Calculation of Desired Coefficients for the Characteristic Equation of Closed Automatic Control System

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Abstract
The localization method has been developed to control linear and nonlinear objects. It is widely used, because it has adaptive properties, which contribute to its widespread use. One of the essential steps of this method consists in setting of the desired equation of the closed system. This equation determines the dynamic error of the system, i.e. the deviation of its output from the prescribed value during the transient process. The order of the system and the controller together define the desired order of this equation. The coefficients should be set based on the requirements for dynamic errors, which most often are typical. Therefore, it is logical to expect that the coefficients of the desired equations are also typical, but they are not. Each researcher uses his method of selecting these coefficients. The reasons for these methods are not always sufficiently justified. Development of science-based and, therefore, a reliable way of solving this problem would solve it once for the all times, so that not resolve it further. It would have saved the researchers of excess labor for this choice and would enhance the quality of the result, if the choice would be optimal. This paper offers not only the methodology of the choice with numerical optimization of the corresponding filter coefficients based on adequate criterion, but also gives the results of this procedure for the most frequent cases of the characteristic equations corresponding to the order from the second to the sixth. If necessary, this technique can be extended to higher order. Numerical simulations confirm the results.

Keywords
control, regulation, dynamic accuracy, static accuracy, characteristic polynomial of a system, polynomial of desired dynamics

1. Introduction

The design of automatic control systems demands agreements of the result to certain static and dynamic requirements to their behaviour in a locked state. The static requirements are allowable amount of residual static error. Dynamic requirements are set mostly in the form of allowable overshoot the permissible duration of the transient process, and so on. However, in some cases, these requirements are given in the form of desired transient processes equation. A polynomial equal to zero formally describes this desired equation. It takes place in the denominator of the equivalent low-pass filter having the same transient process (response) as the locked system. Therefore, the requirement also can have the set of the demands to dynamic properties of a closed system in the form of the desired polynomial (the denominator of the transfer function of the filter) or a desired characteristic equation (the equation in which the desired polynomial is equal to zero). In addition, some techniques, such as the localization method [1, 2], use the characteristic polynomial of the desired type, i.e. without specifying of the desired type of the polynomial such techniques cannot be used at all.

With all this case, when desired characteristic polynomial is given in the form of a set of roots of the polynomial are rare. In more cases the area is given in which these roots should be located. In some cases, this polynomial is given by the choice of its coefficients. In this case, coefficients of Newton binomial are the most commonly used, that is actually, this polynomial has all the same negative real roots. The reasons for such a choice is not sufficient, since with such a choice it is only reliably known that the system with such characteristic polynomial is stable, but it is not the best one. It is obvious that system having such polynomial has not good performance. Scientifically based choice of the characteristic polynomial is not found in the literature, with the exception of few publications, such as [3].

The paper [3] gives base to approach to solving of this problem. However, the result of solving this problem in this publication may be revised based on the latest achievements in the field of simulation of closed dynamic systems, including using VisSim software used in this publication.

This paper critically discusses the previous results, it is proposed updated and more effective method for calculating of the desired polynomial coefficients, from which it is obvious that the choice of the coefficients from Newton binomial or through a region of the roots location is much worse than the proposed method.

The solution of this problem for polynomials of the third to fifth order has the greatest practical importance. For first-order polynomials, this problem is obvious, since there is no choice in fact. For the second-order polynomial, the problem is to choose a single coefficient, which can be done by many methods with approximately the same result. However, with increasing order, this task is significantly more difficult. With all this, for too high order of the polynomial, this task loses practical significance, since in practice it is always sufficient to limit the order of not more than fourth (sometimes fifth). However, the proposed technique without any problems can be spread on a large enough order (sixth and more).

2. Locked System and Its Transfer Function

Linear locked dynamic automatic control system is described, as a rule, with a model, which coincides with the model of the low pass filter (LPF). If the system is prone to oscillations but stable, the equivalent filter in the denominator contains the corresponding polynomials with complex conjugate roots, the real part of which is negative. If all the roots are real and negative, then the system is stable. However, if the system is stable, it does not have to contain only real negative roots because complex conjugate roots with negative real part can also be included in the set of roots of its characteristic polynomial.

If the system is not only stable, but also astatic, the static gain of this filter is equal to one. Since only such systems usually are desired, hence in the desired characteristic polynomial in any case zero power term should be set equal to unit.

