ABOUT THE INDEPENDENT SETS OF PERIODICALLY MESHING TEETH IN GEAR DRIVES PART I

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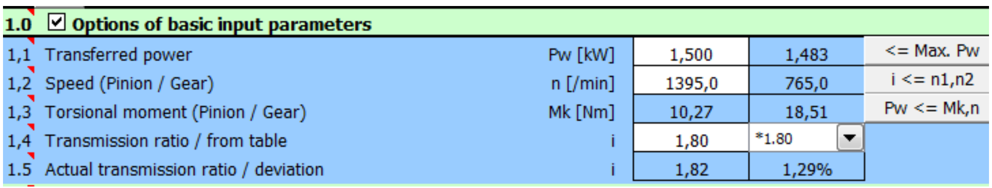
Abstract. This article provides a detailed study in regard to the independent sets of teeth of engaged gear wheels that periodically come into mesh during the operation of a gear drive. Taken into consideration is the importance of a proper choice of the teeth number of gear wheels, and a methodology for calculating the teeth number in power gear drives in terms of optimal operation is proposed. Further the paper discusses the influence of independent sets of meshing teeth on the strength and performance parameters of gear drives.

Keywords: gear transmissions; independent sets of meshing teeth; teeth number choice in power gear drives

1. Introduction

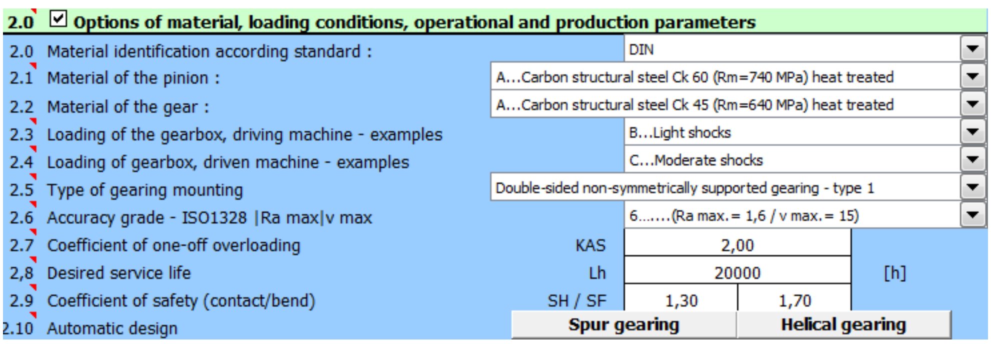
On of the key factors that affect the performance characteristics of gear drives that form part of power trains is the optimal choice of teeth number of the engaged gear wheels [13, 16]. Reducing the number of teeth of engaged wheels leads to smaller and lighter gear drives, which is of importance for the overall space consumption and cost of the power train. An unfavourable effect of reducing the teeth number, particularly when it comes to gear drives with straight teeth (Figure 8), is the increase of specific sliding [7]. In this case intense friction is observed, especially in those points of the active teeth surface that are most distant from the pitch point – the roots and the tips of the teeth (Figure 5), which is a prerequisite for accelerated wear and heavier warming [1, 3, 7, 13]. Another disadvantage of reducing the teeth number is the subsequent diminishing of the transverse contact ratio ε_{α}, which leads to a smaller load capacity and less smooth operation of the gear drive [1, 3]. The above facts will be illustrated with a practical example of a cylindrical spur gear drive from the construction of a real industrial installation. The gear drive has been computer calculated with the specialized in the area of machine elements software system MITCalc (Figures 1÷7) after which the construction design has been optimized in the CAD environment of Autodesk Inventor (Figure 8) [5, 9] with regard to the following:

- predefined power and kinematics parameters – Figure 1;
- selected calculation standard, pinion and gear material, dimensions and further operational and production parameters – Figure 2;
- type and geometry of the standardized gear cutter tool – Figure 3;



1.0	☑ Options of basic input parameters				
1,1	Transferred power	Pw [kW]	1,500	1,483	<= Max. Pw
1,2	Speed (Pinion / Gear)	n [/min]	1395,0	765,0	i <= n1,n2
1,3	Torsional moment (Pinion / Gear)	Mk [Nm]	10,27	18,51	Pw <= Mk,n
1,4	Transmission ratio / from table	i	1,80	*1.80	
1.5	Actual transmission ratio / deviation	i	1,82	1,29%	

