Comparison between Casting Solidification Simulation in Cylindrical Coordinates and in Cartesian Coordinates

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Abstract
Two software modules for solidification simulating of eutectic alloy castings were done at Transylvania University of Brasov. The first of them 3D-SIM EUT is based on finite difference mathematical model in Cartesian coordinates. The second SIM 2D-CIL-EUT, use a mathematical model in cylindrical coordinates. It is specialized for solidification simulation of casting with rotational symmetry. Simulation in cylindrical coordinates has the advantage that it allows the solidification simulation of spatial parts with rotational symmetry by 2D simulation, thus resulting a much shorter time of simulation. In addition, it allows the exact representation of the circular perimeter of parts with rotational symmetry, and a simpler division of system casting-mould in finite elements. After creating software in cylindrical coordinates, its operation and accuracy of the results were verified by several methods. In this paper, the verification was done by comparing to 3D-SIM EUT software in Cartesian coordinates that has been previously verified [1]. The final conclusion of the study is that the software based on the mathematical model in cylindrical coordinates provides results consistent with previously conducted software in Cartesian coordinates. In exchange for the studied part, the simulation duration was 40 times smaller.

Keywords
casting, solidification, simulation, cylindrical coordinates

1. Introduction

The Transilvania University of Brasov has used more software modules to simulate the solidification of metal alloys cast parts (SIM 3D-EUT, for eutectic alloy castings, and 3D-SIM-SOL for solid solution alloys). These are based on 3D mathematical models, with finite differences with Cartesian coordinates [1, 2]. Recently, there was achieved the specialized software module for simulating the solidification of cast parts with rotational symmetry (SIM 2D-CIL-EUT). This software has the particularity of being based on a 2D mathematical model with cylindrical coordinates [3, 4]. The software with cylindrical coordinates has the advantage of simulating volume solidification of parts with rotational symmetry using 2D simulation. The 2D simulation provides a considerable reduction in the duration of the simulation because the number of volume elements the cast part - casting mould system is divided in is much smaller. In addition, computing relations involved in the mathematical model are simpler and the volume of calculations is more reduced. Depending on the configuration and the dimensions of the part and the pace of the division network of the cast part - mould system, the duration of the solidification simulation of a cast part can be reduced tens or hundreds of times [5, 6, 7, 8]. Another advantage of the software with cylindrical coordinate consists in a faster division of the system in finite elements and a more accurate rendering of the contour of parts with rotational symmetry.

After its execution, the software was tested in terms of operation and accuracy of results. The verification was carried out using three methods:

- by comparing the results with those obtained with other software;

- experimentally, by thermal analysis;
- experimentally, by casting some sample parts and analysing the position of the hot spots.

The software for solidification simulation of cast parts with rotational profile, made based on a 2D mathematical model in cylindrical coordinates, can be used for the following purposes:

- for concrete applied studies on the solidification of rotational symmetry parts, produced by casting shops, to design and optimize casting technologies;
- for conducting fundamental research on the influence of geometrical, constructive and technological factors on the solidification of cast parts in order to establish general rules of designing casting technologies for such parts.

In the first case, there is determined the position of the hot spots in cast parts and the need to use feeders, coolers and insulating materials. In addition, there is determined the number, position and dimensions thereof.

Fundamental theoretical studies are designed to determine correlations between various influencing factors and to establish general and simplified rules for the rapid design of casting technologies, available for standard dimensions of parts [9].

2. Purpose of the Paper

Any software for simulation of technological processes requires verification of operation in terms of the accuracy of the mathematical model, of the software structure and veracity of results.

An accessible method for verifying the veracity of the results provided by the software for cast parts solidification simulation consists in comparing the results obtained with the software in question against the results obtained with some other software, recognized as accurate. This methodology was applied in the case of the 2D software with cylindrical coordinates developed at Transilvania University of Brasov. This paper presents the results of the verification using the method of comparing the results with those provided by the SIM 3D-EUT software, previously verified.