In general, the transfer function of low pass filter of order N can be described by the relation:

$$W(s) = \frac{1}{P_n(s)} \tag{1}$$

Here s is Laplace transform argument, and $P_n(s)$ is polynomial of the form [3]:

$$P_n(s) = 1 + a_1 s + a_2 s^2 + \cdots + s^n \tag{2}$$

Here, the coefficient of the term with the order power (2) should be put equal to unit for definite task, as in [3], as it is always possible to ensure with the appropriate time and frequency scales. The response of the system transfer function of the form (1) can be calculated with simulation, for example, with the use of the program VisSim, which also allows plotting of the relevant graphics.

3. Task of Choosing Characteristic Polynomial

From (2) it is seen that the first and last coefficients of the polynomial are specified as unit ones, that is the task of choosing these coefficients is absent. Consequently, for polynomials of zero-order and first-order the discussed problem cannot be set. For a second order polynomial, the task is in choosing of the only coefficient. In general, for a polynomial of order N it is necessary to choose $N - 1$ positive coefficients. Zero and negative coefficients may be excluded from consideration in advance based on relation of the polynomial coefficients with the roots. At the procedure of automatic calculation of the coefficients, zero or negative coefficients should not be in the result of it.

Along with zero static error, which is ensured by the strict equality of the free term coefficient of the polynomial (2) to unit, the system must meet the following requirements:

1. It is unacceptable significant inverse overshoot, i.e. the initial developing of the process in the wrong direction.
2. The significant overshoot is very undesirable.
3. Multiple oscillations about a steady level are undesirable in the transient process.
4. Bad is not the monotony of the transition process, which is sharp and multiple changes of its direction and speed, even if the overall development trends is in proper direction.

With so many requirements, which list may be even extended, the task of choosing coefficients of the polynomial becomes difficult and has ambiguous results.

Paper [3] uses the following approach:

1. The software simulates low-pass filter based on the relevant order model with given coefficients.
2. The software calculates the transient processes according changing of the control system error $e(t)$ in the time which is the difference between the unit prescribed value and the response at the output of this filter.
3. The software calculates some cost function, depending on the error, for example, the integral of the squared error multiplied by the time in a whole degree of M:

$$F_C(T, M) = \int_0^T e^2(t) t^M dt \tag{3}$$

4. Since there is uncertainty in the choice of the degree M, the software perfumes solution of this problem for different values of integer M, from one to seven, and higher, if necessary.
5. The researcher analyzes the results by simple comparison of the graphs of transient processes (visual analysis). On this basis, he made the conclusion on the best value of M. Thus, the choice is made for technique for solving of the problem of finding the coefficients of the polynomial, which is confirmed by simulation for polynomials of the order from the third to the fifth.
6. The paper [3] gives the values of these coefficients of polynomials. It designed and built graphics corresponding transients.

Results are summarized in Tables 1-4. In the bottom row, the coefficients found without the use of a criterion, but with subjective way, are given.

Table 1. Results of the calculation of the coefficients for the second-order polynomial

Value M	a_1	Label
2	1.34	A
4	1.53	B
6	1.66	C
Subjective choice (SC)	1.66	D

Table 2. Results of the calculation of the coefficients for the third-order polynomial

Value M	a_1	a_2	Label
2	1.455	2.04	A
4	1.85	2.185	B
6	1.96	2.185	C
SC	2.3	2.1	D

Table 3. Results of the calculation of the coefficients for the forth-order polynomial

Value M	a_1	a_2	a_3	Label
2	2.4	3.1	1.7	A
4	2.3	3.1	1.73	B
6	2.3	3.1	1.85	C
SC	2.6	3.4	1.95	D

Table 4. Results of the calculation of the coefficients for the fifth-order polynomial

Value M	a_1	a_2	a_3	a_4	Label
2	2.8	5	4.2	2.75	A
4	2.93	4.79	4.275	2.65	B
6	3.0	4.6	4.35	2.38	C
SC	3.4	5.3	5	2.8	D

4. Critics of the Results Known

Paper [3] did not paid sufficient attention to the reverse overshoot. The simulation results are reflected on the charts, selecting fixed boundaries axes. The lower boundary of the ordinate was taken zero.

