ABOUT THE INDEPENDENT SETS OF PERIODICALLY MESHING TEETH IN GEAR DRIVES PART II

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Abstract. This article relies on the concept of independent sets of meshing teeth, which affect numerous performance parameters of gear drives. The influence of these contact sets on the noise- and vibration activity, as well as their effect on the reliability and longevity of power gear drives are taken into consideration. Conclusions are made regarding the application of the above concept to the construction design of various gear drives.

Keywords: independent sets of meshing teeth; power gear drives; performance parameters of gear drives

1. Introduction

Part I of the paper dedicated to this subject of research [3] clarifies the problem of proper application of the concept of independent sets of meshing teeth in gear drives.

Part II focuses on gear drives which have more than one independent set of meshing teeth ($N_A > 1$). The concept of independent sets of meshing teeth is known in the literature [3, 11]. In most cases however, the latter lacks fundamental explanations about its essence, the choice of these sets and their effect on the performance parameters of gear drives. As a result in the design process of gear drives this important principle is often neglected or underestimated. The present article provides a detailed explanation, with special attention given to the influence of the number of independent sets of meshing teeth on gear drive operational properties.

2. Exposure

2.1. Gear drives with more than one independent set of meshing teeth ($N_A > 1$)

The process of gear drive operation is linked to the following circumstances:

1) The meshing of two teeth in an engaged gear drive is accompanied by friction between the active teeth surfaces. This leads to uneven wear, which declares itself in the removal of a certain amount of material from the surfaces of the contacting teeth. As a result the teeth profiles change in shape. From [3] it is known, that the teeth from each independent mesh set only ever contact teeth from the same set.

2) Because of inaccuracies in manufacturing of the gear drive (gear wheels, shafts, bearings, housing, etc.), the tooth profile shape, as well as the mutual disposition of two contacted teeth, will not be identical for all teeth.

Based on the above two statements it can be concluded that in a gear drive which features a factor $N_A > 1$ (N_A - the number of independent sets of meshing teeth), each separate meshing set exhibits an individual pattern and grade of wear of the teeth in this set, independent of the pattern and grade of wear of the teeth in the rest of the meshing sets. Therefore the factor N_A can also be interpreted as the number of independent wear patterns in a gear drive. In some cases gear drives with $N_A > 1$ are prone to deterioration of their performance quality. Example: in a gear drive of the type shown in Figure 1 in the starting position tooth №1 of the small wheel contacts tooth №1 of the large wheel (Figure 2 – Phase 1) [3]. The contact sequences of the teeth from each independent set are presented in Table 1.

If in starting position the initial contact is between tooth №1 of the small wheel and tooth №2 of the large wheel (Figure 2 – Phase 2) there will be a shift in the meshing sequence in each independent set. The contacting teeth numbers in this case are shown in Table 2.

The third possible option, which features the same number of independent meshing sets for the gear drive, but once again with different contacting teeth numbers in each set, is presented in Table 3. In this case in the starting position the initial contact is between tooth №1 of the small wheel and tooth №3 of the large wheel (Figure 2 – Phase 3).

Each of the depicted engagement options of the gear drive can be achieved if the drive is disassembled and then assembled in the specified position, i.e. in a different contact phase. Every gear drive can be assembled in exactly that many contact phases as is the number of independent sets of meshing teeth for this drive. Because of this in western literature N_A is also known as *assembly phase factor*.

The above situation gives rise to the following question – what would happen if the gear drive is disassembled (e.g. for inspection, maintenance, bearing change, etc.) and then unintentionally assembled in a contact phase different from the one present prior to disassembly?

