APPROXIMATING COMPUTER SYSTEM OPERATION TECHNOLOGIES UNDER EXTERNAL ACTION THROUGH THE BRUSSELATOR MODEL WITH PERTURBATION IN THE FORM OF DYNAMIC CHAOS

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Abstract. In this article, the research of the Brusselator mathematical model and its implementation in the form of the chaotic attractor is described. The hypothesis about the possibility of approximating computer system operation technologies under external action through the Brusselator model with perturbation in the form of dynamic chaos is presented and then proven through mathematical simulation. Special system and mathematical software was developed for this experimental research. The results received were evaluated and their reliability was proven by testing the data through simulation.

Keywords: computer systems and networks, Brusselator mathematical model, dynamic chaos, perturbation of the computer system

1. Introduction

Modern computer systems and networking (CSN) are important elements, the backbone of the global information infrastructure (GII). The key functions of processing, storing and exchange of data, informational and subscriber exchange between elements of GII and others are assigned to CSN. It is worth noting that the high level of consumers' demand for CSN in the general process of GII functioning entails a constant "pressure" from malicious elements that seek to introduce a certain imbalance in the normal functioning of the system, which has a certain effect on the process of designing such systems.

It is at this stage of the development that the necessary functional and informational security reserve is laid; it must be provided with various methods and means of access distribution and data protection in CSN.

In general, when solving problems in CSN design it is necessary to take into account the numerous requirements for security established by certain international documents [1, 11] and national standards [3]. In this regard, it is important to use the capabilities of a systematic approach, both at the stage of formalizing the design goals and while ensuring their effective solutions.

The analysis of several sources [5-10] has shown that when solving problems in CSN design, at different stages of the structure synthesis various approaches were and are often used, which have to do with the theoretical justification and mathematical description of various processes which take place in CSN [8, 9], as well as with heuristic research and formalizing the technologies used in their operation [6, 7]. Authors often use these methods separately, without taking a full advantage of their joint use, both as a means of the mathematical formalization of the processes under study, and as practical experience (engineering intuition) of the developers.

The analysis of literature [6, 7, 9, 10] dealing with mathematical simulation of CSN in conditions with external influence has shown the prospects of using approaches based on the key concepts of the nonlinear dynamics theory [4, 5]. These approaches make it possible to use modern knowledge of one of the new areas of science – dynamic chaotic processes together with the practical experience of the developers based on the created simulation models. The main mathematical concepts which describe the operating process of complex technical objects with the help of stochastic dynamic systems implementing stable multifold trajectory indicators as a limit cycle are presented in works [7, 9].

2. The Brusselator mathematical model

One type of such dynamic system models is the brusselator mathematical model:

$$\dot{z}_1 = a-(b+1)z_1 + z_1^2 z_2 , \qquad (1)$$

$$\dot{z}_2 = bz_1 - z_1^2 z_2 , \qquad (2)$$

where z_1 and z_2 are NOT known and reflect the dynamics of the state indicators studied in the process of system operation; $a > 0$ and $b > 0$ are indicators, which define the system's initial states.

Today this model is mostly used to explain effects which take place in chemical reactors. The model's dissipative structure properties, which are also present in CSN, were discovered while carrying out this study. This fact can be proven through the comparative analysis (Figure 1) of the phase portrait of the computer system Central Processing Unit (CPU) usage indicator, received experimentally with the Brusselator phase portrait, calculated through the mathematical model when $a = 0.4$ and $b = 1.2$.

