DETERMINATION OF ERRANT RUN-OUT OF THE AXIS OF ROTATION OF OBJECTS, PERFORMING ACCURATE ROTATIONAL MOVEMENTS

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Abstract. This article discusses problems of the accuracy of the axes position of objects accomplishing rotary motion. Different schemes to estimate the errant run-out of the axes and faces of object tables are proposed, as well as their capabilities are estimated.

Keywords: rotary motion, radial and axial run-out, errant radial and axial run-out

1. Introduction

Constancy of the rotational axis is a basic requirement towards the objects, accomplishing accurate rotational movement (such as rotary tables of roundness measuring machines, rotary modules of measuring systems and production machines, etc.).

This volatility is estimated by the so called Errant run-out (radial, axial and angular).

The **errant radial run-out** (Δ_{rbb}) is defined as volatility of the position of a point from the instantaneous axes of rotation relative to a virtual reference axis in a plane perpendicular to that axis.

The **errant axial run-out** (Δ_{abb}) is defined as volatility of the position of points from the instantaneous axes of rotation relative to a plane, which is perpendicular to the virtual reference axis.

The **errant angular run-out** (Δ_{abb}) is defined as volatility of the angular position of the instantaneous axis of rotation relative to the virtual reference axis.

The **virtual reference axis** is a straight line in a xyz coordinate system, passing through the centroids of the points of the instantaneous axis of rotation in two distant parallel planes xy [2]. The axis of rotation is oriented along the axis Z.

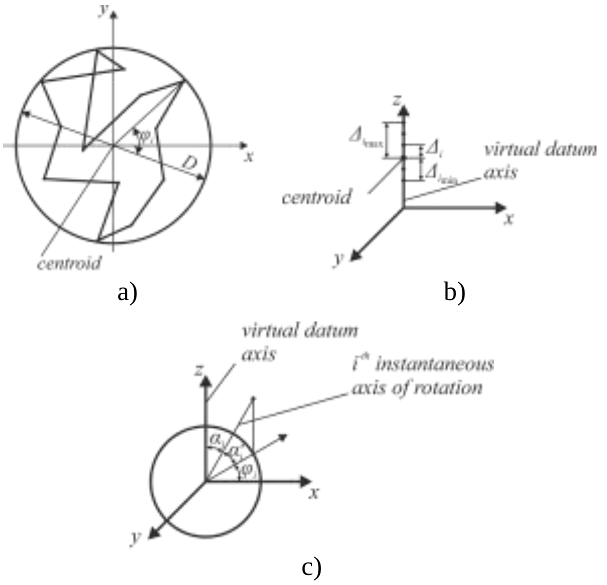
A **centroid** is a point with coordinates, which are arithmetic mean values of the coordinates of the points relative to which it is determined

The errant run-out in a given plane or direction should be considered as the locus of instantaneous axes of rotation of the object and accordingly represents a complex closed curve or a set of points, positioned along the line of measurement (Figure 1, a and b).

The errant run-out can be quantitatively estimated through the size of the available area, or through the magnitude of the deviation from the virtual datum, or through the instant positions of the instantaneous axes relative to the virtual reference axis.

For example, the errant radial run-out can be estimated by the diameter D of the circle described around the curve (Figure 1a), or through the instant deviations Δ_i relative to the centroid, or through the spread of these deviations $\Delta_{i_{\max}} - \Delta_{i_{\min}}$.

It should be noted that the diameter of the enveloping circle D is affected by extreme deviations $\Delta_{i_{extr}}$ and is not sufficiently informative, which greatly limits its use.



a) b)

c)

Figure 1. The errant run-out in a given plane or direction

The errant axial run-out is evaluated by the instant positions of the instantaneous axes of rotation relative to their centroid, or the spread of this position in the direction of the virtual reference axis (the axis z) (Figure 1b).

The errant angular run-out is measured by the angle α_i, which i^{th} instantaneous axis of rotation makes with the virtual reference axis, or by the magnitude of these angles ($\alpha_{i_{\max}} - \alpha_{i_{\min}}$) (Figure 1c).

The position of the instantaneous axis of rotation, or respectively the angle α_i is a function of the angle of rotation φ_i of the object around the virtual datum axis.

