The Influence of Speed on the Wall Thickness of Centrifugally Cast Parts

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Abstract
The paper discusses a study on the vertical centrifugal casting of tubular parts. The necessary speed for the casting of cylindrical parts by this procedure is determined starting from the equation of the interior surface of the liquid alloy during rotation. The study of the influence of speed and part dimensions on the wall thickness of the casting is followed by a comparison of the theoretical and experimental results. The conclusions concern the determination of the optimum speed for a given part.

Keywords
casting, centrifugal casting, leaded aluminium-silicon alloy

1. Introduction

The interest in centrifugal casting has known a steady increase due to its benefits [1, 2, 3, 4, 8]:

-superior mechanical properties of the castings;
- the possibility of obtaining castings of gradient structure and properties;
- high utilisation efficiency of the cast liquid alloy;
- simple and cost efficient installations;
- cost efficient castings.

The interest for this procedure is materialised in the research projects, scientific articles and patents developed in this field [5, 6, 7, 9, 10].

Lately scientific focus has shifted to vertical axis centrifugal casting [2, 10] as it allows:

-the casting of parts lacking a rotational symmetry axis;
- the improved obtaining of parts with gradient chemical composition, structure and properties by means of large rotation radii.

2. Aim of the Paper

In centrifugal casting, the rotational speed of the mould is an important technological parameter. It influences the geometry, compactness, chemical composition and mechanical properties of the alloy in the wall of the castings. In addition, speed can be adjusted easily in practice. Modern centrifugal casting installations allow the modification of this parameter as needed. In the case of centrifugally cast tubular parts, the speed influences the geometry of the interior surface of the liquid alloy. Thus, it influences casting wall thickness. In horizontal axis centrifugal casting at a speed higher the critical, this influence is lesser as the alloy passes at high frequency through the bottom as well as through the top point of the mould. This sequence of positions yields a heat transfer along the radius practically uniform along the entire contour of the casting. Consequently, the solidification rate is uniform along the contour of the casting and a tubular part of uniform wall thickness is obtained. For this reason in centrifugal casting by means of horizontal axis installations there is but a reduced tendency of obtaining parts with unequal wall thickness. The condition for centrifugally casting parts with uniform wall thickness on horizontal axis installations is for the rotational speed to be higher that the critical one. In this case, the critical speed is calculated by equating the centrifugal force acting upon a particle of liquid at the top of the casting's interior surface with the weight of that particle. Thus, the equation for the critical angular speed is obtained:

$$\omega_{CRT} = (g/R_1)^{1/2} \qquad [rad/s] \qquad (1)$$

The critical rotational speed results as:

$$n_{CRT} = 30\ \omega/\pi = (30/\ \pi)\cdot(g/R_1)^{1/2} \quad [\text{rpm}] \tag{2}$$

In centrifugal casting by means of vertical axis installations, the influence of speed on casting wall thickness is real, because the position of the liquid particles within the part does not change during rotation.

Figure 1 shows the schematic of the free surface of a liquid in the case of rotation about a vertical axis. In the case of a non-rotating mould, the free surface of the liquid metal would be horizontal, at level "*h*". This level depends on the volume of liquid alloy (*Vo*) cast into the mould and is given by equation (3):

$$h = Vo/(\pi\cdot R^2) \tag{3}$$

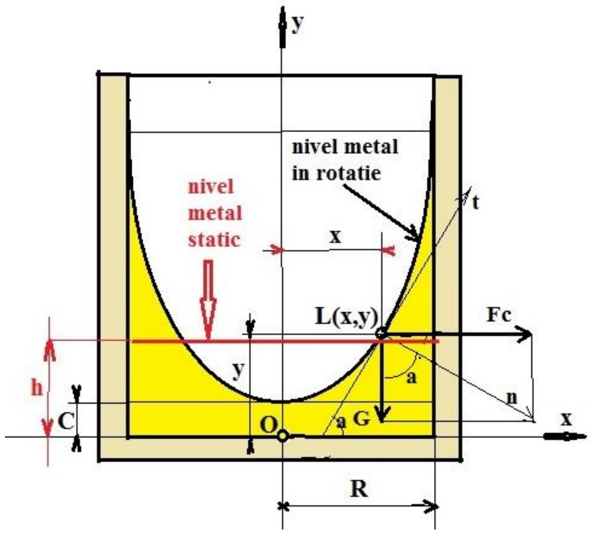

Fig. 1. Free surface of the liquid metal in the case of rotation about a vertical axis