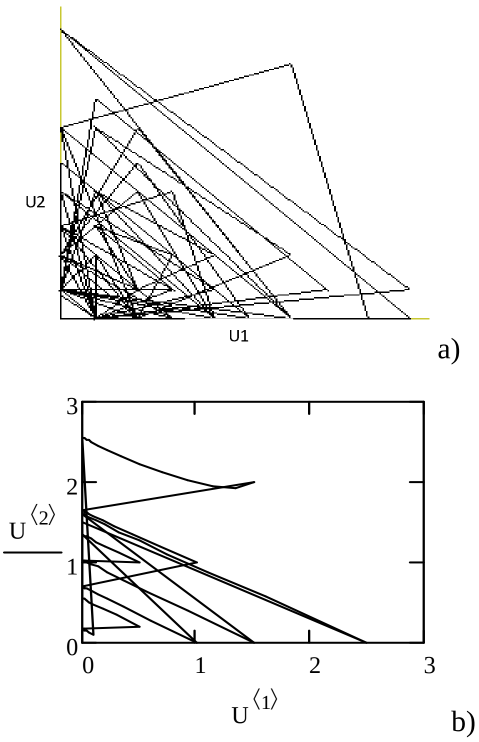

Figure 1. A comparison of the phase portrait indicator of the computer system Central Processing Unit usage with the Brusselator phase portrait

As is seen in this illustration, a certain similarity can be noted between the presented motion trajectories of the index of real CPU usage and the Brusselator. At the same time, this picture also illustrates their distinctive features. This is largely due to the lack of consideration of possible internal perturbations and external disturbance of the system through dynamic chaos [4, 5] in the Brusselator mathematical model. This effect of dynamic chaos is present in the real process of CSN operation. Should this factor be taken into account, the accuracy of the results of mathematical modeling at the stage of CNS design will improve. This is why the problem of approximating computer system operation technologies under external action through the Brusselator model with perturbation in the form of dynamic chaos is relevant today.

Main points. Let's imagine CSN as a controlled object in the form of two subsystems ($Q1$ – static (with fixed parameters), $Q2$ – dynamic (with configurable parameters)) [6, 7], and the matrix **X** with system state coordinates.

Using the system (1, 2) as well as known facts about possible external actions and internal perturbations, the technology of a computer system operation and study the nature of its security from external action are exemplified in the following equation system:

$$\begin{aligned} \dot{x}_1(t) &= A(t) - (B(t)+1)x_1(t) + \\ &+ x_1^2(t)x_2(t) + \varepsilon D(t) + E(t)\chi(t) \end{aligned} \qquad (3)$$

$$\dot{x}_2(t) = B(t)x_1(t) - x_1^2(t)x_2(t), \qquad (4)$$

where $\dot{x}_1(t) \in \dot{X}_1$ is a measurable m-dimensional vector of the $Q1$ object subsystem condition indicator coordinates, $\dot{x}_2(t) \in \dot{X}_2$ is a measurable m-dimensional vector of the $Q2$ object subsystem condition indicator coordinates, $A(t)$, $B(t)$ are continuous matrices with the initial system state, $D(t)$, $E(t)$ are continuous matrices with NON-observable measurement errors, ε is the intensity of internal perturbations, $\chi(t)$ is a m-dimensional vector of uncontrollable external actions.

The distinctive feature of this equation system is the consideration of small internal perturbations $\varepsilon D(t)$ and external malicious actions $E(t)\chi(t)$ in the classic Brusselator model.

For the unperturbed system (3, 4) ($\varepsilon D(t) = 0$) and ($E(t)\chi(t) = 0$) the value $\overline{B}(t) = 1 + \overline{A}(t)$ is a bifurcation point. When the $B(t)$ parameter passes through $\overline{B}(t)$, the state of equilibrium $\overline{x}_1(t) = A(t)$ and $\overline{x}_2(t) = B(t)/A(t)$ loses its stability and a stable limit cycle appears in the system.

The changes in the dynamics of a chaotic attractor of CSN states are examined when periodic and stochastic internal perturbations and external input are added. To do this, a family of phase portraits (attractors) of CSN behavior is created in the Mathcad system. The program code listing is presented in Figure 2.