3. Work Method

The verification was done by simulating the solidification of a part with rotational symmetry, with profiled side. The piece is shown in Figure 1. There was simulated the solidification of this part using the software with cylindrical coordinates under verification and using 3D software in Cartesian coordinates. The latter is known to provide accurate results, being previously checked using different methods. Figure 2 shows the cast part - cast mould assembly for which the verification was conducted. Figure 3 shows the manner in which the cross-section is divided in the part - mould assembly for the two types of coordinates. The thermo-physical characteristics considered in the simulation are shown in Table 1. These correspond to casting the part made of grey iron with eutectic composition [5, 6]. In both cases the pace of the division network was Δ = 5 mm. There was analysed the position of the hotspot, the solidification time of the hot spot, the displacement of the solidification front, the map of isotherms at the time of the solidification of cast parts, the curves of variation of temperature and of the solid fraction in various points of the system.

The simulation results in the two cases are given in Table 2.

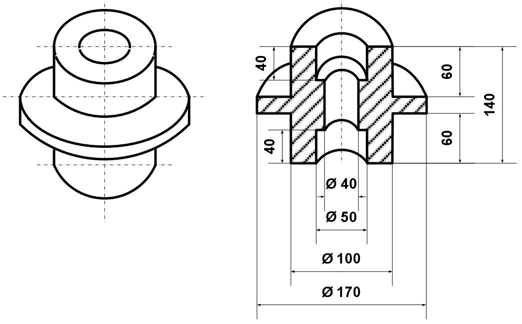


Fig. 1. The cast part subjected to the solidification study by simulating with the two software

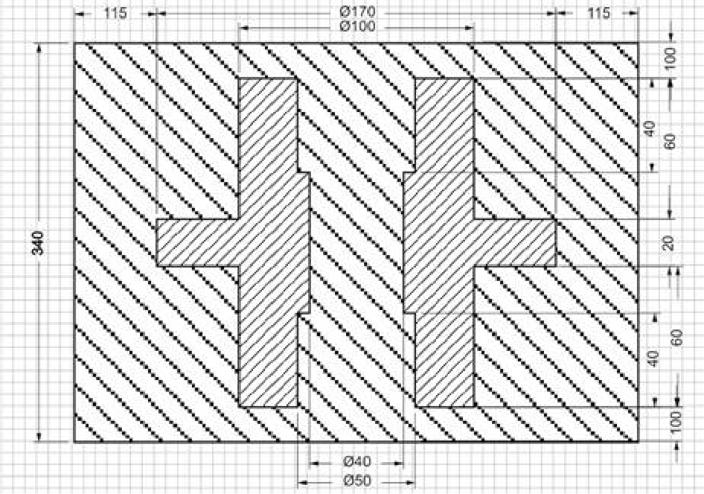


Fig. 2. Casting - mould assembly, subjected to the solidification study
by simulating with the two software

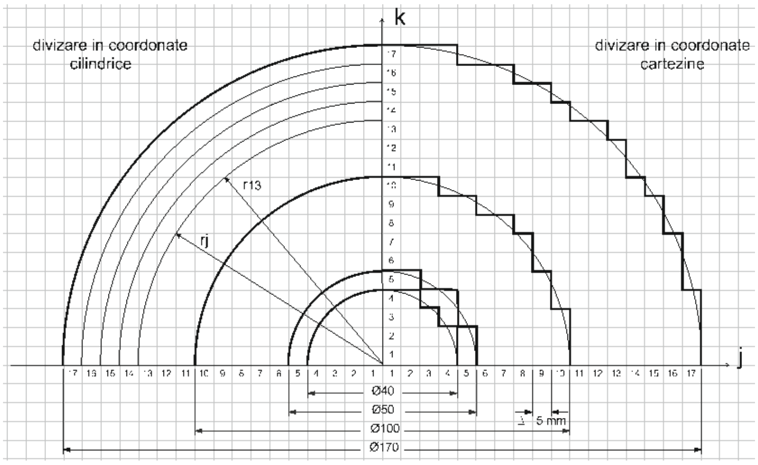


Fig. 3. Division of the cross-section of the part - mould system in cylindrical coordinates (left)
and in Cartesian coordinates (right) with Δ = 5 mm