We carry out a detailed analysis of the transients in these filters, for example by VisSim simulation program. Figure 1 shows the transients in the filters with characteristic polynomials are taken from Table 2. Various graphics are marked with the labels from the last column of this table. It can be seen that the graphs labelled as A, B, C and D are characterized by a reverse overshoot of about 15%. For comparison, the graph marked with the letter E is given, in which the reverse overshoot does not exceed 5%. From this, it can be concluded that all of the above discussed results, with reference to Table 2, are characterized by a sufficiently large reverse overshoot. This value is approximately the same, it was not controlled. That is, this value cannot be reduced with the criterion of the form (3).

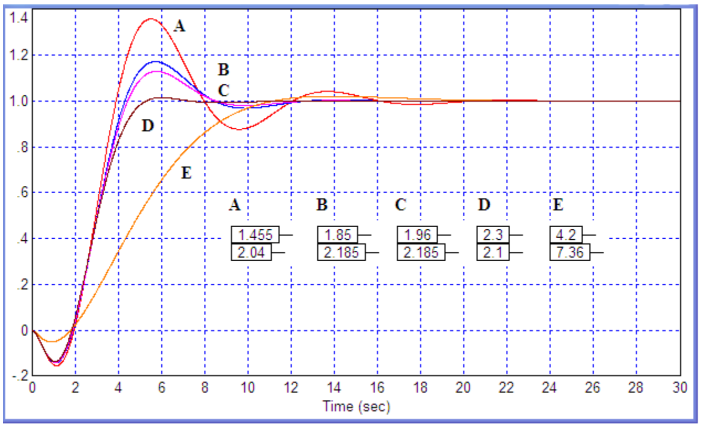

Fig. 1. Transients in filters with different characteristic polynomials coefficients from Table 2

Figure 2 shows similar processes in systems with filters, having in the denominator of the transfer function characteristic polynomials with coefficients from Table 3. Also, the graphics with labels A, B, C and D, are characterized by a reverse overshoot of about 10%. For comparison, the graph labeled with the letter E, almost have no reverse overshoot.

This implies that the considered optimization criteria are not satisfactory to solve the problem and factors presented in Tables 1-4, need to be clarified.

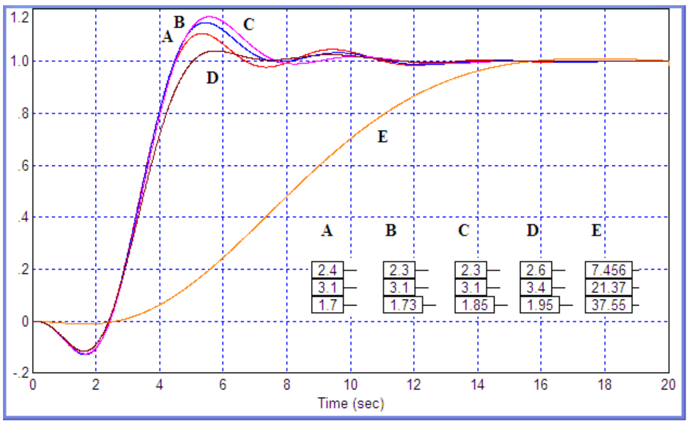

Fig. 2. Transients in filters with different characteristic polynomials from Table 3

5. Problem Solution Proposed

Taking into account the development of closed systems optimization techniques, published in a series of papers [4, 5, 6], we can recommend the following three cost function (4) – (6) and with all this function used in (6) is defined by (7):

$$F_{C1}(T) = \int_0^T |e(t)|dt \quad (4)$$

$$F_{C2}(T) = \int_0^T |e(t)|tdt \quad (5)$$

$$F_{C3}(T) = \int_0^T \{|e(t)t| + f[k, e(t)]\}\, dt \quad (6)$$

$$f[k, e(t)] = k \max\left\{0, \frac{e(t).de(t)}{dt}\right\} \quad (7)$$

We propose to simulate the filter system by means of the open model. Previously, this method was not applied using program VisSim because such modelling did not give a complete adequacy of the resulting transient processes compared these using model in the form of the transfer function. However, as it was found out that adequacy can be easily achieved in the choice of adaptive Bulirsh-Stoer method [4] as a method of integrating, the use of the open model become possible.

5.1. Fourth-order polynomials

Figure 3 shows the structure for modelling and optimization of the filter. In this structure criterion (6), (7) are employed. Simple modifications of this structure can convert this calculator of the cost function into calculator of the new cost function according (5) or (4). The resulting transient process in such a system is shown in figure 4.

