TRACKING CONTROL OF ROBOTIC MULTI-BODY SYSTEMS

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Abstract. This paper presents a simple methodology for obtaining the entire set of continuous controllers that cause a nonlinear dynamical system to exactly track a given trajectory. The trajectory is provided as a set of algebraic differential equations that may or may not be explicitly dependent on time. The method provided is inspired by results from analytical dynamics and the close connection between nonlinear control and analytical dynamics is explored. The results provided in this paper here yield new and explicit methods for the control of highly nonlinear systems. The paper is based on previous work of the authors.

Keywords: nonlinear control, nonlinear dynamical system, multi-body systems

1. Introduction

The main specifically properties in the motion control of the robots systems are the complexity of the dynamics and uncertainties, both parametric and dynamic. Parametric uncertainties arise from imprecise knowledge of kinematics parameters and inertia parameters, while dynamic uncertainties arise from joint and link flexibility, actuator dynamics, friction, sensor noise and unknown environment dynamics.

Robot's motion trajectories are typically specified in the task space in the terms of the time history of the end-effector's position, velocities and acceleration. Operational space (also known as task space) is the space in which high-level motion and force commands are issued and executed. The operational-space formulation is therefore particularly useful in the context of motion and force control systems. On the other hand, in the joint space control methods, is assumed that the reference trajectory is available in terms of the time history of joints positions and orientations of robot arm.

The natural strategy to achieve task space control goes through two successive stages:

- in the first stage, the robot's kinematics in the task space variables is passed into the kinematics corresponding joint space variables, and then;
- in the second stage is designed the control in the joint space.

Because of the complexity of both the kinematics and dynamics of the manipulator and of the task to be carried out, the motion control problem is generally decomposed into three stages:

- motion planning,
- trajectory generation,
- trajectory tracking.

The main problem of motion robot control is to generate the motion in the task space with a given command at joints level. Motion control of robot arm accomplishes the following functions:

- to find of the corresponding movements in joints;
- to generate of control signals for the actuators to produce the input torques;
- to synthesize of programmed paths.

For trajectory tracking, the computed reference trajectory is then presented to the controller, whose function is to cause the robot to track the given trajectory as closely as possible. For design of the tracking controller, we assume that the reference trajectory and path have been pre-computed.

Control of robot manipulators is naturally achieved in the joint space, since the control input are joint torques. Nevertheless, the user specifies a motion in the task space, and thus it is important to extend the control problem to the task space. This can be achieved by different strategies. The more natural strategy consists of inverting the kinematics of the manipulator to compute the joint motion corresponding to the given end-effectors motion.

Thus, the methods used to date primarily rely on linearization and/or PID-type control, and they posit assumptions on the structure of the control effort.

2. Control of nonlinear dynamical systems

Most physical robotic systems are inherently nonlinear. Thus, control of nonlinear systems is a subject of active research and increasing interest. However, most controller design techniques for nonlinear systems are not systematic and/or apply only to very specific cases. The most general results available for nonlinear processes relate to scenarios in which:

- all uncertainty is parametric with a known functional dependence of the state-space model with respect to the unknown parameter, and
- there is no measurement noise nor disturbances.

Under these admittedly restrictive assumptions the results available are quite general. For each possible value of the parameter, one needs to know how to design a (non-adaptive) controller that would stabilize the process if the parameter value was known. Such controller should be able to guarantee input-to-state stability with respect to an appropriately introduced disturbance. For each value of the parameter, one needs to know how to design a (non-adaptive) output estimator for the process that would converge to the process output if the parameter value was known. This is a trivial matter when the whole state of the process can be measured, but can still be challenging for nonlinear systems for which the state cannot be measured.

The main challenge that remains open in the supervisory control of nonlinear systems is robustness with respect to disturbances. Although, the algorithms appear to work well in the presence of disturbances, few stability results are available.

There are few systematic procedures to design controllers/estimators for nonlinear systems that are robust with respect to disturbances. In addition, there are also few results to analyze the closed-loop switched nonlinear systems that arise as one switches among different controllers.