Figure 1. Basic parameters for software calculation of a gear drive



2.0	☑ Options of material, loading conditions, operational and production parameters				
2.0	Material identification according standard :		DIN		
2.1	Material of the pinion :		A...Carbon structural steel Ck 60 (Rm=740 MPa) heat treated		
2.2	Material of the gear :		A...Carbon structural steel Ck 45 (Rm=640 MPa) heat treated		
2.3	Loading of the gearbox, driving machine - examples		B...Light shocks		
2.4	Loading of gearbox, driven machine - examples		C...Moderate shocks		
2.5	Type of gearing mounting		Double-sided non-symmetrically supported gearing - type 1		
2.6	Accuracy grade - ISO1328 |Ra max|v max		6.......(Ra max.= 1,6 / v max.= 15)		
2.7	Coefficient of one-off overloading	KAS	2,00		
2,8	Desired service life	Lh	20000		[h]
2.9	Coefficient of safety (contact/bend)	SH / SF	1,30	1,70	
2.10	Automatic design		Spur gearing	Helical gearing	

Figure 2. Standard, operational and production parameters of the gear drive

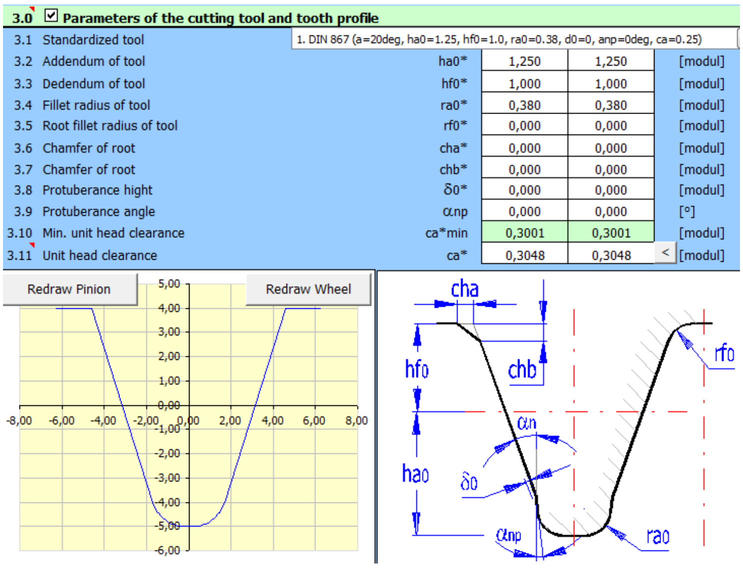


Figure 3. Parameters of the gear cutter tool

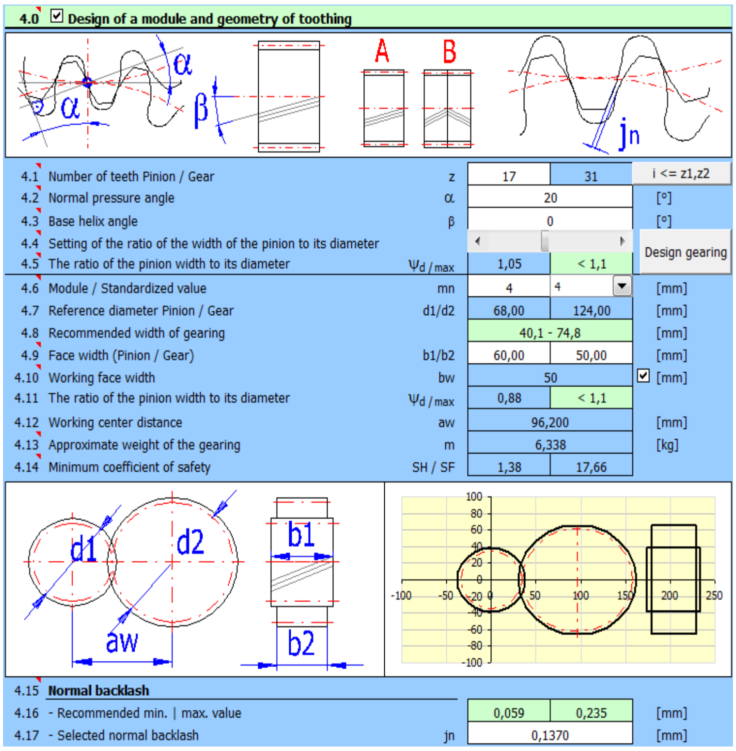
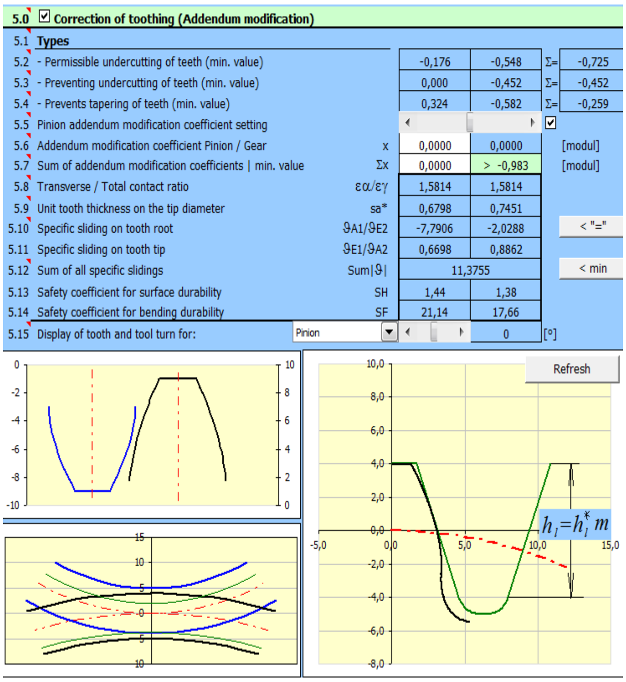