All three types of errant run-out have their systematic and random component, which can be evaluated by repeated measurement and by a corresponding statistical processing of the measurement results.

2. Schemes of measurement - description, nature and comparative analysis

2.1. Scheme No. 1

This scheme is often used in the metrological practice to determine the errant radial and axial run-out. But the methodology included herein below makes it is possible to measure also the errant angular run-out.

The procedure involves the measurement of both the radial and axial run-out of a reference glass hemisphere, located on the top of the turntable of the object under test (Figure 2, a and b), by means of two measuring heads (MH), as well as by subsequent processing of this primary information [4].

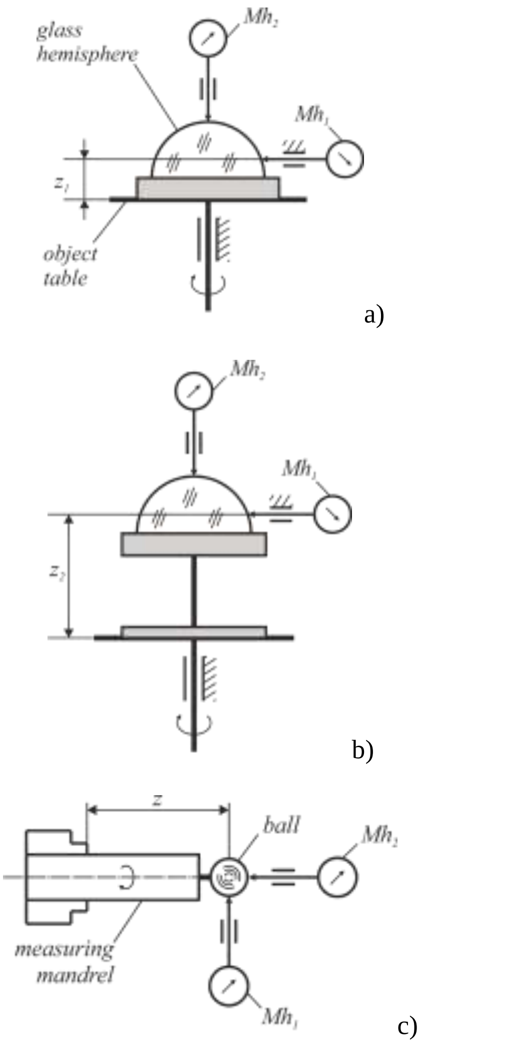


Figure 2. The measurement of both the radial and axial run-out of a reference glass hemisphere

In the general case, the radial run-out includes the following components:

- The deviation from roundness of the measured profile section (EFK);
- The eccentricity of the center of the associated circle of the measured profile (e);
- The errant run-out in a radial direction (errant radial run-out – Δ_{rbb}).

The axial run-out includes the following components:

- The deviation from flatness of the measured profile EFE of the axial surface at a given radius;
- Deviation from perpendicularity EPR of the associated plane of the measured at a given radius profile relative to the axis of rotation;
- Errant axial run-out Δ_{abb} (inconstancy of the axial position of the work-piece during its rotational movement);
- A component related to the parallelism EPA of the instantaneous axes of rotation, respectively the parallel misalignment of the perpendicular to them planes, which serve as a starting instant datum - i.e. a component associated with the angular errant run-out.

In case a standard sphere is used, the component associated with its own shape deviation, can be ignored and then the errant run-out occurs (results in) as the roundness deviation of the measurements hemisphere profile. The errant radial run-out on the graphs showing the results of the measured radial run-out is expressed as the roundness deviation relative to the least squares associated (LSQ) circle, whose center is the centroid of the examined (obtained) values, Figure 3. The errant radial run-out is equal to the roundness deviation of the measured profile Δ_i.

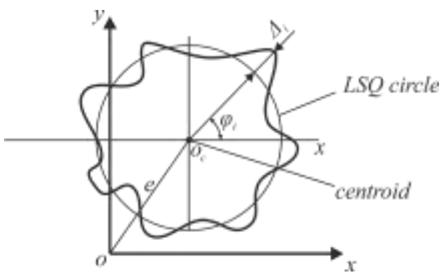


Figure 3. The errant radial run-out expressed as the roundness deviation relative to associated LSQ circle

When measuring the axial run-out with a measuring head (MH) with a flat stylus and a well centered hemisphere, all components except the errant axial run-out are negligible. The latter can be measured directly with the MH_2. Similarly to the errant radial run-out, the errant axial run-out

Δ_{abb_i} at a given angle of rotation φ_i, may be presented in the form of a graph, as the roundness deviation relative to the LSQ associated circle.

