GENERAL MATHEMATICAL MODEL FOR TYPE-DIMENSIONAL SYNTHESIS OF FUNCTION-GENERATING MECHANISMS WITH FOUR-BAR TOPOLOGICAL STRUCTURE

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Abstract. The present work is devoted to the developing of a general mathematical model for type-dimensional synthesis of function-generating mechanisms with four-bar topological structure and rotation of the input and the output. The known cases of particular belt mechanisms and cam mechanisms can be obtained in corollary from the mathematical model.

Keywords: function-generating mechanism, synthesis

1. Introduction

Type-dimensional synthesis [1, 2, 3] of function-generating mechanisms with four-bar topological structure (centrode mechanism, belt mechanism, cam mechanism and four-bar linkage), can be divided into three stages:

- selection of the competing kinds of mechanisms with a four-bar topological structure – performed on the basis of potentialities of mechanisms to generate specified transfer functions;
- synthesis of the competing kinds of mechanisms – performed on the basis of a general mathematical model, by means of which all competing types of mechanisms are being synthesized simultaneously;
- selection of perspective kinematic schemes and final choice of a mechanism – kinematic schemes obtained after the synthesis are estimated by the four criteria (geometry, technology, accuracy and operation). An objective function is introduced and the schemes possessing the highest rate are separated as the final choice is done after constructive development of mechanisms and possibly prototyping and experimental analysis.

Aim of the present work is the development of a general mathematical model for type-dimensional synthesis of function-generating mechanisms with four-bar topological structure and rotation of the input and the output.

2. General mathematical model for type-dimensional synthesis

The synthesis of the competing kinds of mechanisms is done by given position function $\psi(\varphi)$ and its derivatives according general kinematic scheme and general mathematical model (Figure 1) that is based on the relation between transfer functions and mechanisms geometrical dimensions. Variables under synthesis are transfer functions $S_{\varphi}(\varphi)$ and its derivatives as $S'_{\varphi}(\varphi)$ is geometrical equivalent of the projection v_{2_A} onto straight line 2 of the velocity $\overrightarrow{v_A}$ of the point A (Figure 1).

On the first line of the model (Figure 1.b), the position of the relative instant center of velocity *P* is determined toward the coordinate system Oxy. That position depends only on the value of the first transfer function $\psi'(\varphi)$.

Successive positions of the point *P* in the plane Ox_1y_1 of the input link 1 and Cx_3y_3 of the output link 3 are respectively two centrodes of relative motion z_1 and z_3 (Figure 1) representing input and output links of the centrode mechanism (line 7, Figure 1.b).

On the mathematical model second line the value of the slope $k_2 = \tan\alpha$ of the straight line 2, depending on $S'_{\varphi}(\varphi)$ and $\psi'(\varphi)$, is determined.

On the third line of the mathematical model the value of the transfer function $S''_{2_A}(\varphi)$, depending on $S'_{\varphi}(\varphi)$, $S''_{\varphi}(\varphi)$, $\psi'(\varphi)$ and $\psi''(\varphi)$, is determined.

The position of point *A* toward fixed coordinate system Oxy, depending on $S'_{\varphi}(\varphi)$, $S''_{\varphi}(\varphi)$ and $k_2(\varphi)$, is determined on the fourth line of the model.

On the next line the values of the transfer functions $S'_{\psi}(\varphi)$ and $S''_{2_B}(\varphi)$, depending on $S'_{\varphi}(\varphi)$, $\psi'(\varphi)$ and $S'_{\varphi}(\varphi)$, $S''_{\varphi}(\varphi)$, $\psi'(\varphi)$, $\psi''(\varphi)$ respectively, are determined.

On the sixth line of the mathematical model the position of point *B* in the fixed coordinate system Oxy, depending on $S'_{\psi}(\varphi)$, $S''_{2_B}(\varphi)$ and $k_2(\varphi)$, is determined.