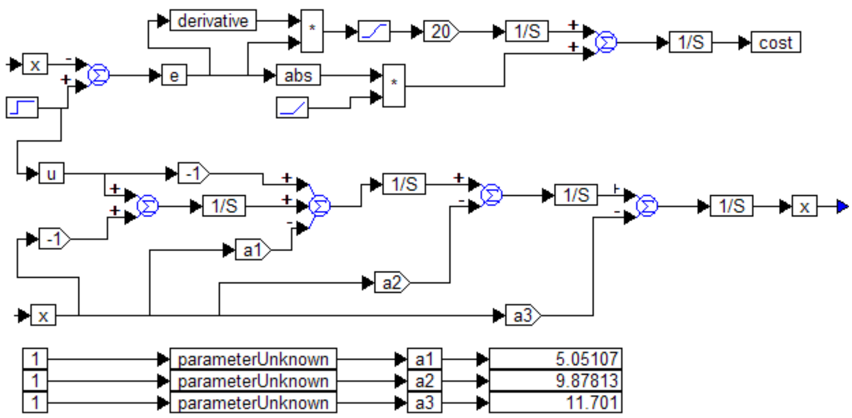

Fig. 3. Structure for optimizing of the fourth-order filter according the cost function (6), (7)

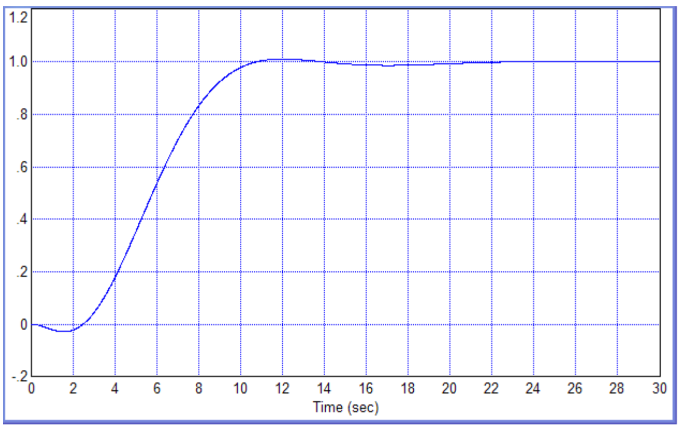

Fig. 4. Resulting transient process in the structure of figure 3 with the weight coefficient k = 20

It is seen from the process shown in figure 4 that the polynomial could be obtained through the optimization with the choice of the criteria (6), (7), which corresponds to the process with little or without any reverse overshoot. This requirement of the absence of reverse overshoot is taken into account by including into the cost function under the integral of the term, which grows if there are areas in which the error and its derivative has the same sign, i.e. the error increases in magnitude. The weight of this term is given with weighting factor. If we put it equal to zero (i.e. the upper branch of the upper adder is unlocked), we obtain the cost function in the form (5). Optimization of the polynomial coefficients in this case gives the values that are shown in figure 5, which also shows the corresponding plot and transient process.

For comparison, figure 6 shows a similar result for the cost function (4). Externally, these results do not differ significantly, even though in the cost function (5) decreasing rate of the error module is essential, and in the cost function (4) the accumulated integral of the error modulus is significant, regardless of at what time this error occurs. Figure 7 shows the results of research results depending on the factor k in (7).

It can be seen that the choice of a large factor can completely eliminate both types of overshoot (upstream and reverse). With a small factor, these overshoots take place, but they can be made irrelevant, which can increase the speed of the transient process. Accordingly, the choice of intermediate values of this factor can achieve the desired compromise between the demands of high performance and small overshoot.

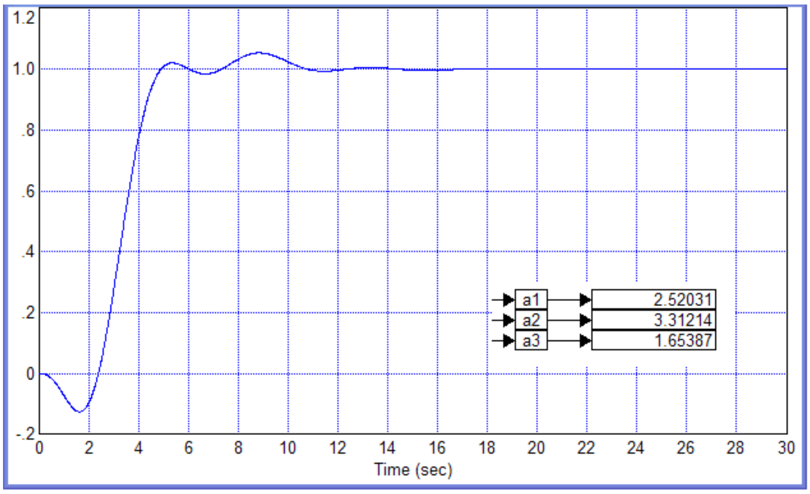

Fig. 5. Resulting transient process in the structure of figure 3 with the weight coefficient $k = 0$, i.e. with cost function (5)