A measuring mandrel with a ball is used for measuring the errant radial and axial run-out of the spindle of machine tools (Figure 2c) [3]. Usually there is standardization of the heights z_1 and z_2 of the cross-section, in which the measuring of the semi-sphere is performed, or of the length z of the measuring mandrel, on which the ball is located (Figure 2b).

It should be noted that the errant radial run-out is due to two factors: the parallel shift of the instantaneous axis of rotation and the angular deviation from their virtual datum axis. If necessary, these two components can be determined separately by assessing the angular errant run-out. In many cases, namely the evaluation of angular errant run-out is essential in determining the functional and metrological capacities of certain measuring systems containing rotary modules.

The following proposed methodology allows the determination of the angular errant run-out using the results from the measurement of the radial run-out of different hemisphere profiles at different height z from the table top (Figure 2, a and b).

As is seen on Figure 4a, the angle, which makes the i^{th} instantaneous axis of rotation with the virtual datum axis, can be determined through the difference between two sections, located at distance Δ_z, using the formula:

$$\alpha_i = arctg \frac{\Delta_{i_{z_2}} - \Delta_{i_{z_1}}}{\Delta_z} \qquad (1)$$

Analogically to the errant radial and axial run-out, the angular errant run-out can be presented in the form of a graph for assessing the deviation from roundness of which Δ_{α_i} represents the departure from roundness relative to LSQ associated circle (Figure 4b).

The depiction of errant radial, axial and angular run-out as graphs to for assessing the circularity deviation allows us to estimate both their local values as a function of the angle of rotation φ_i, and their maximum dissipation in the form of a spread.

Plotting of graphs is performed by standard programs.

The advantages of scheme No. 1 are:

- The possibility for determination of both errant radial and axial run-out, as well as errant angular run-out;
- High precision of the original measurement data;
- The simplicity of the measuring equipment;
- The possibility for automating the measurement, processing and presentation of the results, which allows measurement of the errant radial and axial run-out in both static, as well as dynamic mode.

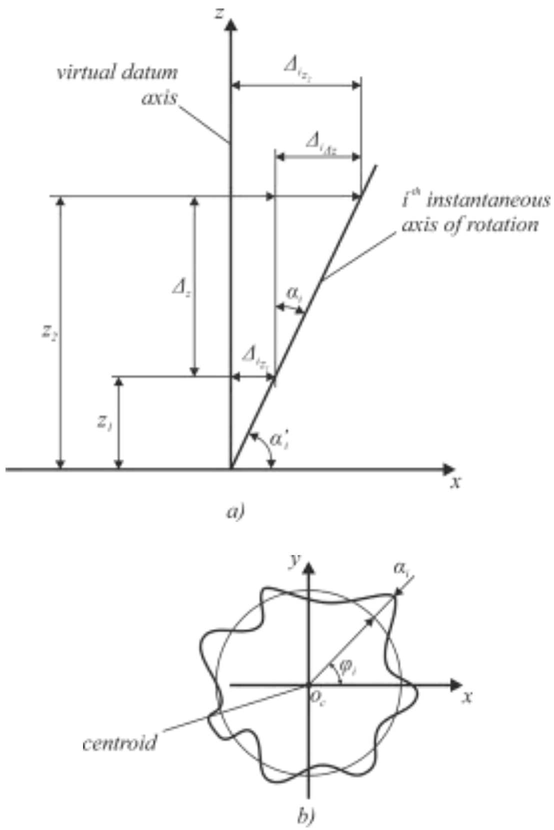


Figure 4. The angular errant run-out presented in the form of a graph

The shortcomings should refer the need for consecutive (non-simultaneous) measurement of the radial run-out of different heights of the measured hemisphere profiles, which hinders precise assessment of the random component of errant angular run-out.