Table 1. The geometrical and thermo-physical characteristics used for the simulation in the case of the parts from Figure 1

No.	Description	Physical symbol	Unit of measure	Value
1	Meshing step of the mould	Δ	m	0.005
2	Time step	τ	s	0.5
3	Temperature of the environment	Tex	^{0}C	20
4	The coefficient of thermal exchange with the environment	αex	W/m^2/K	20
5	The solidus temperature of the cast alloy	Tsme	^{0}C	1150
6	The coefficient of thermal conductivity of the mould	λsfo	W/m/K	0.8
7	The coefficient of thermal conductivity of the solidified alloy	λsme	W/m/K	40
8	The coefficient of thermal conductivity of the liquid alloy	λlme	W/m/K	30
9	The specific heat of the mould	Csfo	J/kg/K	1170
10	The specific heat of the liquid alloy	Clme	J/kg/K	850
11	The specific heat of the solidified alloy	Csme	J/kg/K	750
12	Mould density	ρfo	Kg/m^3	1550
13	Density of the liquid alloy	ρme	Kg/m^3	7000
14	The solidification latent heat of the cast alloy	Lme	J/kg	220000
15	The initial temperature of the mould	T_0fo	^{0}C	20
16	The initial temperature of the liquid alloy	T_0me	^{0}C	1320

Table 2. Results of the simulation in cylindrical coordinates and Cartesian coordinates in relation to the part in Figure 3

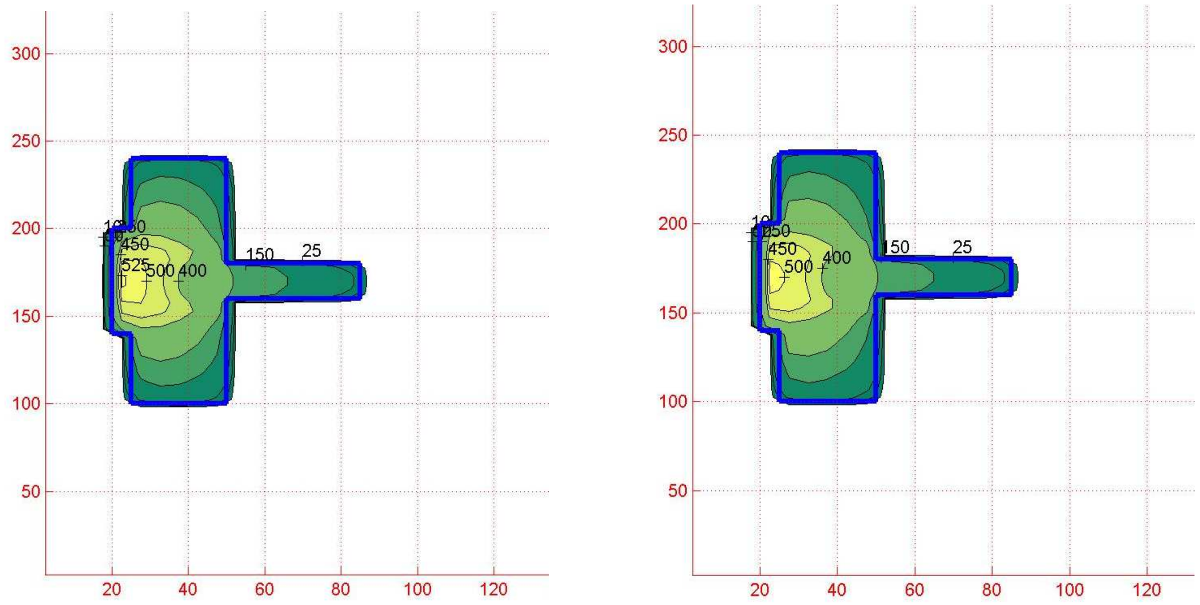
No.	Size analysed	Symbol (unit of measurement)	Step network, Δ = 5 mm	
			Cylindrical model	Cartesian model
1	The coordinates of the thermal node	(i, j)	(34.5) and (35.5)	(34.5) and (35.5)
2	Start of solidification in the thermal node	T start (s)	138.5	144.0
3	The time for the complete solidification or the thermal node (Complete solidification time of the hotspot)	T_fin. (s)	527.5	516.5
4	The temperature in the thermal node after 100 s	Tc100 (^{0}C)	1171.43	1169.84
5	The temperature in the thermal node after 600 s	Tc 600 (^{0}C)	1056.24	1048.50
6	Maximum mould temperature in the axis of the central core	Tfomax (^{0}C)	1095.32	1089.97
7	Volume of casting	V (cm^3)	1163.96	1162.0
8	Total simulation time	t (min)	6	240