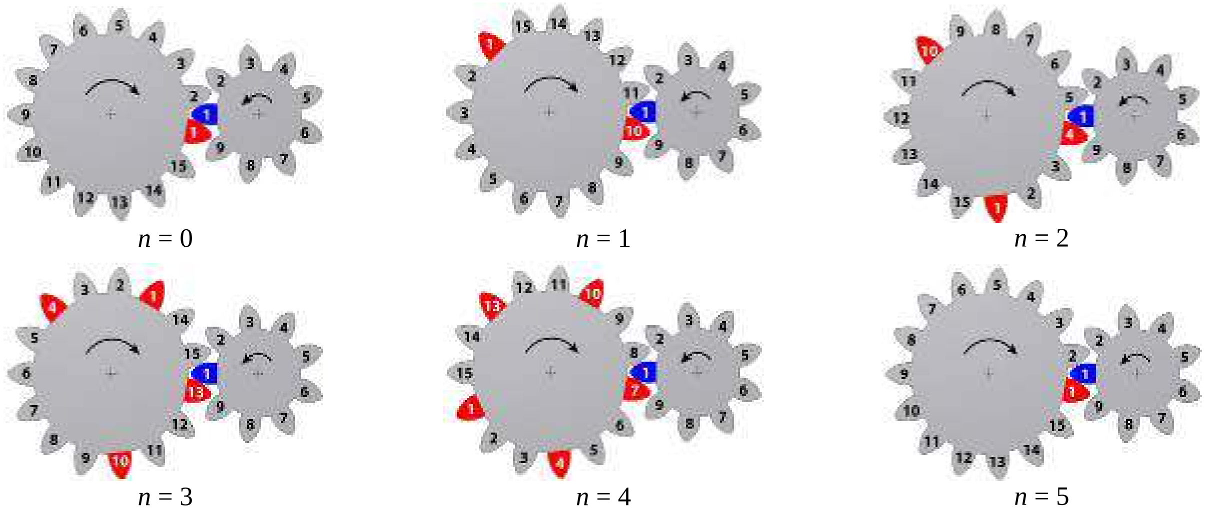

Figure 1. Teeth meshing sequence at every full revolution of the small wheel of the gear drive

Table 1

I	Teeth of the small wheel №	Teeth of the large wheel №
A	1-7-4	1-10-4-13-7
B	2-8-5	2-11-5-14-8
C	3-9-6	3-12-6-15-9

Table 2

II	Teeth of the small wheel №	Teeth of the large wheel №
A	1-7-4	2-11-5-14-8
B	2-8-5	3-12-6-15-9
C	3-9-6	1-10-4-13-7

Table 3

III	Teeth of the small wheel №	Teeth of the large wheel №
A	1-7-4	3-12-6-15-9
B	2-8-5	1-10-4-13-7
C	3-9-6	2-11-5-14-8

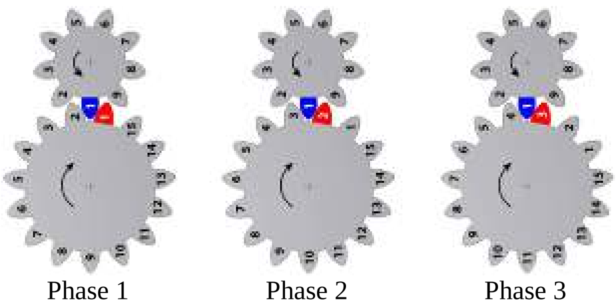

Figure 2. Initial contact phases for a gear drive with $z_1 = 9$, $z_2 = 15$

During prolonged operation of the gear drive in a particular contact phase each of the independent sets of meshing teeth will develop such a wear pattern, which at the utmost compensates the manufacturing and assembly inaccuracies. This process is known as running-in. Since assembling the gear drive in a contact phase different from the original one leads to teeth swapping between the independent meshing sets, then the favourable wear patterns developed by this time will no longer be legitimate and each newly formed teeth set will have to run-in all over again [5, 8]. Depending on the operating conditions, the types of materials and the applied chemical and thermal treatment in the gear wheel manufacturing, the time necessary for the running-in process can vary from several tens to several hundred hours. During this period the gear drive will express elevated noise and vibration levels. As a confirmation of this fact Figure 3 and Figure 4 show a comparison between the vibration frequency spectrum of a gear drive under normal operation and after assembly phase change.