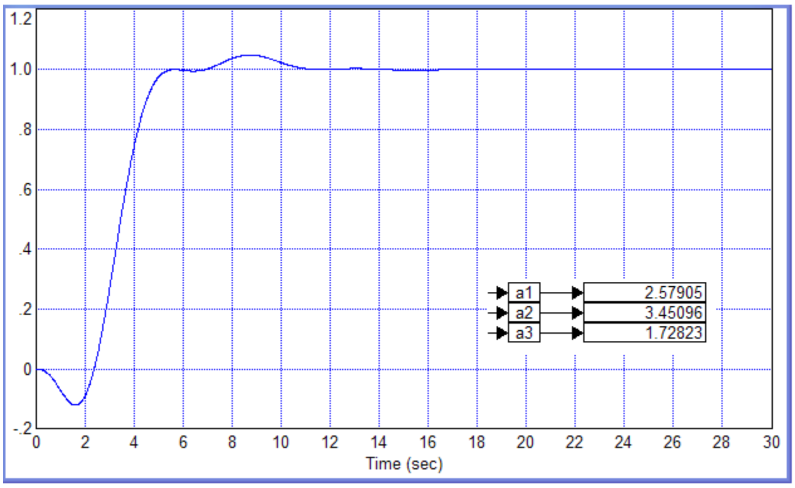

Fig. 6. Resulting transient process in the structure of figure 3 according the cost function (4)

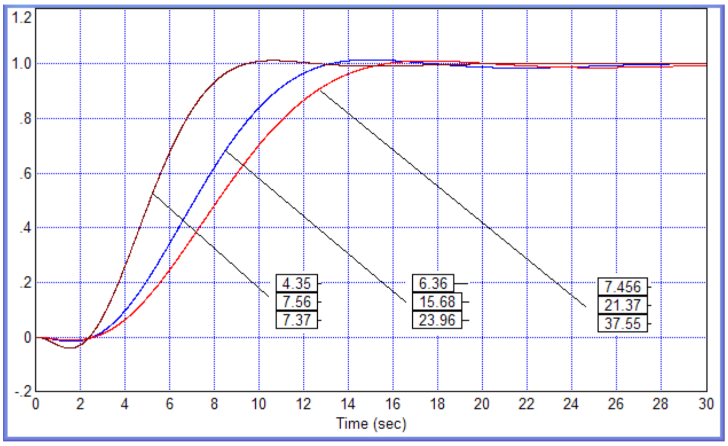

Fig. 7. Resulting transient process in the structure of figure 3 with a weight factor k = 10 (black line), k = 20 (blue line) and k = 100 (red line)

5.2. The third-order polynomials

The task for the third-order polynomial is solved by a simplified of the structure of figure 4. It should be deleted extra coefficient, extra integrator and an extra adder from it, and then restore the integrity of the loop. It is necessary also to remove the excess block "parameter unknown". The resulting structure is shown in figure 8. The results of optimization with the cost function (6), (7) using this structure are shown in figure 9. On this basis, we can make similar conclusions that the choice of weight factor can provide a compromise between the desired speed and a small overshoot.

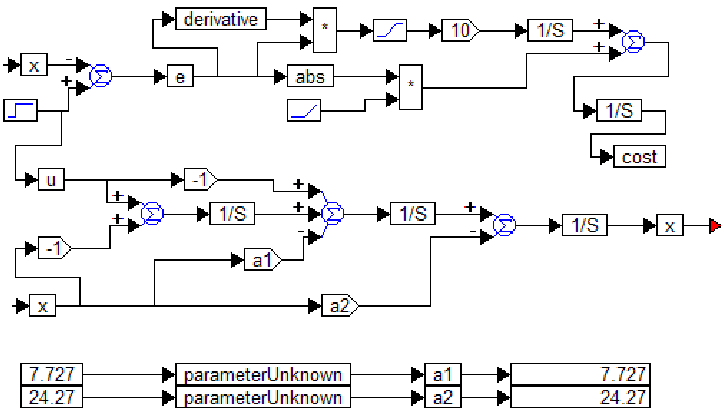

Fig. 8. Structure for optimization of the forth-order filter according the cost function (6), (7)

