ESTIMATION OF MONTHLY ÅNGSTRÖM-PRESCOTT EQUATION COEFFICIENTS FOR BRAŞOV URBAN AREA, ROMANIA

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Abstract. The current trend of energy economy is marked by an attempt to use new energy sources. A new approach - generated by the fact that fossil energy carriers are limited - leads to attempt to use renewable energy sources: atomic energy, solar energy, wind power.

Solar radiation represents the most important climatological factor behind a series of solar energy applications such a photovoltaic systems for electricity generation, solar collectors for heating, solar air conditioning climate control in buildings and passive solar devices. Thus, the determination of solar radiation data is important for studies.

The objectives of the present paper are to develop a specific model for the prediction of monthly average of total radiation from sunshine hours, using a simple linear model suggested by Ångström–Prescott, to determine a model for diffuse radiation and to validate this models by means of three important statistical indicators: Mean Bias Error (MBE), Root Mean Square Error (RMSE) and t-statistic.

Keywords: Ångström–Prescott equation, total and diffuse radiation, MBE, RMSE, t-statistic

1. Problem description

The solar radiation simulation (total and beam components) and the determination of some mathematical expressions of this, represent problems of maximum interest for the solar renewable systems design. The beam function is the input data

- needed for the design or simulation of a solar energy conversion (either thermal or photovoltaic), for the design of PV tracking systems, and also
- used in simulation software to validate new energy concepts, from simple domestic hot water systems to the design and simulation of buildings and their equipment, including control strategies, occupant behaviour, alternative energy systems (solar, photovoltaic, etc.).

1.1. Depression of Braşov urban area - Geographical and climatic description

Braşov city is located in an intra-mountain depression and it represents an area of discontinuity between the Oriental and Meridional Carpathians. Considering its location in the climate sector frame, the depression of Braşov should be dominated by a relative moderate thermal regime.

However, the physical and geographical conditions extend the following feature: temperature inversion caused by the occurrence of radiative-orographic fogs and cloudiness. Thus, during the winter, the depression plains get cooler than the higher places and during the summer the depression lowland of Braşov gets warm more intensively than the surrounding slopes. Because of the surrounding mountains, the possible duration of the direct sunshine is diminished [1].

This region exhibits some typical features with respect to the topology, the climatology and the environment. The built-up area is low in comparison with that of the neighbouring mountains, which circle the basin area. Even in the sunny days the “visible” sky is delineated by the surrounding heights and the possible duration of direct sunshine and the amount of diffuse radiation are diminished. Moreover, in the urban area it is ascertained a more significant attenuation of the solar radiation due to the atmospheric pollution [1].

At this point, it must be mentioned that the variations of solar beam and total radiation for Braşov area present the following features [2]:

- the maximum daily values are recorded around 13 solar hour and not at 12 solar hour (situation that corresponds to the maximum value of the solar altitude angle obtained during a day);
- the real radiation variation is not a symmetrical curve towards 12 solar hours; in the morning the radiation has a slow increase to the maximum value and in the afternoon it decreases slowly; for a clear sky day, around sunset, the radiation values are quite high.

In this context, the obtaining of some more precise theoretical results concerning the energy conversion solar systems design requires the accurate mathematical modelling of the solar radiation relation. In the same time, the geographical and climatic features of every site as well as the influence of the urban conditions on some climatological parameters must be taken into consideration.

1.2. Meteorological Data

The meteorological data measurement was carried out for Braşov area, altitude: 790 m, longitude: 25.35^o and latitude 45.39^o. The local weather station Delta-T is positioned on the roof of the Transilvania University of Braşov. The data sets have been collected since October 2005 until now and they comprise: total solar radiation [W/m^2], diffuse solar radiation [W/m^2], air temperature [^{o}C], wind speed [m/s], wind direction [degrees], relative humidity [%], rainfall [pluviometric mm], sunshine. The horizontal total radiations G, diffuse radiation D, as well as all recorded data are related to 10 minutes range, in a continuous way. The Delta-T weather station is a complete system of instrumentation for automatically measuring and recording the weather. All sensors are mounted on a 2 meters mast, except for the rain gauge. An environmental data logger (Delta-T Logger) initiates readings, controls the sensors and stores data. Stored readings can be collected with a portable computer or printer, without interrupting logging [2].