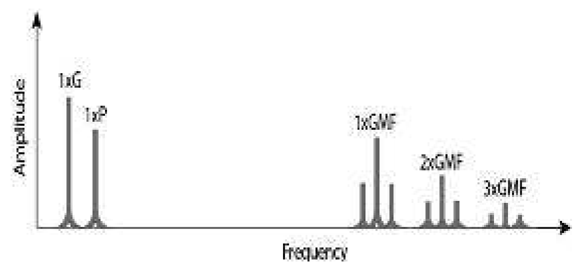

Figure 3. Vibration frequency spectrum under normal operation

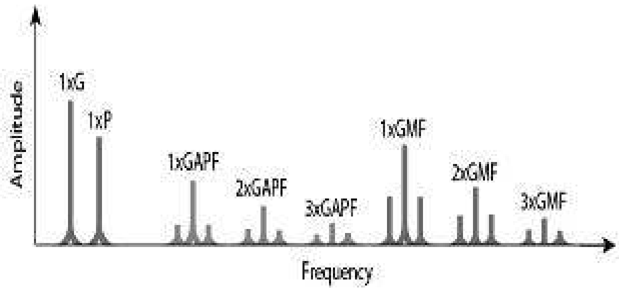

Figure 4. Vibration frequency spectrum after assembly phase change

The normal vibration frequency spectrum of the gear drive is characterized with peaks by the following frequencies:

- rotation frequency of the shaft of the small wheel n_P (*P* – pinion – small gear wheel), in [s^{-1}]:

$$n_P = \frac{n_1}{60};\tag{1}$$

- rotation frequency of the shaft of the large wheel n_G (*G* – gear – large gear wheel), in [s^{-1}]:

$$n_G = \frac{n_2}{60};\tag{2}$$

- *gear mesh frequency* (*GMF*) n_{GMF} and its 2x and 3x harmonics:

$$n_{GMF} = z_1 \times P = z_2 \times G\ .\tag{3}$$

Teeth profile change due to wear of contact surfaces leads to the occurrence of vibrations with frequencies that are subharmonic to n_{GMF}. In vibration analysis the fundamental frequency of these vibrations (their 1x component) is denoted with the abbreviation *GAPF – Gear Assembly Phase Frequency:*

$$GAPF = \frac{n_{GMF}}{N_A}\ .\tag{4}$$

The presence of vibrations with frequencies equal to *GAPF* and its harmonics is the most obvious change in the vibration spectrum of a gear drive that has undergone an assembly phase swap [5, 11]. In order to prevent reassembly in a wrong phase, the wheels of gear drives with factor $N_A > 1$ must be marked before being taken off the shafts when carrying through inspection and maintenance procedures. This is usually done by marking two adjacent teeth of the large wheel with X, and the tooth of the small wheel that goes in-between – with O (Figure 5) [7, 11]. When necessary, the wheels of gear drives with factor $N_A > 1$ must be replaced in couples, it is not acceptable to replace only the small or only the large wheel.

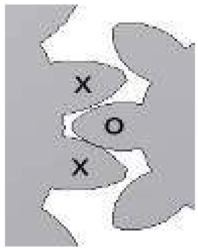

Figure 5. Marking of gear drives with $N_A > 1$

For gear drives with more than one independent set of meshing teeth especially problematic may be the case when one or more teeth have been manufactured with a defect (imperfection) in their shape, since as operation time advances this might lead to severe performance deterioration. The occurrence of a defect in tooth shape may happen on every stage of the manufacturing process – from thermal treatment to final grinding, whereas the cause of the defect can span over a wide range of possibilities: low material quality, processing errors, cutter tools wear, incorrect adjustment of the production machines, etc. [3, 7, 10]. In order to make a conclusion about the importance of the studied problem, the wear mechanism in gear drives with more than one independent set of meshing teeth needs to be taken into detailed consideration. Let W_Σ denote the total wear volume of the teeth of a gear drive with a defect tooth for an amount of time *t*. This volume is provisionally divided in two components:

$$W_\Sigma = W_{\Sigma N} + W_{\Sigma D},\tag{5}$$

where:

- $W_{\Sigma N}$ is the normal total wear volume of the teeth caused by regular operation of the gear drive when there are no teeth with shape faults;
- $W_{\Sigma D}$ is the „additional“ total wear volume of the teeth, caused by the presence of a defect tooth.

