THE PLANE MOTION OF THE MONOWHEEL VEHICLE

Dan BOTEZATU
Transilvania University of Brasov, Romania

Abstract. The monowheel vehicle present a number of challenges to the designer and several compromises have to be made in order to develop a functional one-wheeled vehicle. The present paper will detail the motion equations of the monowheel in a plane motion.

Keywords: motion, equilibrium, monowheel

1. Introduction

The monowheel [1] consists of an inner frame (1) and a wheel (2). The inner frame (1) has three small wheels (4) that make contact with the wheel (2). The wheel (2) is the actual rotating wheel and has a solid rubber tire. The rider sits inside the inner frame that also contains the driving roller (3), the engine, the clutch, the propulsion mechanism and the petrol tank.

Let us consider the following simplified model from Figure 1:

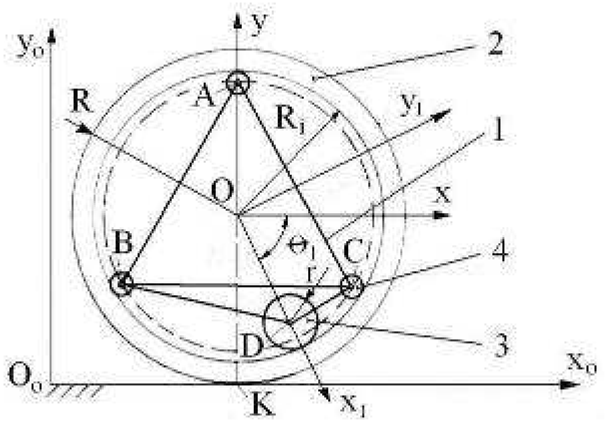

Figure 1. Simplified model

Here it is considered the following reference frames in regards with our objective, to find the monowheel equations of plane motion, as follows:

- the $O_0x_0y_0$ reference frame that is fixed;
- the Oxy transported reference frame that is contained in the wheel's symmetry plane (2), having the Ox-axis always horizontal;
- the Ox_1y_1 reference frame that is solidary with the inner frame (1), having Ox_1-axis passing through point D.

Having in mind the "Rolling Friction" chapter covered in [2] that can be used as starting point in the analysis of this particular rolling friction scenario, it can be observed that the rolling motion of the wheel (2) could be described merely an inside driven mechanical system.

2. The plane motion

2.1. Generalities

During the plane motion of the entire vehicle, the inner frame (1) has a translation with the centre O, and the wheel (2) has a relative rotation about the Oz-axis.

Let us develop the equations of motion for the rectilinear motion of the monowheel vehicle. For this is made the assumption that the rider (see Figure 2) will not move any part of his/her body meaning that his/her centre of mass will remain in the symmetry plane of the whole ring.

Figure 2. CAD monowheel prototype

Therefore four cases will be studied:

- the vehicle is at rest, but very close to the starting point;
- the vehicle starts and there is an accelerated motion (with substantial acceleration);
- the vehicle has a steady plane motion;
- the vehicle decelerates.

After a short review of the theoretical part (see figures 3 and 4) and also [3], it can be noticed that for this one-wheeled vehicle, the emphasis of the deformability of each element from the wheel-ground couple is very important for the well definition of each motion state.

So, due to the fact that the ground as well as the wheel is deformable, the contact appears on a contact area.

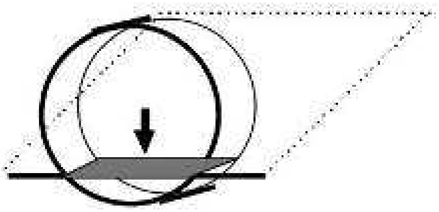

Figure 3. Wheel-ground contact area

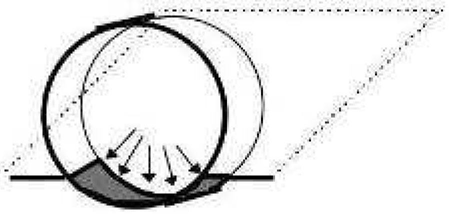

Figure 4. Wheel-ground contact surface

Inside the contact area, the pressures on the wheel are asymmetrical distributed due to the existence of the permanent eccentricity *e* or *l* (see Figure 5). These make the centre of the mass of the whole mechanical system considered or of the inner frame (1) to be able to oscillate on some circular arcs of constant radii about the geometrical centre, i.e. the centre of mass of the wheel (2).

