MODELLING AND SIMULATION OF PRESSURE LIMITING VALVE BASED ON INPUT AND OUTPUT DATA

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Abstract. The article presents an original mathematic model of a direct drive pressure limiting valve, establishing the status of each variable as input or output data, respectively, underling the two stages of the valve: before and after opening. It is deduced time and complex mathematical model, and transient behaviour of the valve is analysed after the numeric simulation done in Simulink-Matlab. There are considered the two stages of the valve, before and after opening, highlighting their different frequencies and different upstream pressure and piloting pressure of the spool.

Keywords: pressure limiting valve, modelling, simulation

1. Introduction

The pressure limiting valves are designed to control the upstream pressure. The classic case is the usage as constant pressure supply valve of hydraulic circuits that have resistive flow control [1, 3]. From constructive point of view there are direct control valves and piloted valves.

Figure 1 shows a direct control pressure limiting valve with the following characteristics:

– cylindrical hydraulic spool, 1 represents the movable element;
– damping is obtained using the hydraulic resistor, 2, which filter the pressure oscillations;
–setting the valve opening pressure is achieved by pre-stressing of the compression spring 3.

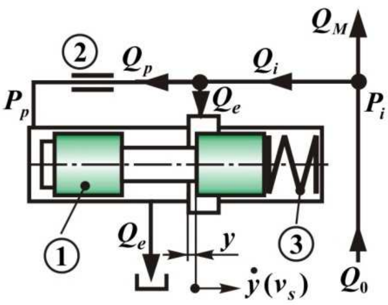

Figure 1. Pressure limiting valve

To simplify the mathematical model the valve normal position, hydraulic spool is critical centre.

2. Mathematical modelling of direct control limiting pressure valve

2.1. Time mathematical model

Mathematical models presented in literature focused on valves [3, 5], does not specify very exactly the input and output data of these hydraulic components. There are considered, as inputs, the pre-stressing force of valve spring and the valve flow.

Based on automatic control systems [2, 4], the control inputs are those parameters that enter information into the system, and the outputs outwards transfer information, regardless their spatial position relative to the system. Input and output parameter is unique only for basic subsystems (e.g. pure hydraulic capacity or resistance [2]). For other systems, the inputs and outputs are strictly determined by the connecting (adjacent) systems.

Into a hydraulic circuit, pressure limiting valve is connected to constant flow hydraulic pump, which has as output the Q_0 flow and to flow control negative element (throttle or flow regulator), which has as input the P_i pressure and as negative feedback through valve the motor flow, Q_M. Considering the above assumptions, results that one of the valve input is the inflow $Q_i = Q_0 - Q_M$. The second input of the valve is the pre-tension, Y_p, of the compression spring, input that generates the opening pressure of the valve. This becomes a reference but it may be considered also as initial condition that exists by default even before activating the valve.

Obviously the valve output is pressure P_i, which depends on flow rate Q_i and hence on the load of hydraulic motor.

Mathematical modelling of the pressure limiting valve is based on the following hypotheses:

– hydraulic resistor that filter the flow, has laminar flow, thus is satisfied the electro-hydraulic analogy;
– is neglected the dry friction of the valve spool;
– is considered linear the spool hydrostatic force and the spool flow.

Based on these hypotheses, the transient state operating of the pressure limiting valve with direct control is described by the equations (1). The equations represent:

– flow continuity equation written for the oil volumeupstream of valve;
– differential pressure on laminar resistor for

pressure oscillations damping;
– flow continuity equation written for pilot circuit of the hydraulic spool;
– dynamic equilibrium equation written for cylindrical spool;
– relation of the flow through valve.

$$\begin{cases} Q_i - Q_e - Q_p = C_{HN} \cdot \dot{P}_i \\ P_i - P_p = R_h \cdot Q_p \\ Q_p = A_s \cdot \dot{y} + C_{Ha} \cdot \dot{P}_p \\ m_{Rs} \cdot \ddot{y} + c_{Rs} \cdot \ddot{y} + K_e \cdot (Y_p + y) + K_{Fh} \cdot y = A_s \cdot P_p \\ Q_e = K_Q \cdot y \end{cases} \quad (1)$$

Besides these equations there should be written the two stages of the valve, before and after opening:

$$\begin{cases} P_p < P_D = K_e / A_s \Rightarrow y = 0 \\ P_p > P_D \Rightarrow y > 0 \end{cases} \quad (2)$$

In equations (1) and (2) have used the following notations: Q_i – output flow of the valve; Q_p – flow of the hydraulic resistor; C_H – hydraulic capacity of the oil volume between valve – pump and speed control throttleof the motor; R_h – hydraulic resistance of the damping resistor; A_s – pilot surface of the spool; C_{Ha} – hydraulic capacity of the piloting chamber; y – valve opening; m_{Rs} – movable mass of the valve; c_{Rs} – viscous friction coefficient of the valve; K_e – elastic constant of compression spring; K_{Fh} – hydrostatic force coefficient of the valve; K_Q – flow coefficient of the valve.