Figure 4. Geometric and strength parameters of the gear drive

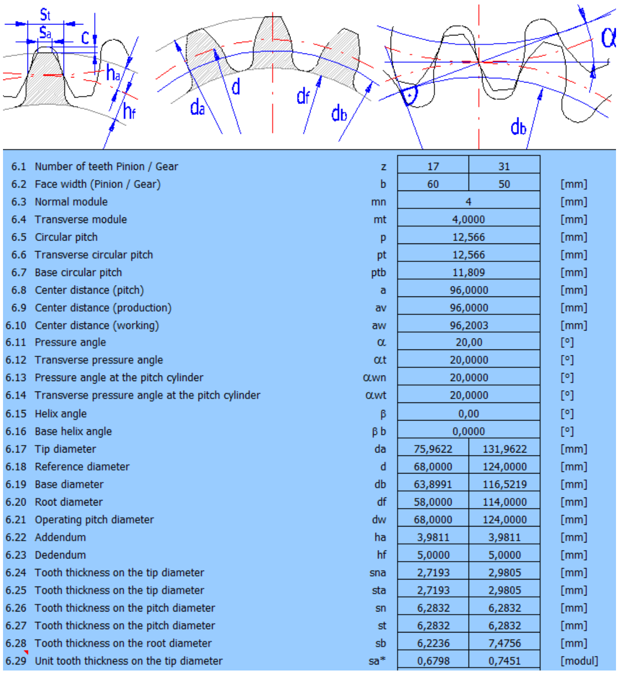
- determined permissible geometric and strength parameters – Figure 4;
- automatically calculated quality characteristics for the entered addendum modification coefficients (x_1, x_2 and Σx), from which is seen (Figure 5) that the specific sliding (ϑ) is biggest at the roots of the teeth (point 5.10 of the software algorithm) and at the tips of the teeth of the two engaged wheels (point 5.11 of the software algorithm);
- calculated geometry (Figure 6) and load capacity of the gear drive (Figure 7).

The negative effect of the drawbacks associated with reducing the teeth number in gear drives can be shrink down by achieving an optimal combination between wheel materials, applied thermal treatment, selected addendum modification coefficients (x_1; x_2; Σx - points 5.6 and 5.7 on Figure 5), high production accuracy and quality of the teeth surface and last but not least the number of independent sets of meshing teeth of the engaged wheels.



5.0	☑ Correction of toothing (Addendum modification)					
5.1	Types					
5.2	- Permissible undercutting of teeth (min. value)		-0,176	-0,548	Σ=	-0,725
5.3	- Preventing undercutting of teeth (min. value)		0,000	-0,452	Σ=	-0,452
5.4	- Prevents tapering of teeth (min. value)		0,324	-0,582	Σ=	-0,259
5.5	Pinion addendum modification coefficient setting				☑	
5.6	Addendum modification coefficient Pinion / Gear	x	0,0000	0,0000		[modul]
5.7	Sum of addendum modification coefficients | min. value	Σx	0,0000	> -0,983		[modul]
5.8	Transverse / Total contact ratio	εα/εγ	1,5814	1,5814		
5.9	Unit tooth thickness on the tip diameter	sa*	0,6798	0,7451		
5.10	Specific sliding on tooth root	ϑA1/ϑE2	-7,7906	-2,0288		< "="
5.11	Specific sliding on tooth tip	ϑE1/ϑA2	0,6698	0,8862		
5.12	Sum of all specific slidings	Sum|ϑ|	11,3755			< min
5.13	Safety coefficient for surface durability	SH	1,44	1,38		
5.14	Safety coefficient for bending durability	SF	21,14	17,66		
5.15	Display of tooth and tool turn for:	Pinion		0	[°]	