4. Results

In Figures 4-5 are shown the solidification front displacement maps in the cross-section of the part and the isotherms map at the time of the solidification of the hot spot, for the two simulations. In Figures 6 - 7 are shown the temperature and solid fraction variation curves in the thermal node of the part, and Figure 8 shows the temperature variation curves in the centre of the inner core of the part) in the point located half-way the axis length. Figure 9 shows the distribution of temperature along the radius of the cast part – casting mould assembly, at time t = 500 seconds.

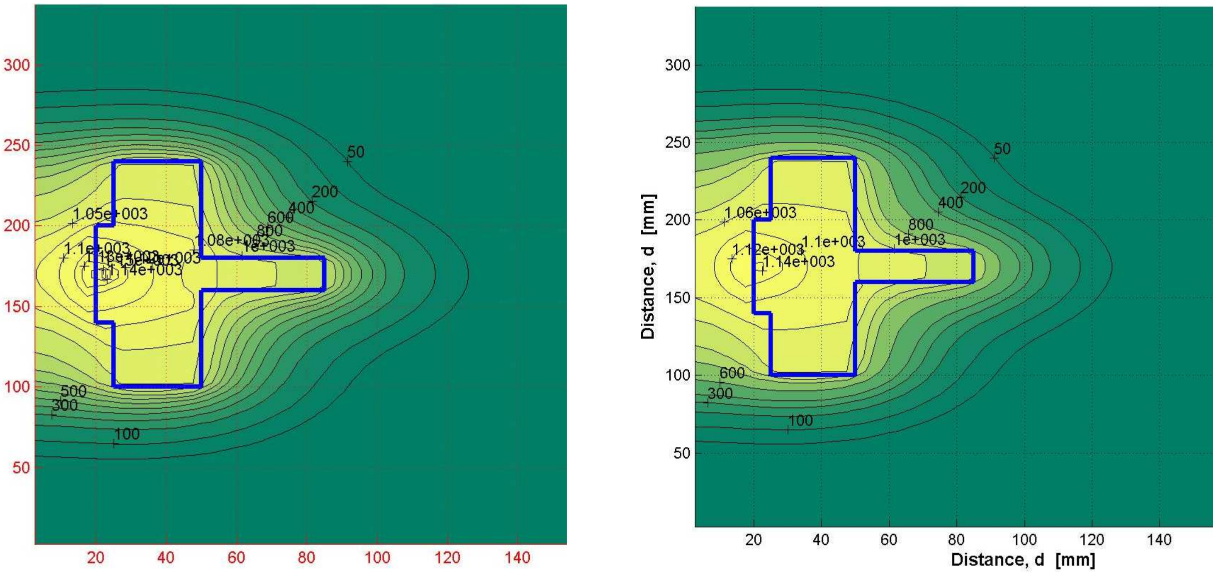
The analysis of the results led to the following observations:

- The results on the part solidification and temperature distribution in the mould - cast part system for the two types of simulation are very similar;
- The temperature in the thermal node at different times (t_1 = 100s, t_2 = 600s) is of the same order of magnitude, the differences being less than 1%;
- The maximum temperature, reached on the axis of the core in the two types of simulation, is also of the same order of magnitude, the differences being below 1.0%;
- In the case of simulation in cylindrical coordinates, the temperatures in various points of the system (hot spot, core axis) are greater by values of less than 10 °C order of magnitude;



