2 with time as

$$\varepsilon_2=\varepsilon_{2\infty}\cdot e^{-\frac{E_2}{\eta}t}=\frac{\sigma_0}{E_2}\cdot e^{-\frac{E_2}{\eta}t}. \qquad (16)$$

Nature of the change $\varepsilon_2(t)$ in the form of (16) is also shown in figure 2a (right part). If at some point in the process of reducing ε_2 according to the law (16) the external load is being applied to a model again (at a certain value of deformation of ε_2), then the solution of equation (12) for this case is provided by

$$\varepsilon_2=\left(\varepsilon_{2_0}-\frac{\sigma_0}{E_2}\right)\cdot e^{-\frac{E_2}{\eta}t}+\frac{\sigma_0}{E_2}. \qquad (17)$$



a) b) c)

Figure 2. Dependence of relative deformation on the time within the instantaneous application of a load: a - the initial deformation is 0, b - the initial deformation is different from 0; c - "Creep value" at $t \prec \frac{\eta}{E_2}$

The value ε_2 increases again, while striving with $t \to \infty$ to the same asymptotic value $\varepsilon_{2\infty} = \sigma_0 / E_0$. In addition, the "elastic" part of the deformation $\varepsilon_1 = \sigma_0 / E_1$ appears again.

Should be considered that for the small values of "elastic" modulus E_2 or at the large values of "parameters of viscosity" η, during the long loading time, the condition is carried out:

$$\frac{E_2}{\eta} t \prec 1. \qquad (18)$$

Part of the deformation ε_2, approximately linearly, increases with time, according to formula (13)

$$\varepsilon_2 \approx \frac{\sigma_0}{E_2} \cdot \left[1 - \left(1 - \frac{E_2}{\eta} \cdot t\right)\right] = \frac{\sigma_0}{\eta} \cdot t. \qquad (19)$$

This phenomenon is analogous to "steady creep" in the solids (Figure 2c). The dependence $\varepsilon_2(t)$ of this type should be observed for food products with high viscosity and low elasticity (for example, candied fruits). It should be noted that the "true" creep value in this case is absent, because plastic deformation does not occur and the model M.2 is mechanically reversible.

3. Conclusions

These relations completely describe the deformation behavior of the model M.2 at a constant external load. They are in a good agreement with the experimentally observed deformation behavior of such products as candied fruits. Comparison of the results, obtained by the formulas with the experimental curves allows determining values of models E_1, E_2, η parameters. In fact, E_2 is easily determined by the value of the "elastic" part of the deformation (11).

The value E_2 can be estimated by the limiting value $\varepsilon_{2\infty} = \sigma_0 / E_2$. If the value E_2 is known already, the parameter η can be easily determined by analyzing the "curve of discharge" (16) or formula (19) for "creep value".

Thus, the theoretical dependences, obtained this way will allow calculating the shear characteristics of candied fruit in terms of their production.

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