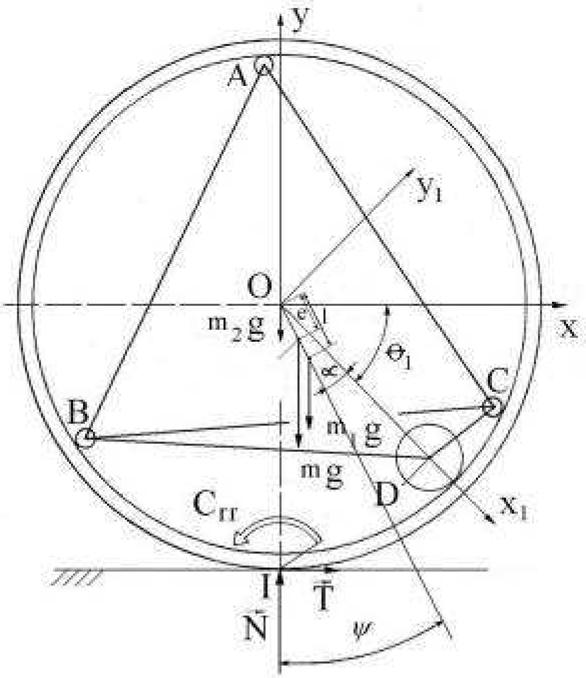

Figure 5. Steady plane motion

Considering the specifics of the monowheel vehicle, one might say that when this mechanical system tends to rotate, it produces an asymmetric distribution of these pressures (see Figure 6) and therefore the displacement of their resultant reaction. If conventionally, we shall represent this reaction *N* applied at the theoretical contact point *I* we must introduce a couple besides it. This couple, named rolling resistance couple, C_{rr}, has the following expression:

$$C_{rr} = s \cdot N \qquad (1)$$

where *N* is the normal reaction of the ground and the arm *s* represents the maximum value of the displacement of the reaction *N*, in conditions of impending rolling motion, while *b* represents all the values comparable with *s*.

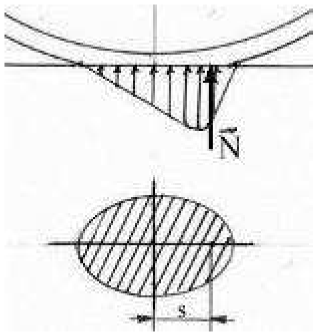

Figure 6. Asymmetric distribution of pressures

2.2. The vehicle is at rest, but very close to the starting point

In the first test case chosen for this vehicle, we shall study the equilibrium equations when the vehicle is at rest but very close to start to move.

Although this period of time, when the vehicle equilibrium is changing, is very short but extremely important for this study, we will separate it into two different intervals:

a. the vehicle is at rest, far from the limit of the starting situation;

b. the vehicle is at rest but it got the limit of the starting situation.

a. In Figure 6 it can be seen that there is an angle ψ_0 that defines the position of the inner frame (1) relative to the vertical passing by O.

This position is an equilibrium one, the possible rolling moment defined by $(G_1 + G_2) \cdot b_0$ being balanced by the rolling resistance couple C_{rr}, the b_0 arm being much smaller than the distance *s*. Therefore there is the angle ψ_0 only when $b = b_0$.

The motion equations are the following:

$$\begin{cases} N - G = 0 \\ (G_1 + G_2) \cdot b_0 - C_{rr} = 0 \\ b_0 = e \cdot \sin \psi_0 \end{cases} \qquad (2)$$

ψ_0 being the angle between the vertical line and the mass centres' line, corresponding to the rest case.

b. Just like the bicycler pushes the pedal but the bicycle doesn't start yet (the moment exists but it is not strong enough), the monowheel is experiencing this too, meaning that there is a limit of equilibrium, and $\vec{T}$ just passed over the static equilibrium state.

This subdivision of rest implies the functioning of the driving leading pulley (3) without getting an arm $b > s$. This means that $0 < b \leq s$ governs the current test case.

The motion equations are the following:

$$\begin{cases} T - (m_1 + m_2) \cdot a = 0 \\ N - G = 0 \\ (G_1 + G_2) \cdot b - C_{rr} - F^i \cdot (R - e \cdot \cos\psi_S) = 0 \\ b = e \cdot \sin\psi_S \end{cases} \quad (3)$$

Therefore the angle ψ_S covers the $b \leq s$ scenario, where s stands for start. The inertia force F^i in this case is very small.

$$\begin{aligned} &(G_1 + G_2) \cdot b - C_{rr} - F^i \cdot (R - e \cdot \cos\psi_S) = 0 \\ &\Rightarrow (G_1 + G_2) \cdot e \cdot \sin\psi_S - (G_1 + G_2) \cdot s - \\ &- \frac{G_1 + G_2}{g} \cdot a \cdot (R - e \cdot \cos\psi_S) = 0 \Rightarrow \\ &\Rightarrow a = \frac{(G_1 + G_2) \cdot (e \cdot \sin\psi_S - s)}{(G_1 + G_2) \cdot (R - e \cdot \cos\psi_S)} \cdot g \Rightarrow \\ &\Rightarrow a = \frac{e \cdot \sin\psi_S - s}{R - e \cdot \cos\psi_S} \cdot g \Rightarrow a = a(\psi_S) \end{aligned} \quad (4)$$

$$\begin{cases} a = 0; \quad for \quad 0 < \psi_S \leq \psi_{\lim} \\ a = \frac{e \cdot \sin\psi_S - s}{R - e \cdot \cos\psi_S} \cdot g; \quad for \quad \psi_S > \psi_{\lim} \end{cases} \quad (5)$$