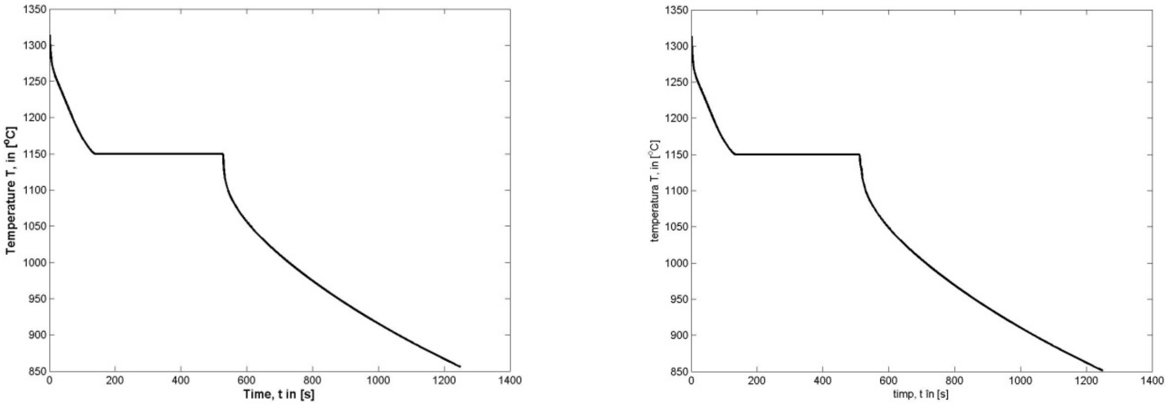
a) cylindrical coordinates Δ = 5 mm b) Cartesian coordinates Δ = 5 mm

Fig. 4. Displacement map of the solidification front in the part cross-section



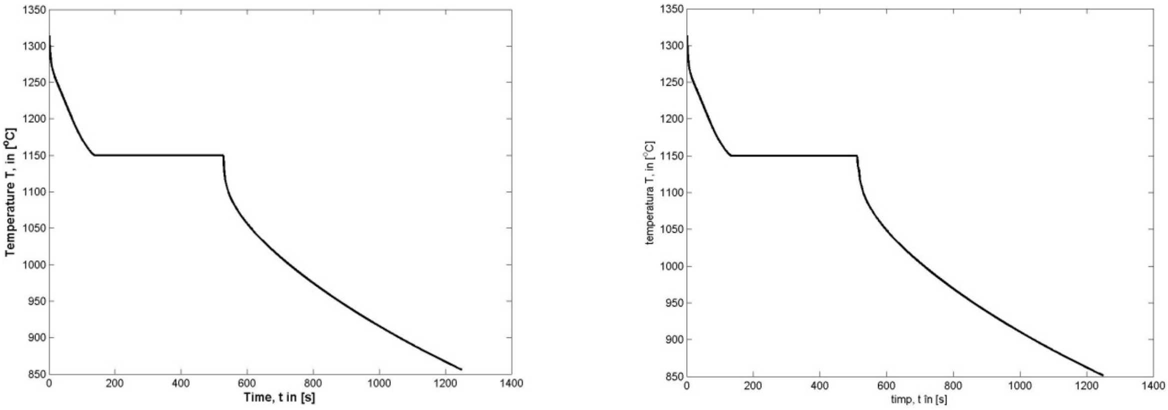
a) cylindrical coordinates Δ = 5 mm b) Cartesian coordinates Δ = 5 mm