The scale of $W_{\Sigma D}$ depends on the size of the defect. In most cases it is small and leads to insignificant increase of W_Σ, which does not cause a substantial effect on the performance characteristics of the gear drive:

$$W_{\Sigma D} < W_{\Sigma N}\ .\tag{6}$$

The main disadvantage of gear drive with factor $N_A > 1$ is that $W_{\Sigma D}$ is distributed only over a limited number of teeth, namely those in the same meshing set as the defect tooth [4]. For example, for the gear drive from Figure 1 in Phase 1, if it is assumed that tooth №1 of the small wheel is defect, then $W_{\Sigma D}$ will be distributed only over teeth № 1, 4, 7, 10 and 13 of the large wheel (Figure 6).

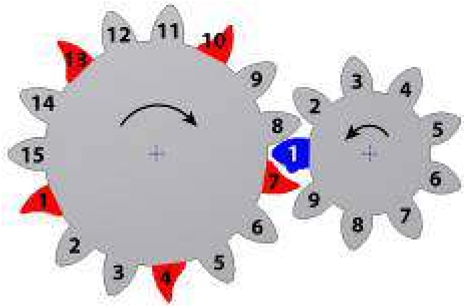

Figure 6. Uneven distribution of $W_{\Sigma D}$

No matter which of the teeth of the small wheel has a defect, the additional wear caused by this defect will be distributed only over 5, and not over all 15 teeth of the large wheel. In this situation the faulty tooth becomes the reason for a significant increase of the individual wear volume W of the teeth that come in contact with it:

$$W = W_N + W_D ; \qquad (7)$$

$$W_D < W_N . \qquad (8)$$

Promoted wear of the affected teeth leads to increase in tooth profile deviation and growth of the backlash of the gear drive, which in turn causes high impact loading, accompanied by elevated noise and vibration levels. The decrease of the cross-section area of the teeth due to high wear reduces their bending strength and raises the fracture probability, which diminishes the gear drive reliability [1]. In the case of surface hardened wheels – case-hardened, induction-hardened, etc., intense wear causes rapid thinning of the surface layer and developing progressive pitting. This in turn is a prerequisite for teeth decay [7]. Therefore power gear drives with $N_A > 1$ that contain a defect tooth are prone to performance quality deterioration, or even complete operation incapability over time. The higher the factor N_A, the less is the number of teeth over which the total wear volume $W_{\Sigma D}$ distributes, consequently failure probability will rise. When there is a defect in the small wheel, $W_{\Sigma D}$ will distribute over z_2/N_A number of teeth of the large wheel, and vice versa – when there is a defect in the large wheel, $W_{\Sigma D}$ will distribute over z_1/N_A number of teeth of the small wheel [11]. Teeth on one wheel affected by a defect tooth of the other wheel afterwards negatively affect the rest of the teeth they mesh with. For example, when the gear drive from Figure 1 has a defect tooth №1 of the small wheel, it initially affects teeth № 1, 4, 7, 10 and 13 of the large wheel. But as operation time advances these teeth will also affect the wear of teeth № 4 and 7 of the small wheel.

Aside for the manufacturing process, a defect in tooth shape may also occur in the following cases [7, 10]:

- solid particles passing through the gear drive, due to contaminated and poorly filtered lubricant, inadequate sealing of the housing from the environment, insufficient ventilation of the housing (causes a suction effect);
- the gear drive is under the influence of vibrations from other machines or in the case of improper transportation, when the teeth rub and slam each other;
- the gear drive is stored without the necessary protection from moisture and teeth surface corrosion becomes probable;
- the gear drive is operated under frequent overloads, switching, passing trough resonant frequencies, etc.