2.3. The vehicle starts and there is an accelerated motion (with substantial acceleration)

During the $b > s$ case, the inner frame (1) has a rotation motion to the wheel (2) due to the contact of the leading pulley (3) with the interior surface of the wheel.

Following this relative rotation motion, the b arm will overrun the limit value of s.

$$\begin{aligned} &a \sim a_0 = R * \varepsilon \\ &b = e * \sin\psi_a \end{aligned} \quad (6)$$

ψ_a, being the angle corresponding to this case with substantial acceleration.

In this situation we have to take into account the aerodynamic drag force.

The motion equations become the following:

$$\begin{cases} T - W - F^i = 0 \\ N - (G_1 + G_2) = 0 \\ (G_1 + G_2) \cdot b - C_{rr} - \\ - W \cdot (R - e \cdot \cos\psi_a) - F^i \cdot (R - e \cdot \cos\psi_a) - \\ - J_2^0 \cdot \varepsilon_2 = 0 \\ C_{rr} = s \cdot N \end{cases} \quad (7)$$

Therefore the angle ψ_a covers the $b > s$ scenario.

$$\begin{aligned} &(G_1 + G_2) \cdot b - C_{rr} - W \cdot (R - e \cdot \cos\psi_a) - \\ &- F^i \cdot (R - e \cdot \cos\psi_a) - J_2^0 \cdot \varepsilon_2 = 0 \Rightarrow \\ &\Rightarrow (G_1 + G_2) \cdot e \cdot \sin\psi_a - (G_1 + G_2) \cdot s - \\ &- \frac{G_1 + G_2}{g} \cdot a \cdot (R - e \cdot \cos\psi_a) - \\ &- W \cdot (R - e \cdot \cos\psi_a) - J_2 \cdot \frac{a}{R} = 0 \Rightarrow a = \\ &= \frac{G \cdot e \cdot \sin\psi_a - G_s - W \cdot (R - e \cdot \cos\psi_a)}{G \cdot (R - e \cdot \cos\psi_a) + \frac{J_2 \cdot g}{R}} \Rightarrow \\ &\Rightarrow a = f(\psi_a) \end{aligned} \quad (8)$$

where W is the aerodynamic drag force.

2.4. The vehicle has a steady plane motion

This constant motion is possible at different constant speeds, where $b > s$ and $a = 0$, the presence of aerodynamics resistance being also taken into consideration.

In order to study this movement (see Figure 6), it's been chosen a single coordinate position of frame (1) using the angle ψ_u, where:

$$\psi_u = \frac{\pi}{2} - (\alpha + \theta_1) \quad (9)$$

In this case, the inner frame (1) makes a translation motion to the ground, and the wheel (2) is uniformly rotating about the inner frame (1).

The motion equations are the following:

$$\begin{cases} T - W = 0 \\ N - G = 0 \\ (G_1 + G_2) \cdot b - C_{rr} - W \cdot (R - e \cdot \cos\psi_u) = 0 \end{cases} \quad (10)$$

ψ_u, being the angle corresponding to the uniform plane motion.

$$\begin{aligned} &C_{rr} = s \cdot N \Rightarrow C_{rr} = s \cdot (G_1 + G_2); \\ &b = e \cdot \sin\psi_u; \qquad W = W(v) \end{aligned} \quad (11)$$

$$\begin{aligned}&(G_1+G_2)\cdot b-C_{rr}-W\cdot(R-e\cdot\cos\psi_u)=\\&=0\Rightarrow(G_1+G_2)\cdot e\cdot\sin\psi_u-(G_1+G_2)\cdot s-\\&\quad -W\cdot(R-e\cdot\cos\psi_u)=0\Rightarrow\\&\Rightarrow(G_1+G_2)\cdot(e\cdot\cos\psi_u-s)=\\&\quad =W\cdot(R-e\cdot\cos\psi_u)\Rightarrow\\&\Rightarrow(b-s)_u=f(v)\end{aligned} \tag{12}$$

2.5. The vehicle decelerates

The $0 \le b < s$ case can be also divided in two subdivisions for a better visualization of the deceleration phenomenon:

a. the vehicle is experiencing low values of deceleration;

b. the vehicle is experiencing maximum deceleration.