2. Method used

2.1. Total solar radiation – empirical formulation

The most studies use for total solar estimation the linear regression given by Ångström (1924); this model correlates the total solar radiation depending on the monthly average of the daily fractional sunshine, n/N (n represents the number of daily sunshine recorded hours and N is the daily sunshine duration) [2, 3, 4]:

$$H / H_{clear\_sky} = a + b \cdot n / N \, , \qquad (1)$$

where, H_{clear_sky} represents the monthly average of the daily total radiation on a horizontal surface [kWh/m^2 / day], calculated considering clear sky assumption; a and b represent the regression constants determined empirically.

In present, the most generally used is the modified version of Prescott (1940); the correlation is:

$$H / H_0 = a + b \cdot n / N \, , \qquad (2)$$

where, H_0 represents the monthly average of the extraterrestrial global radiation on a horizontal surface [kWh/m^2 / day].

For the determination of monthly Ångström–Prescott equation coefficients, the following stages must be carried out:

- in the first stage the daily total radiation on a horizontal surface (H), daily extraterrestrial radiation on a horizontal surface (H_0), daily sunshine number of hours (n) and maximum daily sunshine duration (N) must be calculated;
- then monthly average of daily total solar radiation ($\overline{H}$), monthly average of daily extraterrestrial radiation ($\overline{H_0}$), the monthly average daily sunshine hours number ($\overline{n}$) and the maximum possible monthly average daily sunshine duration ($\overline{N}$, day length) are calculated;
- from regression analysis between $\overline{H}/\overline{H_0}$ and $\overline{n}/\overline{N}$, it is obtained the final estimation for the coefficients a and b for each month.

It is mentioned, the regression coefficients were determined for every month. In this way, the specific climatological and geographical conditions of the site are taken into consideration. Braşov is an urban depression located in a temperate zone with four seasons and important meteorological differences between them.

2.2. Sunshine duration

When the available solar radiation is analysed the sunshine duration must be considered.

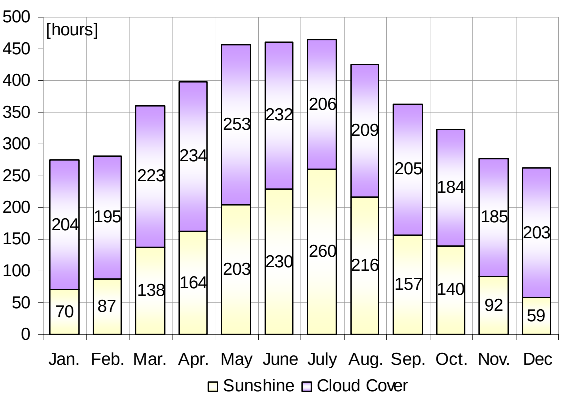

Figure 1. Sunshine duration average for Braşov

As a result of the multiple periodic and non-periodic cloud variations and the frequency of different types of clouds, the sunshine duration varies from year to year and from month to month.

From the analysis of the data recorded with the weather station Delta T, assembled on the roof of the Product Design for Sustainable Development Centre, it results for the period 2006-2010, the annual mean duration of sunshine is 1815 hours, that representing 41.75% from the total number of sunshine hours possible in Braşov.

The sunshine hour's number varies from 1704 in 2006 to 2027 in 2009. Usually, the maximum sunshine duration is recorded for July and the minimum sunshine duration is corresponding to December.

3. Results and discussions

Before calculating the Ångström-Prescott regression coefficients, in Table 1 there are presented a few monthly average daily measured parameters that characterize Braşov urban area.

Table 1. Monthly average daily parameters characteristic to Braşov urban area

Month	$\overline{n}$	$\overline{N}$	$\overline{n}/\overline{N}$	$\overline{H}_0$
January	3.33	8.95	0.25	3.33
February	4.76	10.13	0.30	4.76
March	6.92	11.68	0.38	6.92
April	9.22	13.32	0.41	9.22
May	10.91	14.73	0.45	10.91
June	11.63	15.43	**0.50**	11.63
July	11.25	15.09	**0.56**	11.25
August	9.79	13.87	**0.50**	9.79
September	7.66	12.28	0.43	7.66
October	5.38	10.63	0.44	5.38
November	3.63	9.23	0.33	3.63
December	2.89	8.58	0.22	2.89