In all of these cases the possible defects occur not on random teeth, but on one or more teeth pairs that periodically come into mesh. Every engagement cycle of the defect teeth pair causes a strong vibration [9]. The frequency of this vibration equals the contact frequency of the teeth from the faulty pair. In western literature this frequency is known as *Tooth Repeat Frequency (TRF)*, or the more of then used and equal in meaning *Hunting Tooth Frequency (HTF)*. The *HTF* is calculated with the following formula:

$$HTF = \frac{GMF \cdot N_A}{z_1 \cdot z_2} . \qquad (9)$$

Equation (9) shows that a higher factor N_A is associated with a higher frequency of the vibrations, which aggravates the negative effect of the presence of a defect teeth pair in the gear drive. When a teeth pair is defect the 1x and 2x components of *HTF* appear in the vibration frequency spectrum of the gear drive (Figure 7), as well as sidebands withstanding at ± *HTF* from the frequencies *G, P* and/or *GMF* [5]. In this case the gear drive produces a distinctive growling sound [9].

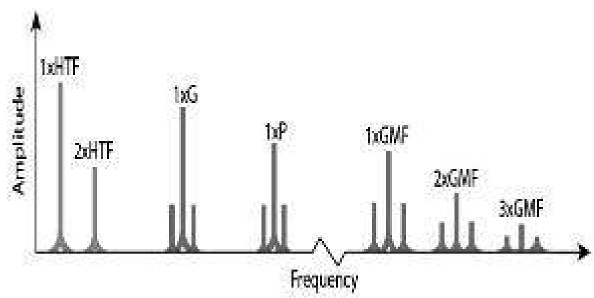

Figure 7. Vibration frequency spectrum when a defect teeth pair is present

2.2. Gear drives with one independent set of meshing teeth (N_A = 1)z

The solution of the described problems lies in reducing the number of independent sets of meshing teeth to one. With factor N_A = 1 the following circumstances are observed:

1) <u>*All teeth are in the same meshing set,*</u> i.e. over time each tooth of the small wheel engages with each tooth of the large wheel (Figure 8). In this case only one wear pattern is established, and it is common to all teeth. The assembly position of the gear drive only affects the meshing sequence of the teeth, but not which teeth contact each other. Therefore the gear drive only has one contact phase and its operation is independent of the mutual disposition of the wheels *(GAPF = GMF)* [8]. The wheels of gear drives with N_A = 1 do not need to be marked before disassembly.

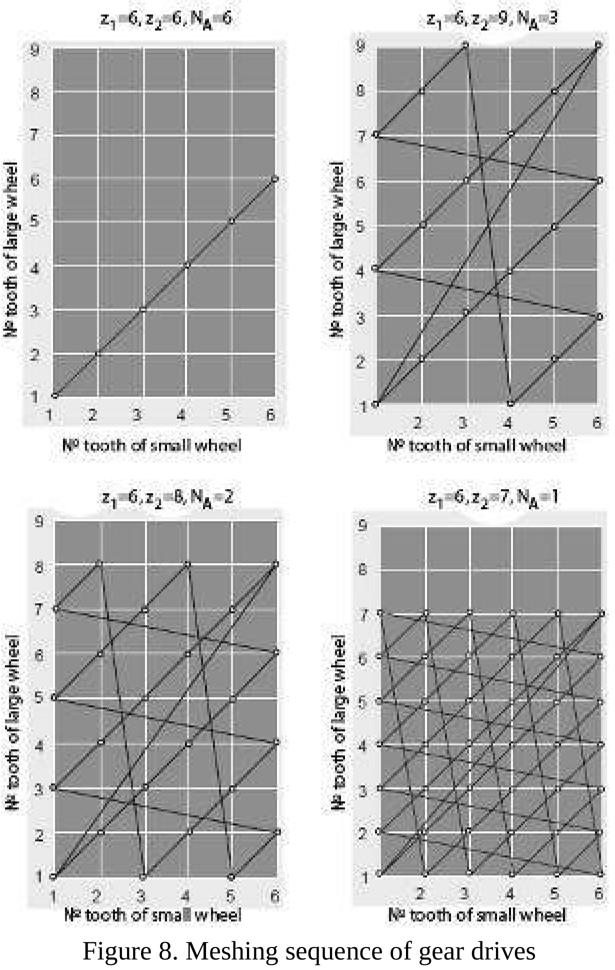

Figure 8. Meshing sequence of gear drives with different values of N_A

2) <u>*Wear due to a defect tooth $W_{\Sigma D}$ distributes over the maximum possible number of teeth.*</u> This eliminates the potential of wear concentration on certain teeth, which minimizes failure probability and performance quality deterioration. Even in the worst case scenario, the time till failure occurs is significantly prolonged, therefore there is a better chance that the changes in the gear drive operation behaviour will be noticed and preventive actions will be taken.