As a result of the execution of this code in the Mathcad system, the Brusselator phase portraits are formed (Figures 3 and 4). The changes in the Brusselator dynamics are examined when nonperiodical and stochastic external input is added. It is known (from [4, 5]), that a periodically perturbed Brusselator can switch to a chaotic mode. For example, an unperturbed limit cycle where a = 0.4 and b = 2.5, when external nonperiodical action $\chi(t) = \cos\beta t$ is added, demonstrates a bifurcation sequence of period increase with a switch to chaos (Figure 3).

$$k := 3.7 \quad \alpha := 0.95 \quad \beta := 0.991 \quad \varepsilon := 0.2$$

$$v := \begin{pmatrix} 0 & 0 & 2.5 & 1.5 & 0.5 & 1 & 1 & 1.5 & 0.1 & 0.5 \\ 0.5 & 1.5 & 0 & 0 & 1 & 0 & 1 & 2 & 0.1 & 0.2 \end{pmatrix}$$

$$t_0 := 0 \quad t_1 := 30 \quad B := 2.5 \quad M := 200 \quad A := 0.4$$

$$D(t,y) := \begin{bmatrix} A - (B+1)\cdot y_0 + (y_0)^2 y_1 + \varepsilon\cos(\beta) + \dfrac{k}{(1-\mathrm{rnd}(1))^{\frac{1}{-0.15\alpha}}} \\ B y_0 - (y_0)^2 \cdot y_1 \end{bmatrix}$$

```
U := | y ← v<0>
     | Z ← rkfixed(y, t0, t1, M, D)
     | Z1<0> ← Z<0>
     | Z1<1> ← Z<1>
     | Z1<2> ← Z<2>
     | for k ∈ 1..last[(v^T)<1>]
     |   | y ← v<k>
     |   | Z ← rkfixed(y, t0, t1, M, D)
     |   | Z2<0> ← Z<0>
     |   | Z2<1> ← Z<1>
     |   | Z2<2> ← Z<2>
     |   | Z1 ← stack(Z1, Z2)
     | Z1
```

Figure 2. Brusselator program code listing

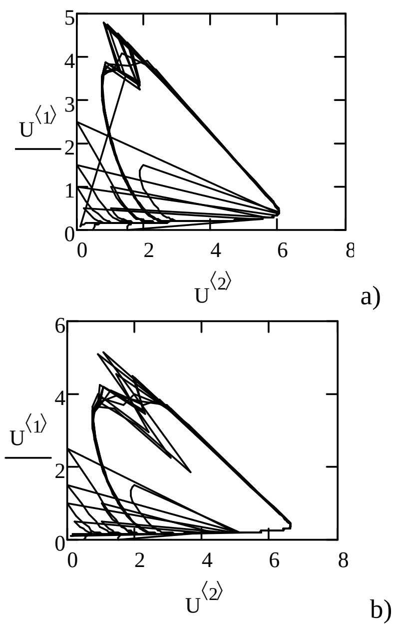

Figure 3. The Brusselator with periodic perturbations when $a = 0.4$, $b = 2.5$, $\varepsilon = 0.02$
a) $\beta = 0.59$; b) $\beta = 0.99$

Figure 4 illustrates the random paths of the system (1), for three values of noise intensity, $E(t) = 0.1$, $E(t) = 0.2$ and $E(t) = 0.3$. Analysis of the graphs in Figure 4 shows that when forming a stochastic sequence with parameters $E(t) = 0.2$, $E(t) = 0.3$ the Brusselator phase portraits gain a visual similarity to the phase portrait of computer system CPU usage indicator (see Figure 1).