2.2. Scheme No. 2

This scheme is inherently similar to scheme No 1, but instead of the reference glass hemisphere it uses a stepped reference shaft, two pins of which have been pre-calibrated pursuant to EFK, and the face surface has been pre-calibrated according to EFE, using a roundness measuring machine (Figure 5). The axial run-out along a path with radius R is measured with the probes MH_1 and MH_2. This path is also used for the calibration of EFE. The need for calibration of EFE is eliminated, when an interferential glass plate has been mounted on the face of the mandrel.

The own deviations of the form (EFK of the necks and EFE of the front surface) are excluded when measuring radial and axial run-out by introducing relevant corrections [3].

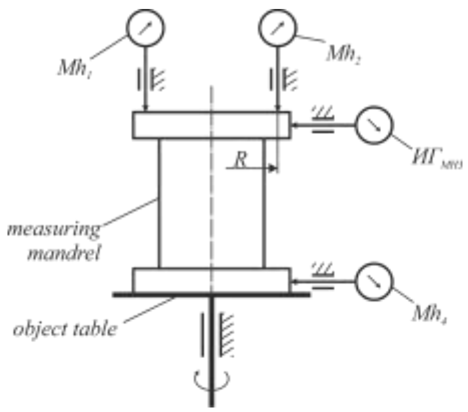


Figure 5. The axial run-out measured along a path with radius *R*

The axial run-out is determined by the halve values of the sums Δ_{A_i} from the readings A_{1_i} and A_{2_i} of MH_1 and MH_2 at a specific angle of rotation φ_i, i.e.

$$\Delta_{A_i} = \frac{A_{1_i} + A_{2_i}}{2}\ . \qquad (2)$$

Thus, the influence of the perpendicularity of the shaft face relative to the axis of rotation is excluded.

A graph is made by these values similar to errant radial and angular run-out, and the local values of the errant axial run-out Δ_{abb_i}, as well as their spread can be estimated by the roundness deviations with respect to the LSQ associated circle.

An advantage of the scheme No. 2, besides the aforementioned of scheme No. 1, is the possibility for simultaneous measurement of the radial and axial run-out of the calibrated radial and axial profiles (surfaces) of the measuring mandrel, which is essential for the assessment of the errant angular run-out under a dynamic mode.

To the shortcomings can be attributed the need for a measuring mandrel, calibrated according to EFK and EFE, which complicates the measurement equipment and affects the accuracy of measurement.

2.3. Scheme No. 3

Both the errant axial and angular run-out can be determined using this scheme (Figure 6).

The axial run-out of the glass interferential plate laid on the object table is measured along paths with radii R_1 and R_2 using two measuring heads MH_1 and MH_2 (Figure 6a). In good centering the measurement line of MH_1 coincides approximately with the axis of rotation i.e. can be assumed $R_1 \approx 0$ and the errant axial run-out can be evaluated directly by its reading A_{1_i}. A graph is built on the values of A_{2_i} on which the local values of A_{abb_i} and their spread is assessed as the deviations from roundness similar to schemes No. 1 and No. 2.

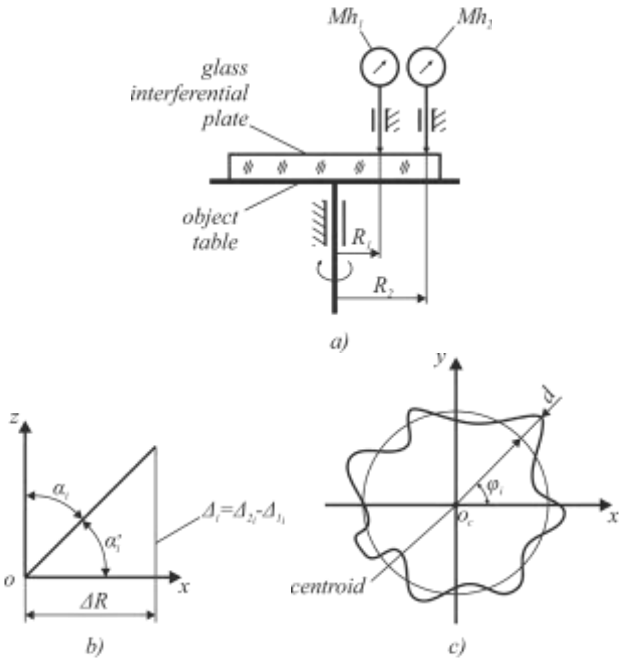


Figure 6. The measurement of the axial run-out of the glass interferential plate l along paths with radii R_1 and R_2

The axial run-out, measured with MH_2, includes both the errant axial run-out as well as the errant angular run-out of the axis.