3) <u>*Before a teeth pair re-enters in mesh, both participating teeth must first mesh with all other teeth of the small/large wheel.*</u> This ensures that the time interval between the repeated contacts of two particular teeth reaches its theoretical maximum. Therefore if a teeth pair is defect, the frequency of the caused vibrations *HTF* will be the lowest possible [8].

To achieve a factor N_A = 1 such number of teeth z_1 and z_2 must be chosen, for which the greatest common divisor is unity. Such appropriate combinations of z_1 and z_2 are for example:

$$\frac{z_2}{z_1} = u \quad \rightarrow \quad \frac{27}{14} \approx 1.93; \quad \frac{41}{15} \approx 2.73 . \qquad (10)$$

On the other hand, combinations that do not meet the above condition are for example:

$$\frac{z_2}{z_1} = u \quad \rightarrow \quad \frac{26}{14} \approx 1.86; \quad \frac{40}{15} \approx 2.67 . \qquad (11)$$

In the literature gear drives with factor N_A = 1 are known under the specific term *hunting tooth gear set* or *hunting ratio gear set* [11]. This name originates from the idea of supplementing an additional tooth to the large wheel so as to reduce the number of independent sets of meshing teeth to unity. Some transmission numbers can not be achieved with combinations that ensure N_A = 1. In particular, such are all integer transmission numbers u = 2, 3, 4, …:

$$\frac{z_2}{z_1} = u \quad \rightarrow \quad \frac{32}{16} = 2; \quad \frac{45}{15} = 3; \quad \frac{56}{14} = 4; ... \qquad (12)$$

Gear drives with integer transmission numbers always have a factor N_A higher than one [1, 4, 10]. Since power gear drives allow a certain deviation from the requested transmission number u [2], in the general case integer transmission numbers can be avoided.

For example, for a requested u = 2 instead of the teeth number combination:

$$u = \frac{z_2}{z_1} = \frac{32}{16} = 2 \quad (N_A = 16) , \qquad (13)$$

the following combination can be chosen:

$$u = \frac{z_2}{z_1} = \frac{33}{16} = 2.0625 \approx 2 \quad (N_A = 1)\,. \tag{14}$$

The acceptable transmission number deviations are summarized in Table 4.

Table 4. Acceptable transmission number deviation for power gear drives

Single-stage	$\max \Delta u = \pm 3\%$
Two-stage	$\max \Delta u_{total} = \pm 4\%$, and $\max \Delta u_1 = \max \Delta u_2 = \pm 2\%$

With single-stage kinematic gear drives that demand precise integer transmission numbers unfortunately it is impossible to achieve a factor $N_A = 1$.

3. Conclusion

This article clarifies the situations in which the presence of more than one independent set of meshing teeth can cause performance quality deterioration of a gear drive, namely:

-elevated noise and vibration levels;
-decreased reliability;
-decreased longevity of the gear drive.

It must be explicitly noted that there are gear drives with factor $N_A > 1$, which provide excellent performance and long life [7], when the following prerequisites are existent: high manufacturing quality, proper lubrication, sealing and ventilation of the gear drive; regular inspections aimed at early prevention of possible problems; observation of the established assembly phase after wheel disassembly; respecting the prescriptions for storage and transportation of the gear drive. Nevertheless, even if the above conditions are met, ensuring a factor $N_A = 1$ is still an excellent practice, whether for single-stage gear drives with output power of a few hundred watts or for complex multi-stage mechanisms transmitting several megawatts. It is recommended to pay special attention to the cases when:

- gear drive failure may lead to stopping the production process, cause damage to other machines or endanger the safety of the personnel;
- the level of noise and vibration of the gear drive affect the production process and the operation of other machines;
- heavy duty operation with frequent overload and switching;
- operation in highly contaminated environment;
- lack or impossibility of regular inspections and control.

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