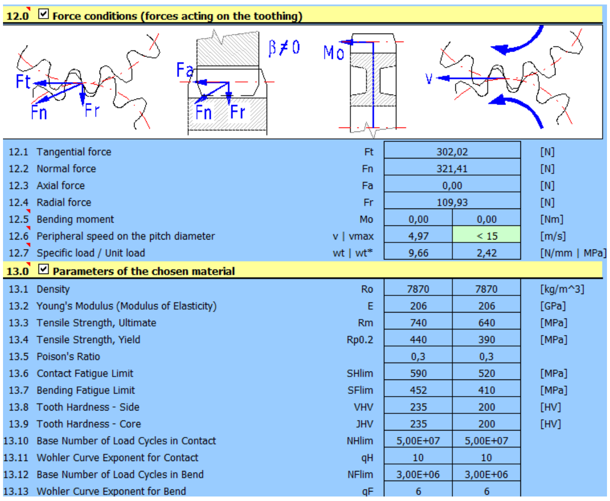
Figure 5. Quality characteristics of the gear drive



6.1	Number of teeth Pinion / Gear	z	17	31	
6.2	Face width (Pinion / Gear)	b	60	50	[mm]
6.3	Normal module	mn	4		[mm]
6.4	Transverse module	mt	4,0000		[mm]
6.5	Circular pitch	p	12,566		[mm]
6.6	Transverse circular pitch	pt	12,566		[mm]
6.7	Base circular pitch	ptb	11,809		[mm]
6.8	Center distance (pitch)	a	96,0000		[mm]
6.9	Center distance (production)	av	96,0000		[mm]
6.10	Center distance (working)	aw	96,2003		[mm]
6.11	Pressure angle	α	20,00		[°]
6.12	Transverse pressure angle	αt	20,0000		[°]
6.13	Pressure angle at the pitch cylinder	αwn	20,0000		[°]
6.14	Transverse pressure angle at the pitch cylinder	αwt	20,0000		[°]
6.15	Helix angle	β	0,00		[°]
6.16	Base helix angle	β b	0,0000		[°]
6.17	Tip diameter	da	75,9622	131,9622	[mm]
6.18	Reference diameter	d	68,0000	124,0000	[mm]
6.19	Base diameter	db	63,8991	116,5219	[mm]
6.20	Root diameter	df	58,0000	114,0000	[mm]
6.21	Operating pitch diameter	dw	68,0000	124,0000	[mm]
6.22	Addendum	ha	3,9811	3,9811	[mm]
6.23	Dedendum	hf	5,0000	5,0000	[mm]
6.24	Tooth thickness on the tip diameter	sna	2,7193	2,9805	[mm]
6.25	Tooth thickness on the tip diameter	sta	2,7193	2,9805	[mm]
6.26	Tooth thickness on the pitch diameter	sn	6,2832	6,2832	[mm]
6.27	Tooth thickness on the pitch diameter	st	6,2832	6,2832	[mm]
6.28	Tooth thickness on the root diameter	sb	6,2236	7,4756	[mm]
6.29	Unit tooth thickness on the tip diameter	sa*	0,6798	0,7451	[modul]

Figure 6. Basic geometry of the gear drive

These measures themselves are once again relied to efficiency and cost, therefore an overall view on the problem reveals a closed loop in which the mechanical design engineer and the production technology engineer are expected to reach an optimal solution regarding the performance of the gear drive. Regarding the concept of independent sets of meshing teeth it should be noted that whenever applicable they are supposed to be arranged in such a way, that each tooth of the small (driving) wheel periodically contacts as many teeth of the large (driven) wheel as possible, since this leads to a significant decrease of the influence of tooth geometry errors on wear, vibration, noise and smoothness of operation of the gear drive, which is of crucial importance for its performance capacity.