Considering:

$$K_{eh} = K_e + K_{Fh}, \quad (3)$$

equation (1) will become:

$$\begin{cases} C_{HN} \cdot \dot{P}_i = Q_i - Q_e - Q_p \\ R_h \cdot Q_p = P_i - P_p \\ C_{Ha} \cdot \dot{P}_p = Q_p - A_s \cdot \dot{y} \\ m_{Rs} \cdot \ddot{y} + c_{Rs} \cdot \ddot{y} + K_{eh} \cdot y = A_s \cdot P_p - K_e \cdot Y_p \\ Q_e = K_Q \cdot y \end{cases} \quad (4)$$

Equations (2) and (4) define the mathematical time model of pressure limiting valve with direct control.

2.2. Complex mathematical model

The mathematical model in complex may be obtained by applying Laplace transformation on equations (4). Thus, the complex mathematical model of the pressure limiting valve with direct control is:

$$\begin{cases} P_i(s) = \frac{1}{T_1 s} \cdot \left[Q_i(s) - Q_e(s) - Q_p(s)\right] \\ Q_p(s) = K_1 \cdot \left[P_i(s) - P_p(s)\right] \\ P_p(s) = \frac{1}{T_2 s} \cdot Q_p(s) - K_2 y(s) \\ y(s) = \frac{K_s \cdot \omega_s^2}{s^2 + 2\zeta_s \cdot \omega_s \cdot s + \omega_s^2}\left[P_p - K_3 \cdot Y_p\right] \\ Q_e(s) = K_4 \cdot y(s) \end{cases} \quad (5)$$

The notations used in equations (5) have the following significance:

$$K_1 = \frac{1}{R_h}; \quad K_2 = \frac{A_s}{C_{Ha}}; \quad K_3 = \frac{K_e}{A_s};$$
$$K_4 = K_Q; \quad K_s = \frac{A_s}{K_{eh}}; \quad T_1 = C_{HN};$$
$$T_2 = C_{Ha}; \quad \omega_s = \sqrt{\frac{K_{eh}}{m_{Rs}}}; \quad (6)$$
$$\zeta_s = 0.5 \cdot c_{Rs}\sqrt{\frac{m_{Rs}}{K_{eh}}};$$

Based on equations (2) and (5) may be drawn the block diagram of the pressure limiting valve with direct control. To be able to draw this block diagram it was considered valve upstream pressure, P_i, as output and as inputs the valve input flow, Q_i, and the pre-stressing of the valve compression spring, Y_p, parameter associated to valve opening pressure, P_D. The block diagram (Figure 2) contains partial transfer functions define as follows:

$$H_1(s) = \frac{1}{T_1 \cdot s}$$
$$H_2(s) = \frac{1}{T_2 \cdot s} \quad (7)$$
$$H_3(s) = \frac{K_s \cdot \omega_s^2}{s^2 + 2\zeta \cdot \omega_s \cdot s + \omega_s^2}$$

Diagram block is characterised by crossover feedbacks and suggestively presents the informational flux during valve function. Thus, when the input flow Q_i is non-zero, the pressure from piloted chamber P_p is increasing. As long as this pressure does not exceed the opening pressure of the valve, $P_p < P_D$, the pilot flow generates only oil compression in piloted chamber but not the movement of the spool. Only after the opening of the valve the Q_e flow crosses through valve and y opening will steadies at a value to ensure, during steady-state, constant upstream pressure, $P_i \cong$ const.