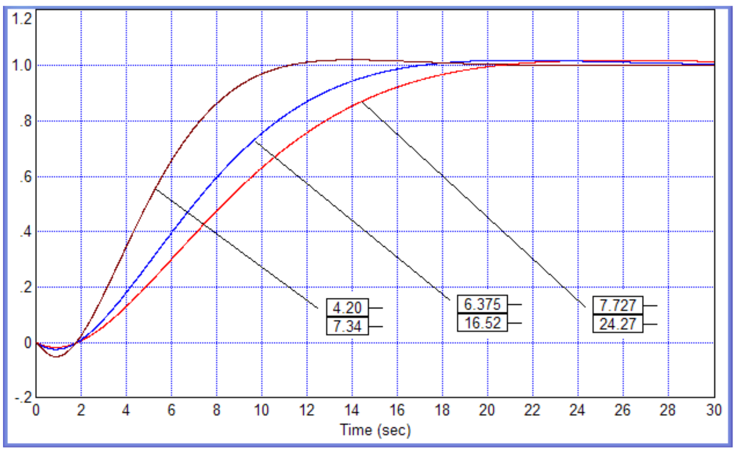

Fig. 9. Resulting transient process in the structure of figure 8 with a weight factor k = 10 (black line), k = 20 (blue line) and k = 100 (red line)

Figure 10 shows the results of optimization with the cost function (4) and (5). It is clear that the results do not differ significantly: in both cases, the reverse overshoot reaches about 15%.

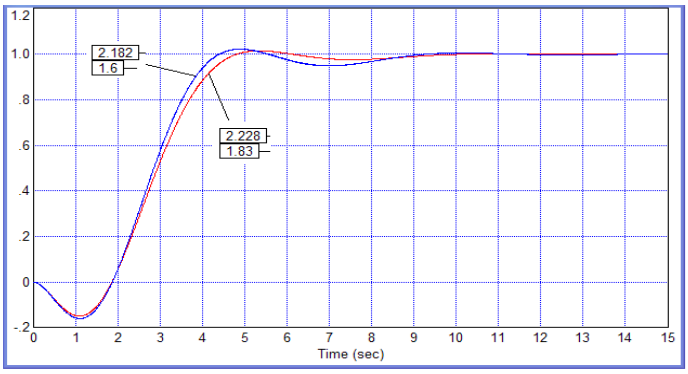

Fig. 10. Resulting transient process in the structure of figure 8 according the cost function (4) (blue line) and the cost function (5) (red line)

5.3. The fifth-order polynomials

The increasing of complexity of the structure of figure 4 solves the task for the fifth-order polynomials. It is necessary to include into it additional factor a_4, an additional integrator and an additional adder, as well as additional block for the optimization of the new unknown parameter, i.e. block "parameter unknown". The resulting structure is shown in figure 11. The results of optimization under the criterion (6), (7) using this structure are shown in figure 12. Results with coefficients k = 10 and k = 50 differs slightly from each other, the according graphics merge together, so additional modelling with factor k = 75 was accomplished.

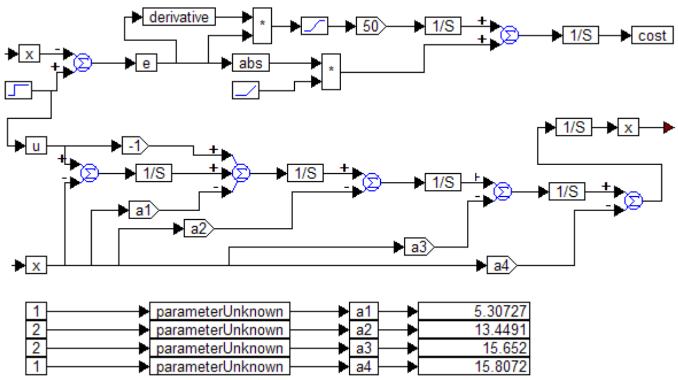

Fig. 11. Structure for optimization of the fifth-order filter according the cost function (6), (7)