12.0	☑ Force conditions (forces acting on the toothing)				
12.1	Tangential force	Ft	302,02		[N]
12.2	Normal force	Fn	321,41		[N]
12.3	Axial force	Fa	0,00		[N]
12.4	Radial force	Fr	109,93		[N]
12.5	Bending moment	Mo	0,00	0,00	[Nm]
12.6	Peripheral speed on the pitch diameter	v | vmax	4,97	< 15	[m/s]
12.7	Specific load / Unit load	wt | wt*	9,66	2,42	[N/mm | MPa]
13.0	☑ Parameters of the chosen material				
13.1	Density	Ro	7870	7870	[kg/m^3]
13.2	Young's Modulus (Modulus of Elasticity)	E	206	206	[GPa]
13.3	Tensile Strength, Ultimate	Rm	740	640	[MPa]
13.4	Tensile Strength, Yield	Rp0.2	440	390	[MPa]
13.5	Poison's Ratio		0,3	0,3	
13.6	Contact Fatigue Limit	SHlim	590	520	[MPa]
13.7	Bending Fatigue Limit	SFlim	452	410	[MPa]
13.8	Tooth Hardness - Side	VHV	235	200	[HV]
13.9	Tooth Hardness - Core	JHV	235	200	[HV]
13.10	Base Number of Load Cycles in Contact	NHlim	5,00E+07	5,00E+07	
13.11	Wohler Curve Exponent for Contact	qH	10	10	
13.12	Base Number of Load Cycles in Bend	NFlim	3,00E+06	3,00E+06	
13.13	Wohler Curve Exponent for Bend	qF	6	6	

Figure 7. Strength parameters of the gear drive

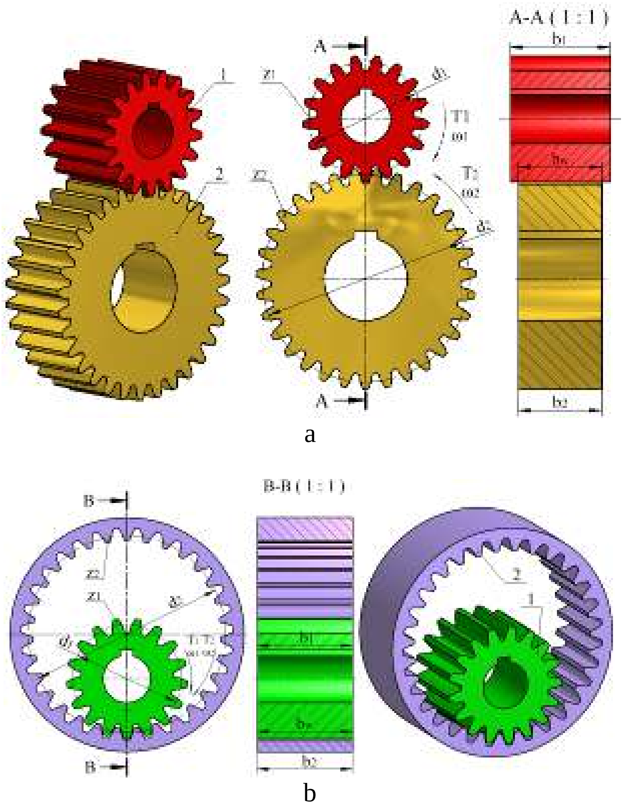


Figure 8. Cylindrical spur gear drive

z_1 = 17; z_2 = 31; m = 4 mm

a) external engagement; b) internal engagement

Strength calculation and geometry definition of cylindrical gear drives is based on the requirement for sufficient contact stress safety of the tooth profiles. Preliminary determination of the number of teeth (z_1) of the small and the number of teeth (z_2) of the large wheel can be done in various ways – through general recommendations formed on previous experience, reference materials, industrial and information catalogues, specialized software CAD systems, as well as via certain methodologies [1, 5, 6, 8, 9]. Below is presented a short algorithm, consisting of four steps, which can be used to calculate a cylindrical spur gear drive [3, 4]:

1) Calculation of the pitch diameter of the small wheel (it is assumed the small wheel is the driving one):

$$d_1 \geq f_H \cdot \sqrt[3]{\frac{T_1}{\psi_{bd_1} \cdot \sigma_{HP}^2} \cdot \frac{u \pm 1}{u} \cdot K_H}\ , \qquad (1)$$

where: f_H - generalized coefficient for preliminary calculation of gear drives; T_1 - torque transmitted from the input (driving) shaft of the small wheel (z_1) (in the software calculated example the torque is designated with *Mk* - Figure 1); ψ_{bd_1} - the ratio of the small wheel width to its diameter; σ_{HP} - permissible contact stress; u - transmission number; K_H - generalized coefficient of gear drive load.