Current systematic approach to design controllers for nonlinear systems is feedback linearization. The basic idea of this technique is to design a control law that cancels the nonlinearities of the plant and yields a closed-loop system with linear dynamics. However, the technique is not robust to disturbances and uncertainties in the robot parameters, can yield to uncontrolled dynamics called zero dynamics and can only be applied to systems verifying certain vector field relations

The development of controllers for nonlinear complex systems has been an area of intense research. Many controllers that have been developed for trajectory tracking of complex nonlinear and multi-body systems rely on some approximations and/or linearization [2]. Most control designs restrict controllers for nonlinear systems to be affine in the control inputs [3]. Often, the system equations are linearized about the system's nominal trajectory and then the linearized equations are used along with various results from the well-developed theories of linear control.

While this often works well in many situations, there are some situations in which better controllers may be needed. This is especially so when highly accurate trajectory tracking is required to be done in real time on systems that are highly nonlinear such the robotic systems.

In the robotics literature [4, 6, 7], trajectory tracking using inverse dynamics and model reference control has been used for some time now, and the methods developed therein can be seen as particular subclasses of the formulation discussed in the present paper. Trajectory tracking in the adaptive control context (which is not the subject of this paper) has also been explored together with specific parameterizations to guarantee linearity in the unknown parameters of a system [5].

3. Controllers that cause a robotic system to track a given trajectory

This paper takes a generally different approach that is based on recent results from analytical dynamics. Here the complete nonlinear problem is addressed with no assumptions on the type of controller that is to be used, except that it will be continuous.

One considers the robot dynamics model, given by the joint-space formulation, usually presented in the canonical forms:

$$\mathbf{M}(\mathbf{q},t)\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau} \qquad (1)$$

$\mathbf{M}$ is an $n\times n$ symmetric, positive-definite matrix and is called the generalized, or joint-space, inertia matrix, $\mathbf{C}$ is an $n\times n$ matrix such that $\mathbf{C}\dot{\mathbf{q}}$ is the vector of Coriolis and centrifugal terms - collectively known as velocity product terms- and $\mathbf{g}$ is the vector of gravity terms. More terms can be added to this equation, as required, to account for other dynamical effects (e.g., viscous friction).

The symbols $\mathbf{q}$, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$, and $\boldsymbol{\tau}$ denote n-dimensional vectors of joint position, velocity, acceleration and effort variables respectively, where n is the number of degrees of motion freedom (DoF) of the robot mechanism.

This equation shows the functional dependencies explicitly: $\mathbf{M}$ is a function of $\mathbf{q}$, $\mathbf{C}$ is a function of $\mathbf{q}$ and $\dot{\mathbf{q}}$, and so on. Once these dependencies are understood, they are usually omitted.

Consider an unconstrained nonlinear mechanical robot system described by the second order differential equation of motion:

$$\begin{aligned}&\mathbf{M}(\mathbf{q},t)\ddot{\mathbf{q}}=\mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t)\\ &\mathbf{q}(0)=\mathbf{q}_0 \qquad \dot{\mathbf{q}}(0)=\dot{\mathbf{q}}_0\end{aligned} \qquad (2)$$

where, $\mathbf{q}(t)$ is the n-vector (n by 1 vector) of generalized coordinates of the robot with n DoF; the dots indicate differentiation with respect to time; and the matrix $\mathbf{M}(\mathbf{q}, t)$ is a positive definite n by n matrix.

Equations (1) and (2) can be obtained using Lagrangean model. The n-vector $\mathbf{Q}$ on the right hand side of equation (2) is a 'known' vector in the sense that it is a known function of its arguments. By 'unconstrained' one means that the components of the initial velocity $\dot{\mathbf{q}}_0$ of the robot system can be independently assigned.

By 'unconstrained' one mean here that the n coordinates, $\mathbf{q}$ are independent of one another, or are to be treated as being independent of each other.

Suppose further that the unconstrained system is now subjected to the m constraints.

One requires that this mechanical system be controlled so that it tracks a trajectory that is described by the following consistent set of m equations:

$$\mathbf{\Phi}_i(\mathbf{q},t)=0 \qquad i=1\cdots h \tag{3}$$

and

$$\mathbf{\Psi}_i(\mathbf{q},\dot{\mathbf{q}},t)=0 \qquad i=h+1,\cdots m \tag{4}$$

One assumes that the mechanical robot system's initial conditions are such as to satisfy these relations at the initial time. The latter set of equations, which are non-integrable, is non-holonomic.