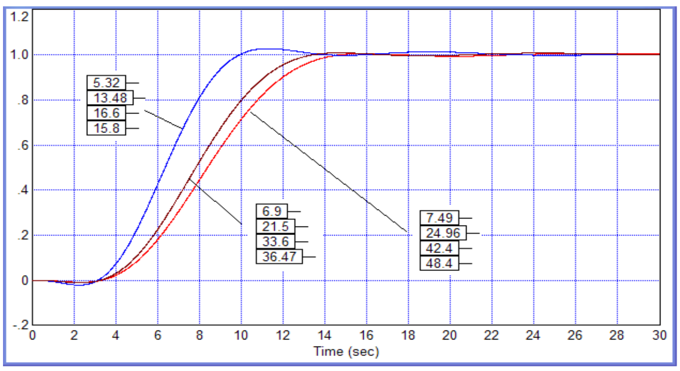

Fig. 12. Resulting transient process in the structure of figure 11, with a weight factor k = 10 (blue line), k = 75 (black line) and k = 100 (red line); at weight factors 10 and 50 the result are almost the same (lines are merged)

On this basis, we can make similar conclusions that the choice of weight factor can provide a compromise between the desired speed and a small overshoot. Figure 13 shows the results of optimization with the cost function (4) and (5).

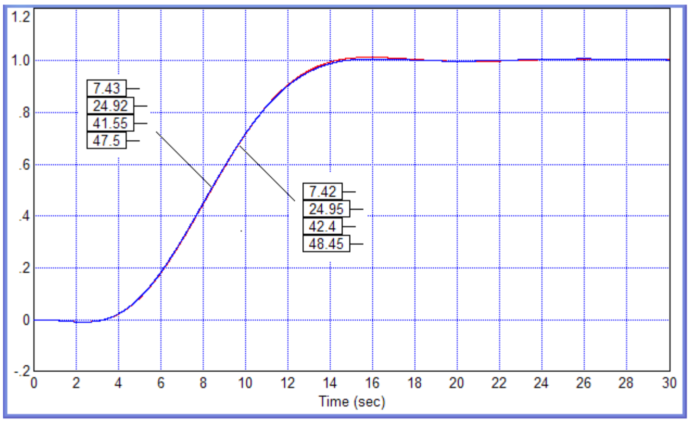

Fig. 13. Resulting transient process in the structure of figure 11 with the cost function (4) and (5) (lines are merged)

These results are indistinguishable from each other. The coefficients are different in the fourth digit and graphics merge. The inverse overshoot is absent, and the coefficients and graphs are similar to these obtained with k = 100.

6. Summary Table of Results

The research results can be summarized in a table similar to that summarized the results obtained in [3]. These results are shown in Tables 5-8. Similar tables can be calculated to create a higher order polynomial (sixth and above), if necessary.

Table 5. Results of the calculation of the coefficients for the polynomials of the third order

Value k	a_1	a_2	Label
10	4.2	7.34	E
50	6.375	16.52	F
100	7.727	24.27	G
0	2.23	1.83	H

Table 6. Results of the calculation of the coefficients for the polynomials of the forth order

Value k	a_1	a_2	a_3	Label
10	4.35	7.56	7.37	E
20	5.05	9.87	11.7	J
50	6.36	15.68	23.96	F
100	7.456	21.37	37.55	G
0	2.52	3.32	1.65	H

Table 7. Results of the calculation of the coefficients for the polynomials of the fifth order

Value k	a_1	a_2	a_3	a_4	Label
10	5.32	13.48	16.6	15.8	E
75	6.9	21.5	33.6	36.47	F
100	7.49	24.96	42.2	48.4	G
0	7.42	24.95	42.4	48.4	H

These polynomials correspond more closely to the definition of “the desired characteristic polynomial of the closed dynamic system”. Transient processes in the systems with such characteristic polynomials are characterized by negligible overshoot or no overshoot at all. This is especially important with regard to the inverse overshoot, to which the paper [3] gives no attention. From prescribed polynomial coefficients, the location of the roots of polynomials in the complex plane can be easily found, if required. It is enough to take into account that in all cases $a_0 = 1$, $a_n = 1$. The calculation of roots of a polynomial from its coefficients in the case of $n = 2$ can be done in known relation, and for the higher order polynomial it can be done by, for example, with MATLAB program. As a result, it is possible to evaluate graphically the desired position of the roots and to build a region of their location. However, to solve the problems of automatic control is not required, because the known methods are based just on the prescribing of the desired characteristic polynomial by means of its coefficients.

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