2) Calculation of the normal module of the wheels of the gear drive:

$$m = \frac{b_w}{\psi_m} = \frac{\psi_{bd_1} \cdot d_1}{\psi_m}\ , \qquad (2)$$

where: b_w - working face width; ψ_m - coefficient depending on the type and load of the gear drive.

3) Determination of the teeth number of the small wheel:

$$z_1 = \frac{d_1 \cdot \cos\beta}{m}\ , \qquad (3)$$

where: β - teeth slope angle in respect to the geometric axis of the wheel.

The minimum allowable number of teeth $z_{\min}$ of gear wheels is limited by the requirements for preventing undercutting at the root and tapering at the tip of the teeth, for known manufacturing technology and cutter tool parameters. In case the gear wheels are manufactured with a rack-type cutter tool the minimum allowable number of teeth is determined by [2]:

$$z_{\min} = \frac{2 \cdot \left(h_l^* - h_a^* - x\right)}{\sin^2 \alpha}\ , \qquad (4)$$

where: h_l^* - limiting height coefficient (the height of the straight-line portion - Figure 5) of the cutter tool profile; h_a^* - addendum height coefficient;

$$x_{\min} = h_l^* - h_a^* - \frac{z \cdot \sin^2 \alpha}{2} \qquad (5)$$

is the minimum shift coefficient, determined by the requirement for preventing undercutting of the teeth; α - pressure angle of the cutter tool profile, which is coincident with the pressure angle of the gear wheel teeth, when measured on the pitch diameter (Figures 3, 4 and 6).

4) Determination of the teeth number of the large (driven) wheel:

$$z_2 = z_1 . u \qquad (6)$$

The gear drive is a three-link mechanism and as such the transmission ratio can be expressed from the kinematics relation between the mobile links (the gear wheels z_1 and z_2):

$$i = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{V_1}{V_2} \gtrless 1, \qquad (7)$$

where: ω_1 and ω_2 are the angular velocities; n_1 and n_2 - the revolutions per minute; V_1 and V_2 - the peripheral velocities of the wheels (Figures 7 and 8).

When determining the transmission ratio the rotation direction of the wheels has to be taken into account. In the case of unidirectional rotation (corresponding to internally engaged gear drives) - Figure 8b, the angular velocities of both wheels have the same mathematical sign:

$$i = \frac{\omega_1}{\omega_2} > 1\ , \qquad (8)$$

on the other side with contra directional rotation (corresponding to externally engaged gear drives) - Figure 8a, the angular velocities have opposite mathematical signs:

$$i = -\frac{\omega_1}{\omega_2} < 1\ . \qquad (9)$$

The transmission number (u) is a ratio of geometric parameters of the gear drive and is therefore always positive:

$$u = \frac{d_2}{d_1} = \frac{z_2}{z_1} > 1\ , \qquad (10)$$

where: d_1 and d_2 are the pitch diameters; z_1 and z_2 are the teeth numbers of the engaged wheels (Figures 4, 6 and 8).

The relation between the peripheral velocity V and the angular velocity ω of the engaged wheels in the gear drive is expressed as:

$$V_{1,2} = \pi \cdot d_{1,2} \cdot n_{1,2}, \quad (11)$$

where: $d_{1,2} = m \cdot z_{1,2}$ - pitch diameters; $n_{1,2} = 30 \cdot \omega_{1,2}/\pi$ - revolutions per minute.

After substitution of (11) in (7), the following formula for the transmission ratio is resulted [2]:

$$i_{1,2} = \frac{\omega_1}{\omega_2} = \pm \frac{z_2}{z_1}. \quad (12)$$

From the above expressions (7÷12) it can be concluded, that the absolute value of the transmission ratio equals the transmission number:

$$|i| = u = \frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1} > 1. \quad (13)$$

The proper choice of the teeth numbers of the gear wheels z_1 and z_2 is directly linked to the number of independent sets of meshing teeth in the particular gear drive. This choice is also one of the main prerequisites for designing gear drives with optimal properties regarding load capacity, longevity, wear, heating, noise and vibration [10].

The concept of independent sets of meshing teeth is known in the literature, but the latter lacks fundamental explanations and detailed information about its essence, application and significance [3, 4, 10÷18]. As a result in the design process of gear drives this important principle is often neglected or underestimated.