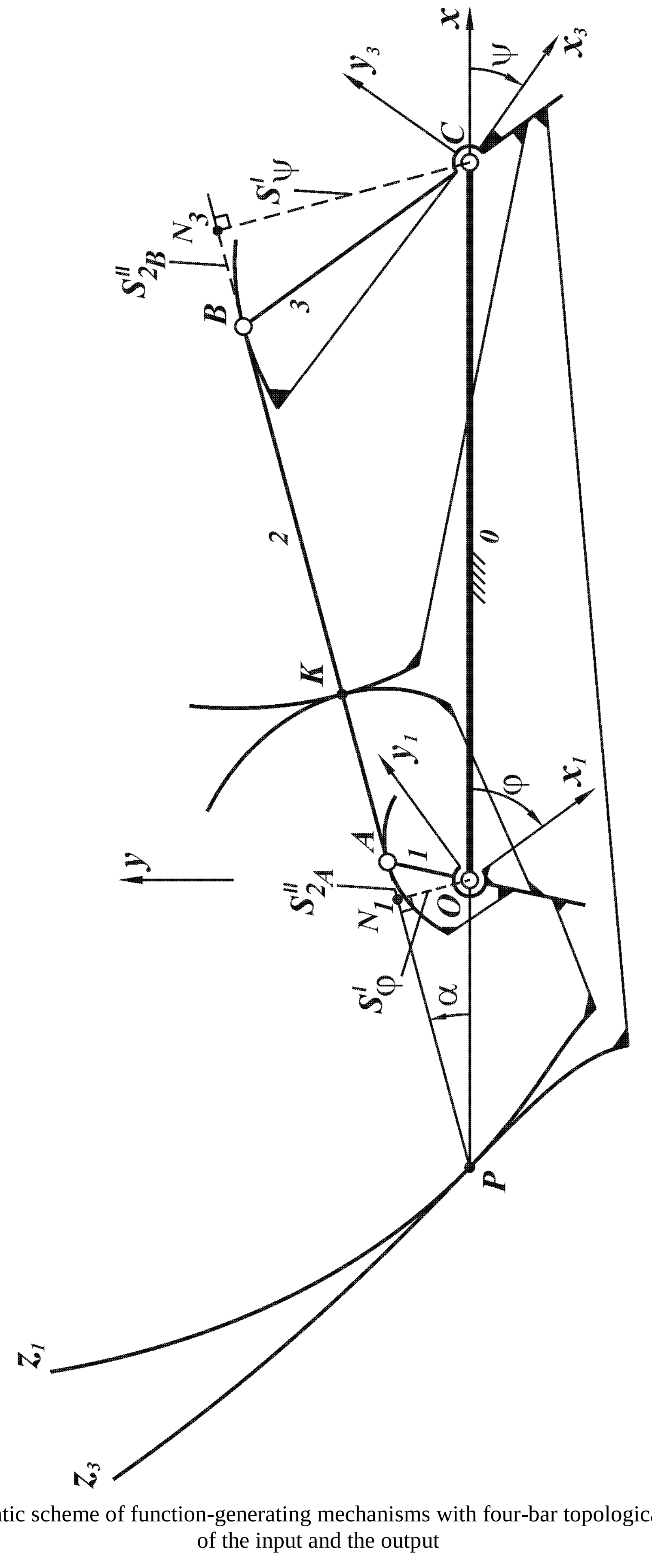

Figure 1. General kinematic scheme of function-generating mechanisms with four-bar topological structure and rotation of the input and the output

№	Kind of mechanism		
	centrode mechanism	belt mechanism	cam mechanism
1	$x_P = L\psi'/(\psi'-1)$, $\quad y_P = 0.$		
2	-	$k_2 = \dfrac{S'_\varphi(1-\psi')}{\sqrt{L^2\psi'^2 - S'^2_\varphi(\psi'-1)^2}}$	
3	-	$S''_{2A} = \dfrac{S''_\varphi}{\operatorname{sign}[1-\psi']\dfrac{S''_\varphi(\psi'-1)\psi' + S'_\varphi\psi''}{\psi'\sqrt{(L\psi')^2 - S'_\varphi{}^2(\psi'-1)^2}} + 1}$	
4	-	$x_A = -S'_\varphi\dfrac{k_2}{\sqrt{1+k_2^2}} + S''_{2A}\dfrac{1}{\sqrt{1+k_2^2}}, \quad y_A = S'_\varphi\dfrac{1}{\sqrt{1+k_2^2}} + S''_{2A}\dfrac{k_2}{\sqrt{1+k_2^2}}.$	
5	-	$S'_\psi = S'_\varphi/\psi'$ $S''_{2B} = \dfrac{S''_\varphi\psi' - S'_\varphi\psi''}{\left(\operatorname{sign}[1-\psi']\dfrac{S''_\varphi(\psi'-1)\psi' + S'_\varphi\psi''}{\psi'\sqrt{(L\psi')^2 - S'_\varphi{}^2(\psi'-1)^2}} + \psi'\right)\psi'^2}$	
6	-	$x_B = L - S'_\psi\dfrac{k_2}{\sqrt{1+k_2^2}} + S''_{2B}\dfrac{1}{\sqrt{1+k_2^2}}, \quad y_B = S'_\psi\dfrac{1}{\sqrt{1+k_2^2}} + S''_{2B}\dfrac{k_2}{\sqrt{1+k_2^2}}.$	
7	$x_{1_P} = x_P\cos\varphi,\quad x_{3_P} = (x_P - L)\cos\psi,$ $y_{1_P} = -x_P\sin\varphi.\quad y_{3_P} = -(x_P - L)\sin\psi.$		
8	-	$x_{1_A} = x_A\cos\varphi + y_A\sin\varphi,\quad x_{3_B} = (x_B - L)\cos\psi + y_B\sin\psi,$ $y_{1_A} = -x_A\sin\varphi + y_A\cos\varphi.\quad y_{3_B} = -(x_B - L)\sin\psi + y_B\cos\psi.$	
9	-	-	$x_K = x_A + (\pm\rho_0 \pm \int_0^\varphi \sqrt{x'_{1_A} + y'_{1_A}}\,d\varphi)$ $\dfrac{1}{\sqrt{1+k_2^2}},$ $y_K = y_A + (\pm\rho_0 \pm \int_0^\varphi \sqrt{x'_{1_A} + y'_{1_A}}\,d\varphi)$ $\dfrac{k_2}{\sqrt{1+k_2^2}}.$
10	-	-	$x_{1_K} = x_K\cos\varphi + y_K\sin\varphi,$ $y_{1_K} = -x_K\sin\varphi + y_K\cos\varphi.$ $x_{3_K} = (x_K - L)\cos\psi + y_K\sin\psi,$ $x_{3_K} = -(x_K - L)\sin\psi + y_K\cos\psi.$