However, before presenting them, let us conclude the movements studied so far like this:

$$0<\psi_0<\psi_s<\psi_u<\psi_a<\frac{\pi}{2} \tag{13}$$

This inequality points out the relative position that inner frame (1) can have about the frame Oxy, and so generating different regimes of motion.

For deceleration, the interval values for the angle ψ_d, are:

$$\psi_u>\psi_d>0 \tag{14}$$

ψ_b being the braking angle.

a. The first subdivision presents us the vehicle along the decelerating motion of low values of deceleration.

The motion equations are the following:

$$\begin{cases}F^i-T-W=0\\N-G=0\\(G_1+G_2)\cdot b-C_{rr}-W\cdot(R-e\cdot\cos\psi_d)+\\\quad +F^i\cdot(R-e\cdot\cos\psi_d)+J_2^0\cdot\varepsilon_2=0\end{cases} \tag{15}$$

ψ_d being the corresponding position angle.

$$\begin{aligned}&(G_1+G_2)\cdot b-C_{rr}-W\cdot(R-e\cdot\cos\psi_d)+\\&+F^i\cdot(R-e\cdot\cos\psi_d)+J_2^0\cdot\varepsilon_2=0\Rightarrow\end{aligned}$$

$$\Rightarrow a=\left[\frac{(G_1+G_2)\cdot(s-e\cdot\sin\psi_d)}{(G_1+G_2)\cdot(R-e\cdot\cos\psi_d)+\frac{J_2\cdot g}{R}}+\frac{W\cdot(R-e\cdot\cos\psi_d)}{(G_1+G_2)\cdot(R-e\cdot\cos\psi_d)+\frac{J_2\cdot g}{R}}\right]\cdot g \tag{16}$$

b. The maximum deceleration is determined by the adherence value between the wheel (2) and the ground.

The motion equations are the following:

$$\begin{cases}(m_1+m_2)\cdot a_b=-T\\N-(m_1+m_2)\cdot g=0\\-N\cdot(s+e\cdot\sin\psi_b)+T\cdot(R-e\cdot\cos\psi_b)=\\=0\end{cases} \tag{17}$$

$$\begin{aligned}&s+e\cdot\sin\psi_b=\mu_{dyn}\cdot(R-e\cdot\cos\psi_b)\Rightarrow\\&\Rightarrow\psi_b=\psi_b(\mu_{dyn},s)\end{aligned} \tag{18}$$

Figure 7 is describing the following:

$$\delta=\frac{v_{slide}}{v_0},\quad 0<\delta<1,\quad \mu_0=\frac{T}{G},\quad \mu_{dyn}=\frac{T}{G} \tag{19}$$

where v_{slide} is the speed of the sliding point of contact of the tyre and v_0 is the speed of the centre of the wheel (2) – point O.

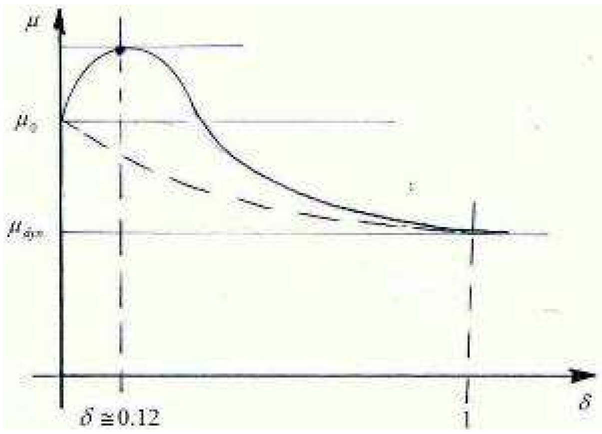

Figure 7. Maximum deceleration

The maximum deceleration happens at $\delta \cong 0.12$ meaning when the value of T is even higher than T at μ_0, where δ is the sliding speed ratio, μ_0 is the relative rest and μ_{dyn} covers the dynamic situation.

3. Conclusions

As the objective of this paper was reached, it can be seen that due to the displacement of the overall mass centre of such vehicle, the problem of the motion is quite challenging.

In further papers as well as in my PhD thesis the results of a continued study about the unsteady longitudinal motion and also the turn of a monowheel vehicle will be presented.

Acknowledgement

This paper is supported by the Sectoral Operational Programme Human Resources Development (SOP HRD), ID134378, financed from the European Social Fund and by the Romanian Government.

References

1. http://www.mcleanmonocycle.com/
2. Deliu, G. (2002) *Mechanics for Engineering Students.* Editura Albastră, ISBN 973-650-082-9, Cluj-Napoca, 2002
3. Deliu, G., Deliu, M. (2009) *Monowheel Dynamics.* RECENT, ISBN 1582-0246, Vol. 10, No. 3(27), p. 245-248

Received in June 2014