In order to control the system so that it exactly tracks the required trajectory i.e. satisfies equations (3) and (4) one must apply an appropriate control n-vector $\mathbf{Q}_C(\mathbf{q},\dot{\mathbf{q}},t)$ so that the equation of motion of the controlled system becomes

$$\begin{gathered}\mathbf{M}(\mathbf{q},t)\ddot{\mathbf{q}}=\mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t)+\mathbf{Q}_C(\mathbf{q},\dot{\mathbf{q}},t)\\ \mathbf{q}(0)=\mathbf{q}_0 \qquad \dot{\mathbf{q}}(0)=\dot{\mathbf{q}}_0\end{gathered} \tag{5}$$

where now, the components of the n-vectors $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$ satisfy equations (3) and (4) at the initial time, $t = 0$.

Throughout this paper, one shall, for brevity, drop the arguments of the various quantities, unless needed for clarity.

The controlled system is described by the relation (5), where $\mathbf{Q}_c$ is the control matrix.

One begins by expressing equation (5) in terms of the weighted accelerations of the system. To control a mechanical system described by equation (2), so it exactly satisfies the trajectory described by the requirements (3) and (4) by choosing the weighting matrix to be any positive - definite n by n matrix: $\mathbf{N}(\mathbf{q},t)=\mathbf{M}^{-1}(\mathbf{q},t)$.

One denotes the acceleration of the uncontrolled system by:

$$\mathbf{a}(\mathbf{q},\dot{\mathbf{q}},t)=\mathbf{M}^{-1}(\mathbf{q},t)\mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t)\,. \tag{6}$$

In equation (4), one identifies the expression:

$$\ddot{\mathbf{q}}_C(\mathbf{q},\dot{\mathbf{q}},t)=\mathbf{M}^{-1}(\mathbf{q},t)\mathbf{Q}_C(\mathbf{q},\dot{\mathbf{q}},t) \tag{7}$$

witch can be viewed as the deviation of the acceleration of the controlled system from that of the uncontrolled system.

From equation (5), one obtains the expression:

$$\ddot{\mathbf{q}}=\mathbf{a}+\ddot{\mathbf{q}}_C \tag{8}$$

One now differentiates equation (3) twice with respect to time t, and equation (4) once with respect to time, giving the set of equations

$$\mathbf{A}(\mathbf{q},\dot{\mathbf{q}},t)\ddot{\mathbf{q}}=\mathbf{b}(\mathbf{q},\dot{\mathbf{q}},t) \tag{9}$$

where $\mathbf{A}$ is an m by n matrix of rank k and $\mathbf{b}$ is an m-vector. With equations (6) and (8) equation (9) can be further expressed as [1]:

$$\mathbf{B}(\mathbf{q},\dot{\mathbf{q}},t)\ddot{\mathbf{q}}_C=\mathbf{b}(\mathbf{q},\dot{\mathbf{q}},t) \tag{10}$$

where $\mathbf{B}$ is an m by n matrix who is calculated by the expression:

$$\mathbf{B}(\mathbf{q},\dot{\mathbf{q}},t)\ddot{\mathbf{q}}_C=\mathbf{A}(\mathbf{q},\dot{\mathbf{q}},t)[\mathbf{N}^{1/2}(\mathbf{q},t)\mathbf{M}(\mathbf{q},t)]^{-1} \tag{11}$$

One can now express the accelerations n-vector $\ddot{\mathbf{q}}$ in terms of its orthogonal projections on the range space of $\mathbf{B}^T$ and the null space of $\mathbf{B}$, so that:

$$\ddot{\mathbf{q}}=\mathbf{B}^{+}\mathbf{B}\;\ddot{\mathbf{q}}\;+(\mathbf{I}-\mathbf{B}^{+}\mathbf{B})\ddot{\mathbf{q}} \tag{12}$$

In equation (12), the matrix $\mathbf{B}^+$ denotes the Moore–Penrose generalized inverse of the matrix $\mathbf{B}$. It should be noted that equation (12) is a general identity that is always valid since it arises from the orthogonal partition of the identity matrix $\mathbf{I}=\mathbf{B}_S^{+}\mathbf{B}_S+(\mathbf{I}-\mathbf{B}_S^{+}\mathbf{B}_S)$.