Fig. 5. Hotspot map at the moment of solidification of the thermal node



a) cylindrical coordinates Δ = 5 mm b) Cartesian coordinates Δ = 5 mm

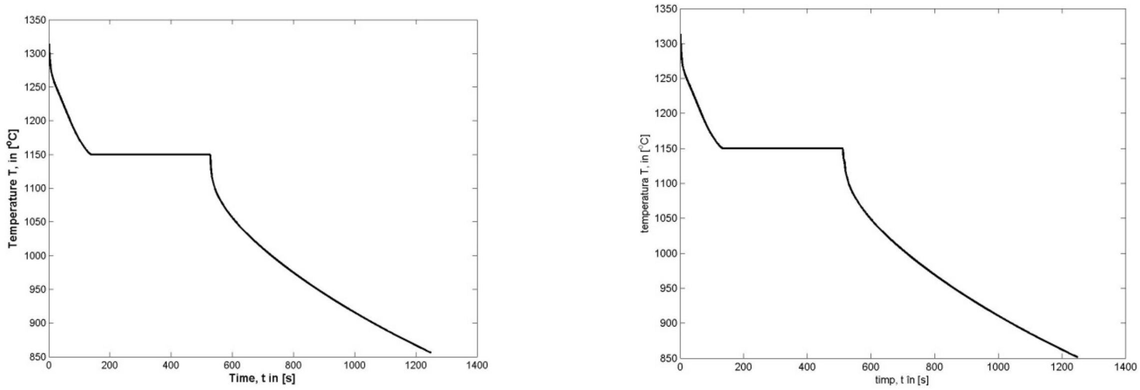
Fig. 6. Temperature evolution in the thermal node (from figure 4) of the part



a) cylindrical coordinates Δ = 5 mm b) Cartesian coordinates Δ = 5 mm

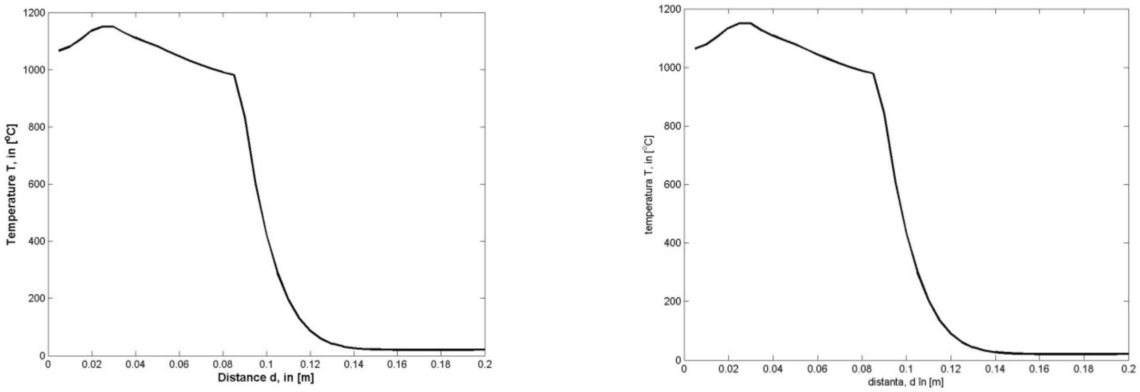
Fig. 7. The evolution of the solid fraction in the thermal node of the part (from figure 4)

- The curves illustrating temperature and solid fraction variation versus time have in various points a similar form, showing similar dynamics of heat transmission and solidification in the two simulated cases;



a) cylindrical coordinates Δ = 5 mm b) Cartesian coordinates Δ = 5 mm

Fig. 8. Temperature evolution on the axis of the mould at the middle of the length of the part in the thermal node (from figure 4)



a) cylindrical coordinates Δ = 5 mm b) Cartesian coordinates Δ = 5 mm

Fig. 9. Temperature distribution along the casting – mould radius at t = 500 seconds (Distance 170 mm from external bottom surface - figure 2)

- Figures 4 and 5 show that the maps that represent the displacement of the solidification front and the distribution of temperatures on the section of the part are also almost identical in the two types of simulation;
- In both simulations the hot spot coordinates are identical;
- The total solidification time of the part in the case of simulation in cylindrical coordinates is greater by a few seconds (11 seconds), representing about 2.2%;
- Differences between results are small and do not affect the information on the dynamics of casting solidification, which is important for the design of casting technologies;
- The duration of simulation in Cartesian coordinates in this case is approx. 40 times higher than in cylindrical coordinates.