Month	$\frac{\overline{H}_g}{\overline{H}_0}$	$\frac{\overline{H}_{diff}}{\overline{H}_0}$	$\overline{H}_g$	$\overline{H}_{diff}$	$\frac{\overline{H}_{diff}}{\overline{H}_g}$
January	0.326	0.20	1.08	0.67	0.62
February	0.371	0.20	1.67	0.96	0.57
March	0.395	0.21	2.73	1.42	0.52
April	0.412	0.20	3.80	1.86	0.49
May	0.437	0.20	4.77	2.23	0.47
June	0.464	0.20	**5.40**	**2.30**	0.43
July	0.477	0.20	**5.53**	**2.24**	0.40
August	0.452	0.21	4.42	2.01	0.46
September	0.415	0.19	3.18	1.49	0.47
October	0.416	0.20	2.24	1.07	0.48
November	0.355	0.19	1.29	0.71	0.55
December	0.279	0.18	0.81	0.51	0.63

It can be noticed that fractional sunshine ($\overline{n}/\overline{N}$) is less than 50 percent throughout the year, excepting June, July, August; in these three months the clearness index ($\overline{H}_g/\overline{H}_0$) is also recording the maximum values (higher than 0.45).

Table 2. Regression coefficients of the Ångström-Prescott equation for Braşov urban area

Month	Equation
January	$\overline{H}/\overline{H}_0 = 0.186 + 0.553(\overline{n}/\overline{N})$
February	$\overline{H}/\overline{H}_0 = 0.201 + 0.546(\overline{n}/\overline{N})$
March	$\overline{H}/\overline{H}_0 = 0.262 + 0.351(\overline{n}/\overline{N})$
April	$\overline{H}/\overline{H}_0 = 0.190 + 0.541(\overline{n}/\overline{N})$
May	$\overline{H}/\overline{H}_0 = 0.170 + 0.598(\overline{n}/\overline{N})$
June	$\overline{H}/\overline{H}_0 = 0.209 + 0.514(\overline{n}/\overline{N})$
July	$\overline{H}/\overline{H}_0 = 0.300 + 0.334(\overline{n}/\overline{N})$
August	$\overline{H}/\overline{H}_0 = 0.176 + 0.549(\overline{n}/\overline{N})$
September	$\overline{H}/\overline{H}_0 = 0.139 + 0.637(\overline{n}/\overline{N})$
October	$\overline{H}/\overline{H}_0 = 0.221 + 0.443(\overline{n}/\overline{N})$
November	$\overline{H}/\overline{H}_0 = 0.189 + 0.499(\overline{n}/\overline{N})$
December	$\overline{H}/\overline{H}_0 = 0.138 + 0.636(\overline{n}/\overline{N})$

Table 3. Regression coefficients of the diffuse radiation equation for Braşov urban area

Month	Equation
January	$\overline{H}_d/\overline{H} = 1 - 1.1474(\overline{H}/\overline{H}_0)$
February	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
March	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
April	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
May	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
June	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
July	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
August	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
September	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
October	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
November	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$
December	$\overline{H}_d/\overline{H} = 0.9714 - 1.1474(\overline{H}/\overline{H}_0)$

It can be also noticed, the diffuse radiation represents, during a year, at least 40 percents from total radiation.

The highest values of the ratio $\overline{H}_g/\overline{H}_{diff}$ are recorded during December and January (over 60%) when the clearness index $K_t = \overline{H}_g/\overline{H}_0$ records the minimum values.

The minimum contribution of the diffuse radiation in the total radiation is recorded during June and July (40-43%) when the clearness index has the highest values.

The regression coefficients of the Ångström-Prescott equation are obtained from the monthly average daily data during 2006-2010, these equations being presented in Table 2.

Considering the estimating equations for the monthly average daily total solar radiation in Table 3 are presented the regression function for the monthly average daily diffuse radiation.

As it can be noticed, for the monthly average daily diffuse radiation, two equations were obtained, namely one equation for January and one equation for the rest of the year.

3.1. Estimating Solar Radiation

In the next step, using the estimation equations proposed for the total and diffuse radiations (monthly average daily values), the graphical comparison between the measured and estimated values is achieved (Figure 2 and Figure 3).