Then in analogical manner to scheme No. 1, the current angle α_i between the instantaneous rotation axes and the virtual reference axis can be determined by the expression (Figure 6b):

$$\alpha_i = 90^\circ - arctg\,\frac{A_{2_i} - A_{1_i}}{R_2}\ . \qquad (3)$$

where A_{2_i} are the current readings of MH_2 measuring the axial run-out of the path of radius R_2.

Using these values of α_i a graph can be built, on which the current values of Δ_{abb} i.e. α_i should be presented as deviations relative to LSQ associated circle (Figure 6c).

The major advantage of scheme No. 3 is the ability to measure the axial and angular run-out with the help of a relatively simple measuring equipment and software.

The inability to determine the errant radial run-out can be referred to the deficiencies.

2.4. Scheme No. 4

This scheme is typically used to determine the errant axial run-out, as well as the deviation from perpendicularity of the table relative to the axis of rotation.

An interferential glass plate with negligible deviation from flatness is set on the table of the rotary module under test (Figure 7a) [1].

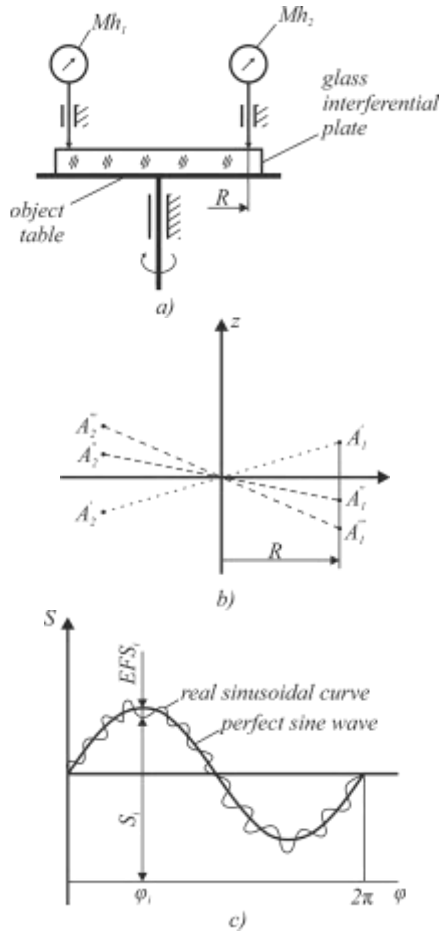


Figure 7. The measurement of the errant axial run-out, as well as the deviation from perpendicularity of the table relative to the axis of rotation

The axial run-out of the plate along a path of radius R is measured using two measuring heads MH_1 and MH_2 located at 180°.

As has already been mentioned, the axial run-out includes the flatness deviation of the plate (which is negligible), the deviation from perpendicularity of the associated plane of the plate relative to the axis of rotation, the errant axial run-out and the errant angular run-out of the axis of rotation.

Determination of the errant axial run-out

The readings of the two measuring heads A_{1_i} and A_{2_i} are recorded simultaneously while rotating the object table at a relevant angle φ_i.

The semi-sum of these readings is calculated:

$$\Delta_{A_i} = \frac{A_{1_i} + A_{2_i}}{2}. \qquad (4)$$

This semi-sum reflects only the errant axial run-out when the flatness deviation of the plate is negligible. The current position of the center of the plate in z-axis direction can be evaluated using the values of Δ_{A_i}, relative to their centroid (average value) and also the spread of these deviations ($\Delta_{A_{i\max}} - \Delta_{A_{i\min}}$) (Figure 1b).

Determining the errant angular run-out

Two ways to evaluate the errant angular run-out are proposed as follows:

1st way: Through the differences in the estimates of the deviations from the perpendicularity of the plate relative to the axis of rotation

The semi-differences $\Delta_{A'}$ and $\Delta_{A''}$ in the readings of the measuring heads are calculated at angles of rotation φ_i and $\varphi_i + 180°$:

$$\Delta_{A_i'} = \frac{A_{1,\varphi_i} - A_{2,\varphi_i}}{2}; \qquad (5)$$

$$\Delta_{A_i''} = \frac{A_{1,\varphi_i+180°} - A_{2,\varphi_i+180°}}{2}. \qquad (6)$$

These values reflect the deviations from the plate perpendicularity relative to the axis of rotation and the errant angular run-out (shake of the object table).