This last observation shows the big advantage of the simulation in cylindrical coordinates relative to simulations in Cartesian coordinates. This advantage manifests itself in the special case of parts with big dimensions and complicated configurations.

In conclusion, the differences between the results from the two types of software are relatively small. The results show that in the case of simulation in cylindrical coordinates solidification is about 2.2% slower than at the simulation in Cartesian coordinates. These small differences can be explained firstly by the way in that the mathematical models underlying the two software applications consider that the heat transmission in the cast part – casting mould system takes place. Thus the mathematical model in cylindrical coordinates considers that the heat transmission along the radius is achieved by a convergent/divergent flow. On the other hand, the mathematical model in Cartesian coordinates takes into consideration that the transmission of heat between the cubic elements into that the assembly is divided (also along the radius) occurs by a parallel flow. In view of these aspects we consider the simulation in cylindrical coordinates as being closer to reality. A further partial explanation for the differences between the results obtained in the two cases is that upon the division in cylindrical coordinates of the part, its volume is greater by about 0.2%.

These small differences are explained through the imprecision of division of the circular perimeter of the part in the case of Cartesian coordinates. From table 2 it can be observed that in the case of division of the mould in Cartesian coordinates the volume of the part is a little smaller (approx. 0.2%). Simulations conducted with various mesh increments have revealed smaller differences between results for smaller division increments.

The final conclusion of this study is that the casting solidification simulation software in cylindrical coordinates ensures accurate results, compatible with the software in Cartesian coordinates, while being much faster and easier to use for the simulation of the solidification of parts with rotational symmetry.

References

1. Ciobanu, I., Munteanu, S.I., Crişan. A. (2004): *Model matematicşi soft 3D bazat pe metoda diferenţelor finite pentru simularea solidificării pieselor turnate din aliajeeutectice*. Metalurgia, ISSN 0461-9579, vol. 56, no. 12, p. 17-24 (in Romanian)
2. Ciobanu, I., Monescu, V., Munteanu, S.I., Crişan, A. (2010): *Simularea 3D a solidificăriipieselorturnate*. EdituraUniversităţiiTransilvania din Braşov, ISBN 978-973-598-678-0, Braşov, Romania (in Romanian)
3. Ionescu, I., Ionescu, Daniela, Ciobanu, I., Jiman, V. (2012): *Mathematical Modeling of eutectic alloy cylindrical castings solidification*. Metalurgia, ISSN 0461-9579, vol. 64, no. 8, p. 10- 20
4. Ionescu, I., Ionescu, Daniela, Ciobanu, I., Jiman, V. (2012): *Computation of the heat Exchange Coefficient in Cylindrical Coordinates Mathematical Modeling of Castings Solidification*. RECENT, ISSN 1582-0246, vol. 13, no. 3(36), p. 307-316
5. Soporan, V., Constantinescu, V. (1995): *Modelarea la nivel macrostructural a solidificăriialiajelor*. Editura Dacia, ISBN 973-35-0526-9, Cluj-Napoca, Romania (in Romanian)
6. Ştefănescu, D. (2009): *Science and Engineering of Casting Solidification*. Department of Material Engineering, University of Alabama, Tuscalosa (USA), Springer US, ISBN 0-306-46750-X, ISBN 978-0-387-74609-8
7. Soporan, V. (1994): *Stadiul actual al modelării prin metode numerice a procesului de solidificare a pieselor turnate*. Metalurgia, ISSN 0461-9579, vol. 46, no. 9, p. 20-23
8. Kurz, W., Fisher, D.J. (1986): *Fundamentals of solidification*. Crystal Research and Technology, ISSN 1176-1176, vol. 21, no. 9, DOI: 10.1002/crat.2170210909, Trans Tech Publication, ISBN 0-87849-523-3
9. Ştefănescu, D. (2001): *Science and Engineering of Casting Solidification*. Departament of Material Engineering, University of Alabama, Tuscalosa, Kluwer Academic/Plenum Publishers, ISBN 0-306-46750-X

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