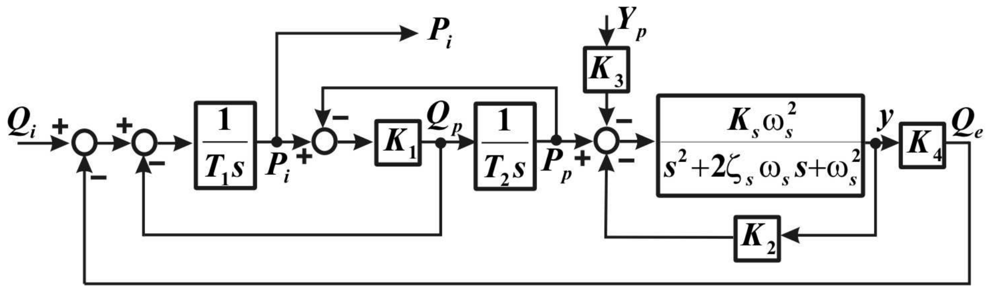

Figure 2. Block diagram of the pressure limiting valve

Using block diagram algebra and considering the numerical values of the functional-constructive parameters there can be determined the following input-output relation (8).

$$P_i(s)=\frac{2.5\cdot 10^{12}\cdot s^3+3.6\cdot 10^{16}\cdot s^2+5\cdot 10^{20}\cdot s+5\cdot 10^{22}}{s^4+1.4\cdot 10^4\cdot s^3+2\cdot 10^8\cdot s^2+5.5\cdot 10^{10}\cdot s+5.3\cdot 10^{13}}\cdot Q_i(s)-$$
$$-\frac{7.4\cdot 10^{20}\cdot s+5\cdot 10^{22}}{s^4+1.4\cdot 10^4\cdot s^3+2\cdot 10^8\cdot s^2+5.5\cdot 10^{10}\cdot s+5.3\cdot 10^{13}}\cdot Y_p(s) \quad (8)$$

Input-output relation defined by equation (8) highlights that the behaviour of the pressure limiting valve is similar to a fourth-order time delay element.

3. Numerical simulation of the pressure limiting valve

Numerical simulation for transient state of pressure limiting valve was done based on simulation diagram from Figure 3.

This diagram was designed in Simulink-Matlab [6] based on the following numerical values of the parameters: K_1 = 7.1×10^{-11}; K_2 = 6.3×10^{10}; K_3 = 1.9×10^{8}; K_4 = 0.48; T_1 = 4×10^{-13}; T_2 = 5×10^{-15}; K_s = 42.2×10^{-9}; ω_s = 1183; ζ = 5×10^{-4}.

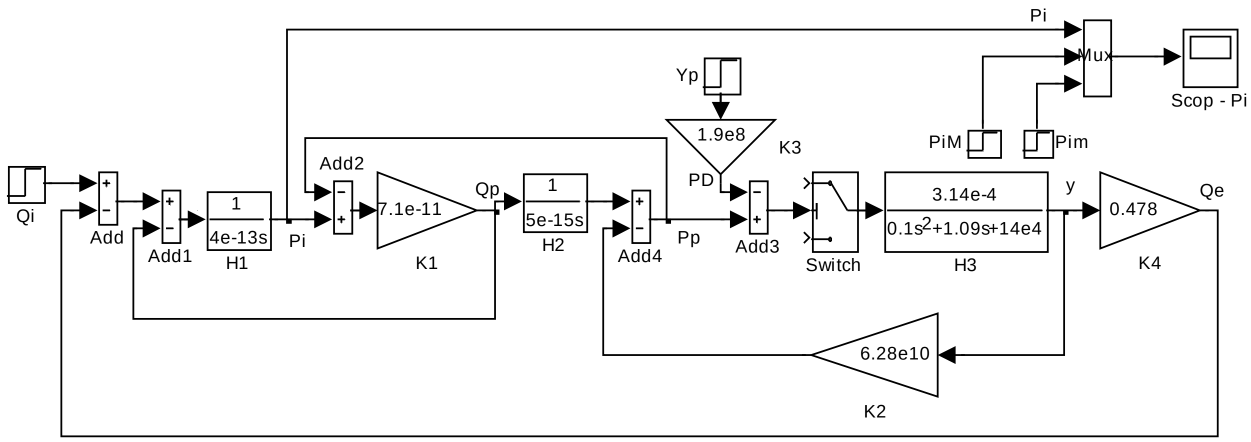

Figure 3. Simulation diagram of the pressure limiting valve

Numerical simulation was done for step inputs: Q_i = 30 l/min and Y_p = 15.7 mm.

In Figure 4 is the indicial response of y opening of the valve.

In Figure 5 is presented the indicial response of upstream pressure P_i, and Figure 6 represents the indicial response of piloting pressure P_p.