In the absence of errant angular run-out the values of $\Delta_{A'}$ and $\Delta_{A''}$ will be equal to the absolute value, but with opposite signs (Figure 7b), i.e.

$$|\Delta_{A'}| = |-\Delta_{A''}|. \qquad (7)$$

The deviation from perpendicularity at a radius R is determined by the maximum value of the difference $\Delta A_{\max} = \Delta A'_{i\max} - \Delta A''_{i\min}$.

In the presence of errant angular run-out:

$$\Delta A_i'' \rightarrow \Delta A_i''';$$

$$\Delta A_i''' = \frac{A_1''' - A_2'''}{2};$$

$$|\Delta A_i'''| \neq |-\Delta A_i'|.$$

The difference $\Delta A_i''' - \Delta A_i'' \Rightarrow \Delta A_i''' + \Delta A_i''$

reflects the errant angular run-out as the angle α_i between two instantaneous axes of rotation, located at 180° from each other - i.e. angles φ_i and $\varphi_{i+180°}$ (Figure 7b): $\alpha_i = arctg\frac{\Delta A_i''' - \Delta A_i'}{R}$.

When the difference $\Delta A_i''' - \Delta A_i'$ is positive, the angle α_i is counterclockwise, and when it is a negative - clockwise.

The current values of the angle between the instantaneous axes of rotation and the virtual reference axis cannot be determined by this procedure, but information for maximum "shaking" of the object table is received, i.e. of $\alpha_{i_{\max}}$.

2nd way: Through the deviation from sinusoidality

In the absence of errant axial and angular run-out and in the presence of a deviation from plate perpendicularity relative to the axis of rotation, the local values of the axial run-out are described by a sine wave.

The presence of errant run-out will involve a deviation from sinusoidality EFS (Figure 7c).

In this case, under deviation from sinusoidality is meant the deviation of the real sinusoidal movements of a point of the interferential plate from the perfect sine-curve due to errant angular run-out.

The deviation from sinusoidality EFS_i at a given rotation angle φ_i of the table is determined with regard to the ideal sine-curve built relative to a perfect sine wave, using the least squares method.

Then the current values of angular errant run-out α_i expressed in angular units α_i is determined by the expression:

$$\alpha_i = 90 - \alpha_i' - arctg\frac{EFS_i}{R},$$

and the spread as a difference between the maximal and the minimal values of α_i:

$$\Delta\alpha = \alpha_{i_{\max}} - \alpha_{i_{\min}}.$$

The major advantage of the scheme discussed is the possibility of accessing not only the errant axial run-out but also the errant angular run-out, while using a simple measurement equipment, both in a static and in a dynamic mode.

The inability to determine the errant radial run-out may be allocated to the shortcomings of the scheme.

2.5. Scheme No. 5

Through a measurement under this scheme the errant axial and angular run-out can be determined.

An interferential glass plate with negligible deviations from flatness is set on the object table of the rotary module. Using three measuring heads, which are located on a circle with a given radius R at 120° interval from each other, the axial run-out of the plate is measured during its 360° rotation (Figure 8a).

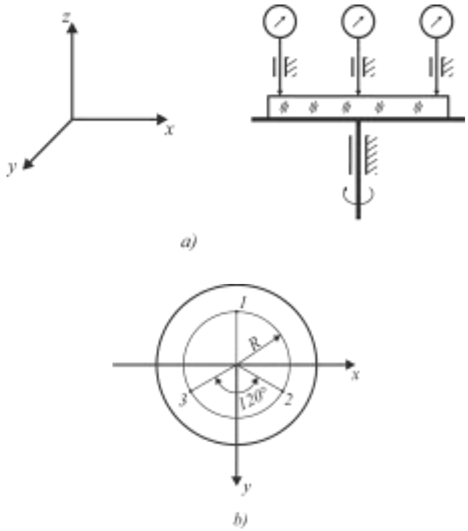


Figure 8. The measurement of the errant axial and angular run-out

Using the readings of measuring heads at a given rotation angle of the table φ_i - i.e. by the coordinates of three points on the plate in a coordinate system xyz, using a standard program and the position of its centroidal, the location of its centroid along axis z (coordinate z_i) is calculated, as well as the position of the normal vector of this plane relative to the axis of rotation. The axis of rotation is oriented along the axis z.