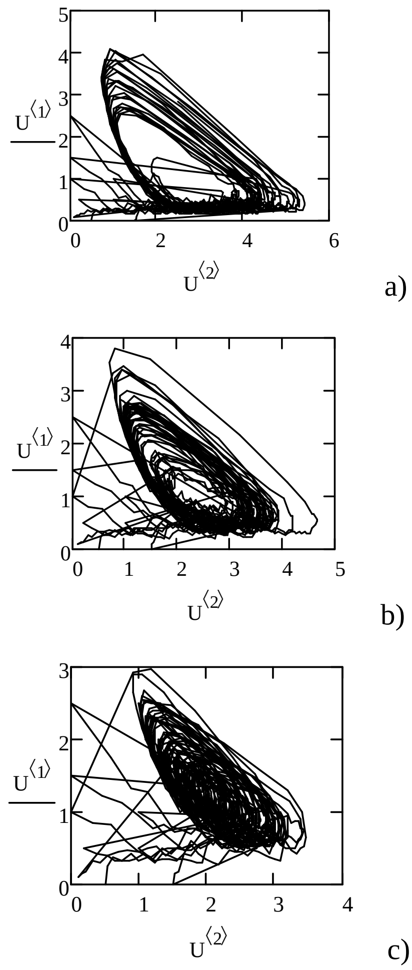

Figure 4. The Brusselator with perturbations the statistic characteristics of which conform to Pareto's principle
a) $E(t) = 0.1$; b) $E(t) = 0.2$ c) $E(t) = 0.3$

To substantiate the validity of the received results, an experiment has been carried out with the help of a specialized program which identified the state of the computer system and produced its phase portraits [7]. In accordance with the conditions of the experiment:

– the computer system carries out some established functional tasks (*LibreOffice* - an office software suite, and a browser are started);
– the action of a Dos-attack on the computer system is imitated through the *Ping* utility, which is initialized from three remote computers;
– the number of experimental values of the CPU usage indicator is $N^* = 2010$.

Estimates of the expectation value $U(\vec{\xi})^{(i)}$ and the dispersion $\hat{D}_{U(\vec{\xi})^{(i)}}$ are obtained ($\hat{\sigma}_{U(\vec{\xi})^{(i)}}$ of the mean-square deviation) for a random value $U(\vec{\xi})^{(i)}$ of CPU usage:

$$\hat{U}(\vec{\xi})^{(i)} = \frac{\sum_{i=1}^{k} \hat{U}(\vec{\xi})^{(i)}}{\mathcal{N}^*}, \tag{5}$$

$$\hat{D}_{U(\vec{\xi})^{(i)}} = \frac{\sum_{i=1}^{k} \left(U(\vec{\xi})^{(i)} - \hat{U}(\vec{\xi})^{(i)} \right)^2}{N^* - 1}, \tag{6}$$

$$\hat{\sigma}_{U(\vec{\xi})^{(i)}} = \sqrt{\hat{D}_{U(\vec{\xi})^{(i)}}}. \tag{7}$$

Using the known expression for the calculation of the probability belief for the deviation of the relative frequency from the constant probability in independent testing [3], the probability belief that the received experimental value of the CPU usage indicator will not "deviate" from the mathematical expectation $\hat{U}(\vec{\xi})^{(i)}$ by more than 1, is found:

$$P\left(\left| \hat{U}(\vec{\xi})^{(i)} - U(\vec{\xi})^{(i)} \right| < 1 \right) = 2\,\Phi\left(1 \big/ \hat{U}(\vec{\xi})^{(i)} \right), \tag{8}$$

where Φ is Laplace's function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2}\, dt$ [3].

The conducted experiment showed that for all researched data types the probability belief that the value of the statistic variable $U(\vec{\xi})$ (CPU usage) will not "deviate" from the mathematical expectation $U(\vec{\xi})$ by more than 1 is $P \approx 0.9$.

3. Conclusion

Thus, in the process of conducting the research on Brusselator mathematical models and their implementation in the form of a chaotic attractor, several visual patterns, common to phase portraits of a computer system CPU usage indicators, were found.

Improvement of the Brusselator mathematical model through considering the periodic internal perturbations and chaotic external action enabled the approximation of the technology of computer system operation in conditions of external action through the Brusselator mathematical model with perturbations on the form of dynamic chaos.

Evaluation of the credibility of the results received was conducted using the Pearson criterion χ^2. Results of the evaluation proved the hypothesis about the possibility of approximating the technology of CSN operation in conditions of external output by the Brusselator mathematical model.

The results of the presented mathematical model can be used in the design and implementation of means of prevention and detection of external influences on CSN as a constituent element, performing the tasks of identifying the state of the subject (the computer system).

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