Therefore, Figure 2 proposes the monthly diagrams for total and diffuse radiation – monthly average daily values; the diagrams are plotted for all months between January 2006 and August 2010.

From the presented diagrams, it can be noticed, the simulated values of the total and diffuse horizontal radiation are very close to the real recorded values.

Figure 3 presents the measured total radiation versus simulated total radiation and also measured diffuse radiation versus simulated diffuse radiation (monthly average daily values).

From the study of these diagrams, it can be noticed that the proposed models work correctly for all values of total radiation values; the graphs reveal a greater spreading (but this occurs in acceptable limits) of the real recorded diffuse radiation values.

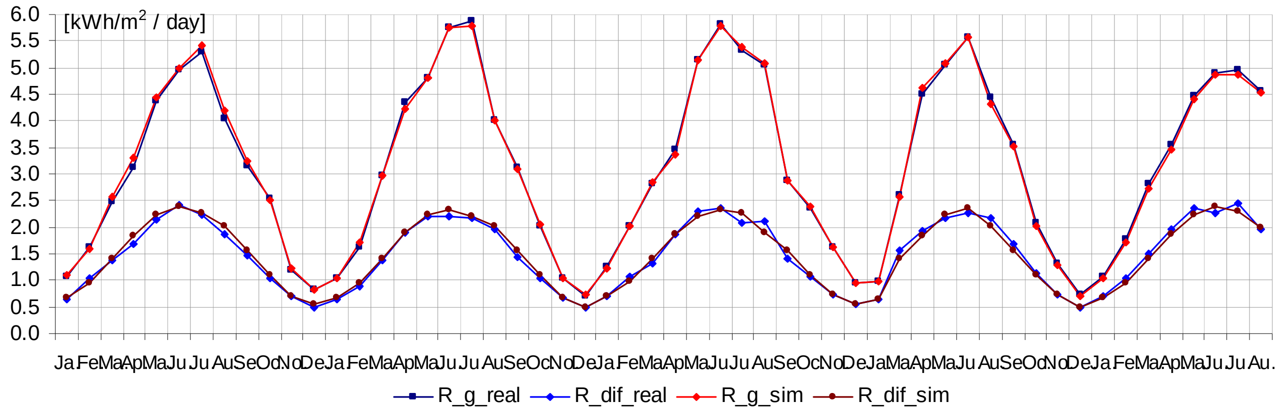

Figure 2. Estimated and real curves of the horizontal total and diffuse monthly average radiations for Braşov (R_g – total radiation; R_dif – diffuse radiation)

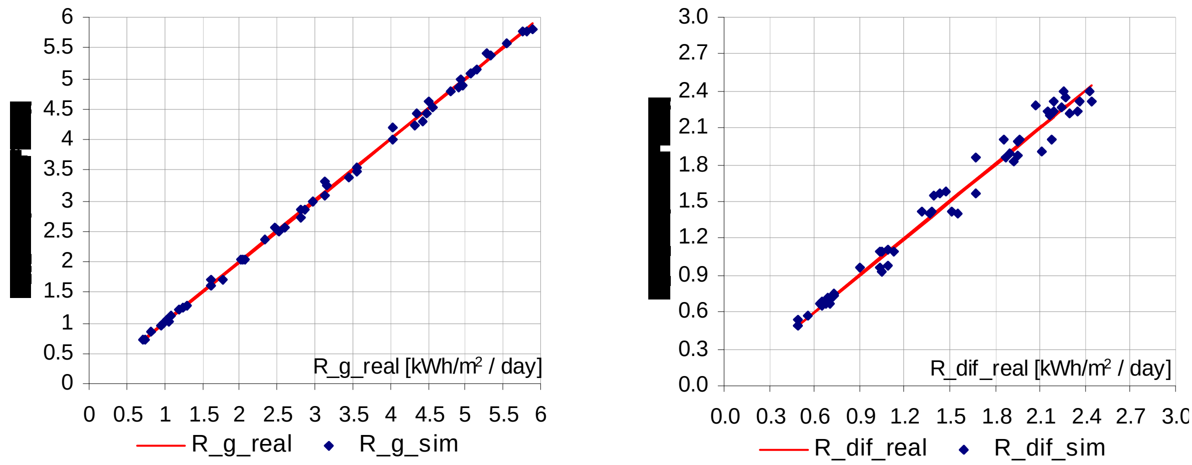

Figure 3. Estimated versus real monthly average daily values for the horizontal total and diffuse radiation