The aim of the present article is: To clarify the significance of the problem regarding the proper application of the concept of independent sets of meshing teeth in engaged gear wheels, as well as to elaborate the problem in detail and designate its effect on strength and performance parameters of gear drives during their operation.

2. Exposure

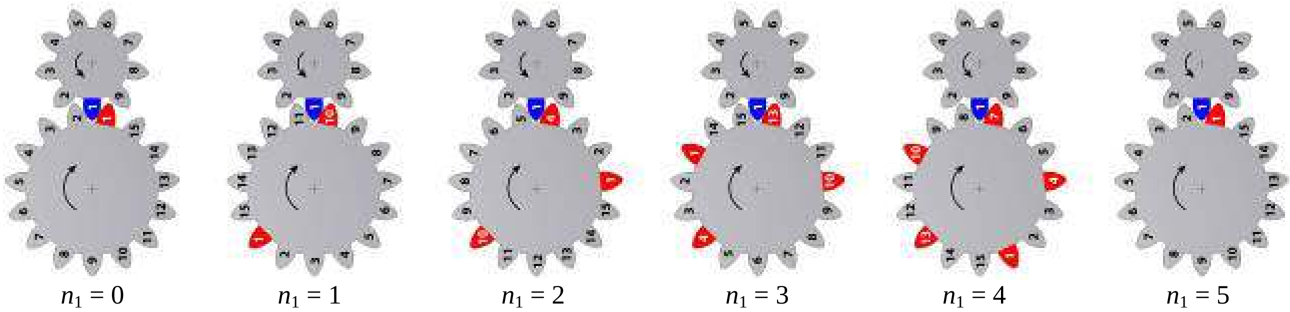
2.1. Independent sets of meshing teeth in gear drives

According to [18] the essence of independent sets of meshing teeth in gear drives can be presented in the following way:

For a cylindrical gear drive, calculated using equations (1, 2, 3 and 6), the values below have been obtained:

- $z_1 = 9$ - number of teeth for the small wheel;
- $z_2 = 15$ - number of teeth for the large wheel;
- $u = 1.67 \approx 1.7$ - transmission number of the gear drive.

Each tooth of both wheels has been assigned a certain number (Figure 9). In the starting position of the gear drive, denoted with $n_1 = 0$, tooth №1 of the small wheel (marked in blue) is in contact with tooth №1 of the large wheel (marked in red). When the gear drive is set in motion in the specified direction, after each full revolution of the small wheel n_1, it is tracked which tooth of the large wheel will contact tooth №1 of the small wheel.



$n_1 = 0$ $n_1 = 1$ $n_1 = 2$ $n_1 = 3$ $n_1 = 4$ $n_1 = 5$

Figure 9. Teeth meshing sequence for a cylindrical spur gear drive with $z_1 = 9$ and $z_2 = 15$ at every full revolution of the small wheel

After the first revolution meshing occurs between tooth №1 and tooth №10, after the second – between №1 and №4, after the third – between №1 and №13, after the fourth – between №1 and №7. After the fifth revolution tooth №1 of the small wheel once again contacts tooth №1 of the large wheel, which corresponds to the starting position of the gear drive. If we continue rotating the wheels of the gear drive, the meshing cycle will repeat in the same manner. The stated above demonstrates that when the gear drive is in operation tooth №1 of the small wheel will periodically mesh with teeth №1, №4, №7, №10 and №13 of the large wheel but will never contact any of the other 10 teeth. Tracking the contact sequence for the rest of the teeth of the small wheel (from №2 to №9) with the teeth of the large wheel reveals a certain relation, which is presented in Table 1. The letters *A*, *B* and *C* designate the independent sets of meshing teeth. When the gears are driven a tooth from one set never comes in contact with a tooth from another set.

Table 1. Independent sets of meshing teeth

	Teeth of the small wheel №	Teeth of the large wheel №
A	**1 – 7 – 4**	**1 – 10 – 4 – 13 – 7**
B	**2 – 8 – 5**	**2 – 11 – 5 – 14 – 8**
C	**3 – 9 – 6**	**3 – 12 – 6 – 15 – 9**

2.2. Calculating the number of independent sets of meshing teeth in the gear drive

The number of independent sets of meshing teeth in a gear drive is designated with N_A. In the example above $N_A = 3$. In mathematical form the factor N_A is equal to the greatest common divisor of z_1 and z_2 [4].