Based on these responses can be done the following interpretations:

1. Valve opening occurs after approx. 2.65 ms and settling time is approx. 30 ms with a 50% overshoot. For this response the oscillations frequency after valve is opened is f_{pD} = 81 Hz and has the same value for each signal;
2. Upstream pressure P_i has a 45% overshoot and steady-state value of 34.5 bar, which is different from the opening pressure, P_D = 30 bar, due to additional compression of the spring but, especially, due to hydrostatic force on the spool;
3. Pressure P_p from piloting chamber, before valve opening, has a frequency of 1000 Hz and after opening f_{pD} = 81 Hz.

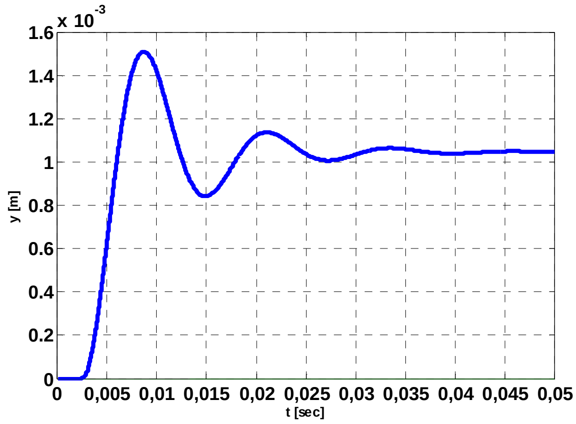

Figure 4. Indicial response of valve opening

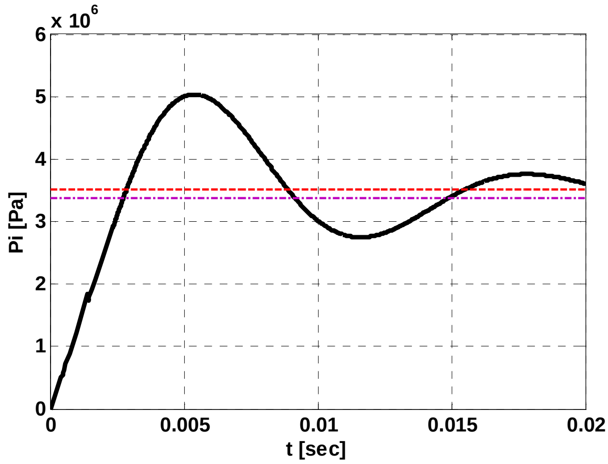

Figure 5. Indicial response of input pressure

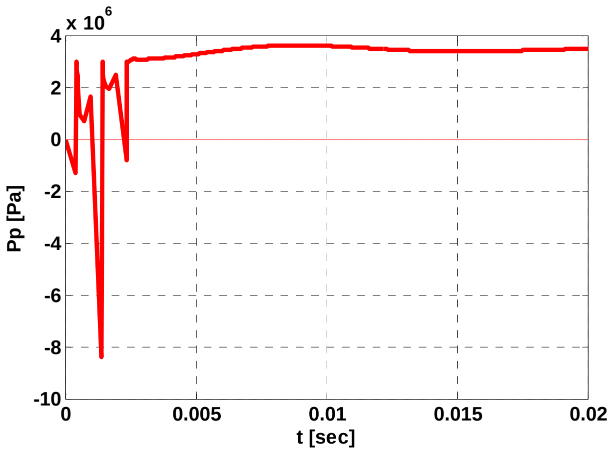

Figure 6. Indicial response of piloting pressure

4. Final conclusions

Simulations and the determined zeros and poles of transfer functions presented in equations (8) allow the following conclusions:

- pressure limiting valve may be associated, precisely enough, with a proportional-derivative second-order delay system, which has damping ratio of 1655 rad/s and natural frequency of 0.8:

$$P_i(s) = \frac{2.5 \cdot 10^{12} \cdot (s+100.7)}{s^2 + 263.2 \cdot s + 2.74 \cdot 10^5} \cdot Q_i(s) - \frac{7.36 \cdot 10^{18} \cdot (s+1375)}{s^2 + 263.2 \cdot s + 2.74 \cdot 10^5} \cdot Y_p(s) \quad (9)$$

- approximation error is around 5%.

In conclusion, the mathematical model presented in this paper provides the following advantages:

- validates the behaviour of pressure limiting valve as second-order system;
- allows the interconnection between valve and adjacent systems, considering their specific inputs and outputs;
- underlines the stages of the valve, before and after opening.

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