3.2. Model Performance Estimation

In this section, the paper proposes the determination of the Root Mean Square Error (RMSE), Mean Bias Error (MBE), the Mean Percentage Error (MPE) and *t-values*, with the purpose of performance estimation for the proposed models [3, 4]. The MBE, MPE, RMSE and *t-statistic* are defined by relations (3), (4), (5) and (6), [3, 4, 5]:

$$MBE = \left(\sum_{i=1}^{N} (Rc_i - Rm_i) \right) / N \text{ , [kWh/m}^2\text{]} \qquad (3)$$

$$MPE = \frac{\left(\sum_{i=1}^{N} (Rc_i - Rm_i) \right) / N}{\left(\sum_{i=1}^{N} Rm_i \right) / N} \cdot 100 \text{ , [\%]} \qquad (4)$$

$$RMSE = \sqrt{\left(\sum_{i=1}^{N} (Rc_i - Rm_i)^2 \right) / N} \text{ [kWh/m}^2\text{]} \qquad (5)$$

$$t = \sqrt{\left((N-1) MBE^2 \right) / \left(RMSE^2 - MBE^2 \right)} \qquad (6)$$

where: Rm_i and Rc_i are the i^{th} measured and model-estimated monthly values of the total or diffuse radiation; N represents the number of observations.

The MBE, MPE, RMSE and *t-values* were determined using measured data of total and diffuse solar radiations on horizontal surfaces corresponding to the period between 2006 and August 2010. The MBE test provides information on the long-term model performance [3, 4]. The MBE values reveal whether a given model has the tendency to underestimate or to overestimate the simulated values; the MBE values closest to zero are desirable. The RMSE values indicate the level of scatter that a model produces, thus providing a term-by-term comparison of the actual deviation between the estimated and real values. The values of the MBE represent the systematic error or bias, while the RMSE is a non-systematic error [3, 4].

Considering the models proposed, the paper presents the MBE and RMSE, for diffuse and total radiations, calculated for the entire database (Table 4, Table 5 and Figure 4).

Table 4. Statistical parameter values (MBE, RMSE, *t-statistic*)

Month	RMSE [kWh/m^2]	MBE [kWh/m^2]	t
January	0.02235	-0.00029	0.02567
February	0.05605	-0.00004	**0.00122**
March	0.06461	-0.00047	0.01439
April	0.12022	-0.00016	**0.00266**
May	0.04057	-0.00034	0.01661
June	0.03454	0.00042	0.02408
July	0.09187	-0.00057	0.01069
August	0.10002	0.00061	0.01227
September	0.05321	0.00015	**0.00477**
October	0.03268	-0.00003	**0.00176**
November	0.01746	0.00009	**0.00881**
December	0.02017	-0.00029	0.02510

Table 5. Statistical parameter values (MBE, RMSE, t-statistic) – Diffuse radiation estimation

Month	RMSE [kWh/m^2]	MBE [kWh/m^2]	t
January	0.02521	0.00706	0.58306
February	0.09454	-0.06410	1.59787
March	0.09888	-0.01220	0.24859
April	0.09534	-0.00278	**0.05837**
May	0.07771	0.00012	**0.00308**
June	0.09380	0.05062	1.28205
July	0.12219	0.03573	0.52968
August	0.13824	-0.02363	0.34701
September	0.12950	0.07218	1.16265
October	0.04208	0.02256	1.10014
November	0.01617	0.00829	1.03489
December	0.02602	0.01259	0.95711