Using equation (10) in the first member on the right hand side of equation (12), and equation (8) to replace $\ddot{\mathbf{q}}$ in the second member, one gets:

$$\ddot{\mathbf{q}}=\mathbf{B}^{+}\mathbf{b}+(\mathbf{I}-\mathbf{B}^{+}\mathbf{B})(\mathbf{a}+\ddot{\mathbf{q}}_C) \tag{13}$$

which, owing to equation (7), yields:

$$\mathbf{B}^{+}\mathbf{B}\;\ddot{\mathbf{q}}_C=\mathbf{B}^{+}(\mathbf{b}-\mathbf{B}\mathbf{a}) \tag{14}$$

The general solution of the linear set of equations (14) is given by [1]:

$$\ddot{\mathbf{q}}_C=(\mathbf{B}^{+}\mathbf{B})^{+}\mathbf{B}^{+}(\mathbf{b}-\mathbf{B}\mathbf{a})+[\mathbf{I}-(\mathbf{B}^{+}\mathbf{B})^{+}(\mathbf{B}^{+}\mathbf{B})]\mathbf{z} \tag{15}$$

After any combination one obtains the second equality:

$$\ddot{\mathbf{q}}_C=\mathbf{B}^{+}(\mathbf{b}-\mathbf{B}\mathbf{a})+(\mathbf{I}-\mathbf{B}^{+}\mathbf{B})\mathbf{z} \tag{16}$$

where the n-vector $\mathbf{z}(\mathbf{q},\dot{\mathbf{q}},t)$ is any arbitrary n-vector. To obtain the second equality above, one used the property that $(\mathbf{B}^{+}\mathbf{B})^{+}=(\mathbf{B}^{+}\mathbf{B})$ in the two members on the right hand side along, with the property so that $\mathbf{B}^{+}\mathbf{B}\mathbf{B}^{+}=\mathbf{B}^{+}$.

The set of all possible controls $\mathbf{Q}_C(\mathbf{q},\dot{\mathbf{q}},t)$ (or controllers) that causes the controlled system to exactly track the required trajectory is explicitly given by

$$\mathbf{Q}_c(\mathbf{q},\dot{\mathbf{q}},t) = \mathbf{N}^{-1/2}\ddot{\mathbf{q}}_c = \\ = \mathbf{N}^{1/2}\mathbf{B}^{+}(\mathbf{b}-\mathbf{Ba}) + \mathbf{N}^{1/2}(\mathbf{I}-\mathbf{B}^{+}\mathbf{B})\mathbf{z} \quad (17)$$

The mechanical robotic system, described by the nonlinear Lagrange equation (1), is explicitly controlled through the addition of a control, *n*-vector $\mathbf{Q}_c(\mathbf{q},\dot{\mathbf{q}},t)$, provided by equation (17), in witch the *n*-vector ***z*** may be chosen still ensures that the description of the trajectory given by equations (3) and (4) is exactly satisfied.

For spatial tests to reduce the number of virtual pairs judicious comparisons, we assume the environment is quite free and the end-effectors move in such a way that the geometric coherence can be preserved, i.e. the assumption that the motion is essentially continuous in time domain.

5. Conclusions

This paper presents the motion in terms of second-order differential equations. This methodology has been inspired by results in analytical dynamics.

The explicit closed-form expression (17) provides the entire set of continuous tracking controllers that can exactly track a given trajectory description, assuming that the system's initial conditions satisfy the description of the trajectory. The explicit closed-form expressions for the controllers can be computed in real time.

Closed-form expressions for all the continuous controllers required for trajectory tracking for nonlinear systems do not make approximations. Furthermore, no approximations or linearization are made here with respect to the trajectory that is being tracked, which may be described in terms of nonlinear algebraic equations or nonlinear differential equations. Moreover, the approach arrives not just at one nonlinear controller for controlling a given nonlinear system, but also at the entire set of continuous controllers that would cause a given set of trajectory descriptions to be exactly satisfied.

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