The fluctuation of the coordinate z_i of the centroid is the errant axial run-out, while the variation of the position of the normal vector to the axis of rotation reflects the errant angular run-out.

Both types of run-out can be expressed with their current values relative to the corresponding centroid, or through their spread.

The proposed by this scheme approach can be successfully used for the assessment of the angular and axial position of any object table (platform), performing linear and angular displacements in the space (Figure 9), both under static and dynamic mode.

Under a static mode, the measured object can be placed within the workspace of a coordinate measuring machine (CMM) and the coordinates of the corresponding points in the coordinate system XYZ can be consequently determined.

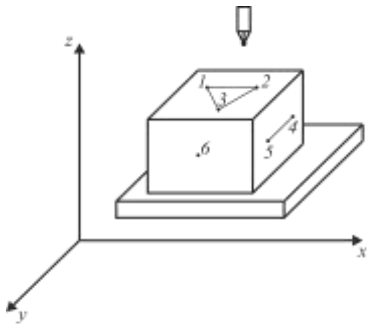


Figure 9. The assessment of the angular and axial position of any object table

The main advantage of the scheme is the ability to determine the errant axial and angular run-out under static, as well as under dynamic mode.

The proposed approach under this scheme can be successfully applied to the assessment of the axial and angular position of any object table (platform) performing axial and angular movements in the space under static and dynamic mode.

The complicated measuring equipment (several measuring heads) and software may be referred to the disadvantages.

The approach, proposed under this scheme, can be successfully used for assessing the axial and angular position of any object table (platform) carrying out axial and angular movements in the space both under a static and under a dynamic mode.

The measurement in static mode involves the use of CMM. A measurement platform together with the mounted on it prismatic body is located within CMM workspace. The coordinates of six pre-marked points of the cube in the coordinate system of the CMM (XYZ) are determined during the discrete movement of the platform (Figure 9). For each static position, using the three points 1, 2 and 3, the defined by it planes are determined, the position of the normal vector to the plane relative to axis *z* is calculated, as well as the centroid of the three points - i.e. the translation Δ_z, rotations φ_x and φ_y of the body relative to axis *x* and axis *y*.

Using the coordinates of points 3 and 4, the translation Δ_x along the axis and the rotation about the axis *z*, i.e. φ_z are determined.

Using the coordinates of point 6, the translation Δ_y can be determined.

In this way complete information of the platform position can be obtained during its discrete movement within the workspace of CMM.

Under dynamic mode the same effect is achieved with simultaneous measurement of the coordinates of the respective points, using measuring heads mounted on a stable fixed initial datum (datum coordinate system) (Figure 10).

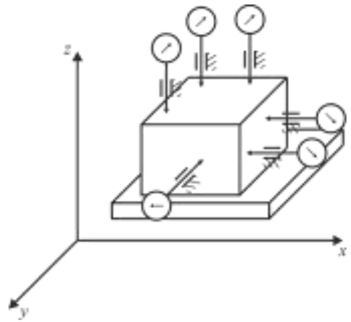


Figure 10. The assessment of the platform position under dynamic mode

For small displacements, when the impact of the change of the points' positions on the cube surfaces can be considered negligible, measurements can be performed simultaneously with the six MH.

When the displacement of the measured points is significant, it is appropriate to determine the position of each side of the cube separately, based on three measured points.

3. Conclusion

1. The constancy of the axis of rotation measured by radial, axial and angular errant run-out is a basic requirement towards the objects, accomplishing accurate rotational movements.

2. The accomplished survey and analysis of various schemes for determination of this inconstancy, allows the selection of the most appropriate way to measure the respective errant run-out for each case.

3. Using appropriate automation and software, all discussed and analyzed schemes allow receiving and processing of the primary measurement information for the determination of errant run-out under static, as well as under dynamic mode.

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