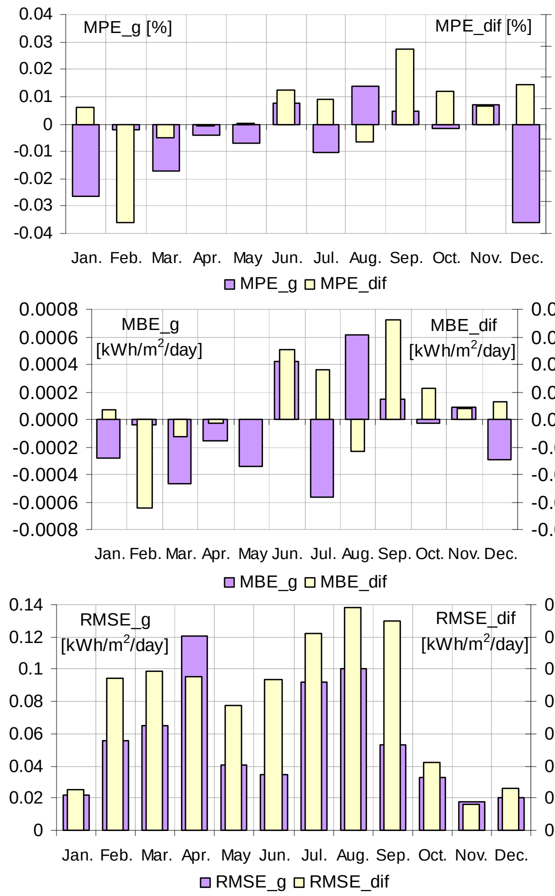

Figure 4. Monthly MPE, MBE, RMSE values

The highest MBE values were obtained for the diffuse radiation models. As it can be seen, the Mean Bias Errors for the diffuse component of the solar radiation are between –0.065 and 0.07 kWh/m^2/day. Regarding the total solar radiation, the MBE values are between –0.0006 and 0.0006 kWh/m^2/day. These values indicate that the percentage error for a single month is between -0.035% and 0.015%. The comparison between the estimated radiation components (total and diffuse) and the measured ones, leads to the conclusion that, the highest MBE values were obtained for the diffuse solar radiation during February and September (the same conclusion can be worded when the errors are expressed as percentages (MPE)).

Regarding the level of scatter that the models produce, the maximum RMSE values are obtained for total radiation during the April, July and August, and for the diffuse radiation during July, August and September. However, this study indicates that the percentage error for a single month never exceeds ±6.5% for the total or diffuse radiation. It must be emphasize that, for a correct estimation, it is not enough to have low monthly MBE and RMSE values.

The comparison between the estimated and the real total and diffuse solar radiations (monthly average daily values) was also carried out according to the *t-value*. The *t-statistic* is given as a function of the widely used root mean square error and mean bias error and helps to assess a model performance more quickly and more reliably [3, 4, 5]. The *t-values* are calculated for both proposed models (Table 4, 5). The highest *t-values* were obtained when the estimation was achieved for the diffuse radiation. The highest value was 1.59, recorded for February and this is lower than the critical *t-value*, obtained from standard statistical tables (*t_critical* at 5% is 2.132). Considering all the months (Table 4 and Table 5), the calculated *t-values* are less than the critical *t-value*, showing that the equations have statistical significance for all months. The best performance can be found by selecting the lowest *t-values*. February, April, September, October, and November are the months with the best performance considering the model for the total radiation; for the diffuse radiation model, the best performance was recorded for April and May. According to our results, the models fitted the data adequately and can be used to estimate the monthly average daily values of total and diffuse solar radiations.

As a conclusion of the comparative study of all diagrams presented, it can be asserted that the proposed models application for Braşov urban area leads to very good results. The superimposed real and estimated diagrams proved that the proposed models are efficient models in simulation of the monthly average daily solar radiation (total and diffuse radiation) for Braşov urban area.

4. Conclusions

This paper proposed an estimation model for the monthly average of the global radiation and an estimation model for the monthly average of the diffuse radiation.

The two models performance was evaluated by calculating the root mean square error (RMSE), the mean bias error (MBE), (and also *mean percentage error (MPE)*) and *t-statistic*. The RMSE, MBE, *MPE* and *t-statistic* are the most used methods for the comparison and performance estimation of the solar radiation models. These calculated statistical indicators concerning the total and diffuse radiations (monthly average daily values) are very good and they validate the two proposed estimation models. This is also proved by the high values of the coefficient of determination across the variables (R^2 = 0.99 for the global radiation model and R^2 = 0.98 for the diffuse radiation model). For all the months, the calculated *t-values* were less than the critical *t-value*, showing that the radiation models have statistical significance for all months.

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