Figure 2. General mathematical model for synthesis of the competitive kinds of mechanisms with rotation of the input and the output

Successive positions of the point *A* in the plane Ox_1y_1 of the input link 1 and the point *B* in the plane Cx_3y_3 of the output link 3 (Figure 1) are outlines of the input and output links of the belt mechanism respectively. At the same time they are evolutes of the input and output outlines of the cam mechanism (line 8, Figure 2).

On the ninth line of the mathematical model the position of point *K*, that belongs to the belt mechanisms and it is a contact point between two cam outlines of the cam mechanism (Figure 1), is determined with respect to the fixed coordinate system Oxy. The variable parameter

$$\rho_0 = \sqrt{\left(x_{A_0} - x_{K_0}\right)^2 + \left(y_{A_0} - y_{K_0}\right)^2}$$

is a distance between point *A* and point *K* when $\varphi = 0$. The following integral

$$\int_0^{\varphi} \sqrt{x_{1_A}^{'} + y_{1_A}^{'}}\, d\varphi$$

is the length of the input evolute 1 (belt mechanism input link) from the initial position A_0 of point *A* when $\varphi = 0$ to the current position *A* with corresponding value of φ. That length is equal to the length of the rolled up, respectively rolled down, onto input evolute 1 belt.

The signs „+“ and „–“ in front of the integrals on the ninth line are determined by concavity of the corresponding segment of the cam 1 of the belt mechanism – if the segment is concave (under straight line 2) or convex (over straight line 2), and by sign of $(\alpha'(\varphi) - 1)$: sign „+“ when the segment is convex and $(\alpha'(\varphi) - 1) > 0$, or the segment is concave and $(\alpha'(\varphi) - 1) < 0$; sign „–“ when the segment is convex and $(\alpha'(\varphi) - 1) < 0$, or the segment is concave and $(\alpha'(\varphi) - 1) > 0$. The sign in front of ρ_0 is determined by the sign of the coordinate difference $\left(x_{K_0} - x_{A_0}\right)$.

Successive positions of the point *K* in the planes Ox_1y_1 of the input link 1 and Cx_3y_3 of the output link 3 are respectively input and output cams of the cam mechanism shown in Figure 1, which are respectively noncircular evolvents of the input and output links of the belt mechanism (line 10, Figure 2).

3. Conclusion

In the present work the general mathematical model for type-dimensional synthesis of function-generating mechanisms with four-bar topological structure and rotation of the input and the output is developed. The model is characterized by the following:

- the synthesis simultaneously generates the outlines of two links with a simple movement of the belt and cam mechanisms (representing generally noncircular cams), in contrast to existing mathematical models for synthesis where one of the links with simple movement is a priori chosen and as a result of the synthesis the other one is obtained;
- the developed mathematical model allows to obtain conditions under which the cams of the cam and belt mechanisms are smooth curves;
- the evolutes of input and output links of the belt mechanism, depending on the derivatives of the variable transfer function $S_\varphi(\varphi)$ and the given transfer function $\psi(\varphi)$ to the fourth order inclusive, can be obtained. This leads to the full disclosure of the mechanism geometry;
- the known cases of particular belt mechanisms and cam mechanisms can be obtained as corollary from the mathematical model.

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