In the algorithm for determining N_A that is presented below the abbreviation *GCD (Greatest Common Divisor)* is used; it is worth mentioning that other abbreviations such as *GCF (Greatest Common Factor)* and HCF *(Highest Common Factor)* are equal in meaning and are often found in literature:

$$N_A(z_1, z_2) = GCD(z_1, z_2). \tag{14}$$

The fastest way to find the *GCD* of two numbers *a* and *b* is to use the Euclidean algorithm. This algorithm is iterative, i.e. the solution is reached after *N* steps [17]. First of all two positive variables r_{n-1} and r_{n-2} need to be defined, with the condition that:

$$r_{n-1} \le r_{n-2}. \tag{15}$$

The initial values (i.e. by $n = 0$) of these variables are namely the numbers *a* and *b* for which the *GCD* is to be calculated:

$$r_{-1} = a; \quad r_{-2} = b. \tag{16}$$

The aim of the algorithm is to increment *n* ($n = 0; 1; 2; \ldots$) and on every step to find an integer coefficient q_n, which when solving equation (17):

$$r_{n-2} = r_{n-1} \cdot q_n + r_n \tag{17}$$

allows fulfilling the inequality:

$$r_n < r_{n-1}. \tag{18}$$

After a certain number of repetitions $n = N$, the remainder r_n equals zero:

$$r_N = 0, \tag{19}$$

at which point the algorithm is stopped.

It can be proven that *GCD(a,b)* is equal to the last remainder that was not zero:

$$GCD(a,b) = r_{N-1} \ne 0. \tag{20}$$

Example: for $a = 9$ and $b = 15$ the Euclidean algorithm is expressed by the following series of equations:

$$\begin{aligned} &(n = 0) \quad 15 = 9 \cdot 1 + 6;\\ &(n = 1) \quad 9 = 6 \cdot 1 + 3;\\ &(n = 2) \quad 6 = 3 \cdot 2 + 0;\\ &\Rightarrow N_A = 3, \end{aligned} \tag{21}$$

which confirms the empirical result for N_A for the discussed gear drive.

The Euclidean algorithm can easily be implemented in all programming languages in order to use it in custom software solutions for gear drive design or to modify existing software products serving this purpose. An example for such a product is MITCalc, specialized in machine elements and mechanical transmissions calculations [5, 9]. One of the possible options for software implementation of the Euclidean algorithm is to use the *mod* (%) operator, which returns the remainder from the division of two numbers:

$$r_n = r_{n-2} \bmod r_{n-1}. \tag{22}$$

The example below demonstrates one possible solution for an algorithmic function that finds the *GCD* of two numbers, implemented in *C* programming language:

```
int GCD(int a,int b)
    {
    int temp;
    while (b!=0)
    {
        temp=b;
        b=a%b;
        a=temp;
    }
    return(a);
    }.
```

Freeware ready-made solutions for finding the *GCD* are accessible on the web, many of them are to be found under the form of a *Java Applet* [19].

In some literature sources the N_A factor is defined as the product of simple multipliers that are common to the number of teeth of both gear wheels z_1 and z_2 [18]:

$$\begin{aligned} &z_1 = 9 = 3 \times 3;\\ &z_2 = 15 = 3 \times 5;\\ &\Rightarrow N_A = 3. \end{aligned} \tag{23}$$

This approach for finding the *GCD* involves the decomposition of z_1 and z_2 and is more complex than the Euclidean algorithm, especially when it comes to its implementation in software code.

3. Conclusion

The knowledge and proper application of the concept of independent sets of meshing teeth in the design stage of the gear drive eliminates the risk of later occurrence of various issues, resolving which would at the very least demand significant financial resources. Achieving $N_A = 1$ only requires a modest change in the value of the requested transmission number of the gear drive, which does not imply labour-intensive and complex calculations, deterioration of performance and quality, increase of manufacturing cost, etc.

The principle is applicable for cylindrical, bevel, hypoid and worm (z_1 equals the number of threads of the worm) drives, regardless of the slope angle of the teeth [11].

Providing a factor $N_A = 1$ is the proper approach when designing power gear drives irrespective of load scale, as well as in the case of responsible gear drives, which require special attention when the following events are probable:

- damage of the gear drive may lead to stop of the production process or endanger the safety of the personnel;
- the level of noise and vibration of the gear drive affect the production process;
- heavy duty operation;
- operation in highly contaminated environment;
- lack of regular inspection.

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