During rotation, the free surface of the liquid becomes curved. In stationary operation (when the entire alloy has the same angular speed ω) the free surface of the liquid alloy has a stationary geometry.

During rotation upon a liquid particle of mass "*m*" located in point *L*(*x*,*y*) on the free surface of the liquid act the centrifugal force *Fc* and its weight *G* (Figure 1). The values of these forces are given by equations (4) and (5):

$$Fc = m\cdot\omega^2\cdot x \tag{4}$$

$$G = m\cdot g \tag{5}$$

The resultant of the two forces acting upon a particle at the surface is normal to that surface. This yields the condition that the projections of the two forces on the direction of the tangent to the surface have to be equal as modulus and of opposing directions. The direction of the tangent is denoted by *Lt* (Figure 1). For a random point "*L*(*x*,*y*)" located on the free surface of the liquid (Figure 1) the moduli of the projections(*Gt* and *Fct*) of the two forces (*G* and *Fc*) on the direction of the tangent to the surface are given by equations (6) and (7):

$$Gt = m\cdot g\cdot \sin\alpha \tag{6}$$

$$Fct = m\cdot x\cdot\omega^2\cdot\cos\alpha \tag{7}$$

where α is the angle of the tangent in relation to axis 0x. The condition for the resultant of these two forces to be null is:

$$m\cdot g\cdot\sin\alpha = m\cdot x\cdot\omega^2\cdot\cos\alpha \tag{8}$$

The gradient of the tangent to the surface contour, or the derivative of the function that corresponds to the surface contour is:

$$\text{tg}\alpha = \text{d}y/\text{d}x = (\omega^2/g)\cdot x \tag{9}$$

The differential equation of the free surface is obtained, and by integration its solution.

$$dy = (\omega^2/g) \cdot x \cdot dx \quad (10)$$

$$y = (\omega^2/2g) \cdot x^2 + C \quad (11)$$

The solution (11) of the differential equation is a parabola with symmetry axis Oy. The integration constant "C" is the ordinate of the vertex of this parabola. The value of this constant can be determined from the equality of the volume *Vo* with the volume of the rotating liquid. In previous papers dealing with centrifugal casting the demonstration stops at equation (11), without revealing the use of this equation for determining the technological parameters necessary for obtaining a casting with a correct geometry.

This paper undertakes to clarify these aspects and discusses the utilisation of equation (11) in the design of a vertical centrifugal casting technology of tubular parts. Further studied is the influence of speed on the geometry of the free surface and on the wall thickness in dependence on the dimensions of the casting in the case of this casting procedure.

3. Work Method

Figure 2 shows the schematic of vertical axis centrifugal casting of a real tubular part. The part is characterised by exterior radius R, length L, and wall thickness (at the top) s_2. Because of the parabolic shape of the interior surface, the wall is thicker at the bottom of the casting $s_1 > s_2$. The following relationships can be formulated (Figure 2):

Interior radius at the bottom:

$$R_1 = x_1 = R - s_1 \quad (12)$$

Interior radius at the top:

$$R_2 = x_2 = R - s_2 \quad (13)$$

Mean wall thickness:

$$s_{\text{med}} = (s_1 + s_2)/2 \quad (14)$$

Mean interior radius:

$$R_{\text{med}} = (R_2 + R_1)/2 \quad (15)$$

Difference of wall thickness of the centrifugally cast part:

$$\Delta s = (R_2 - R_1) = x_2 - x_1 = s_1 - s_2 \quad (16)$$

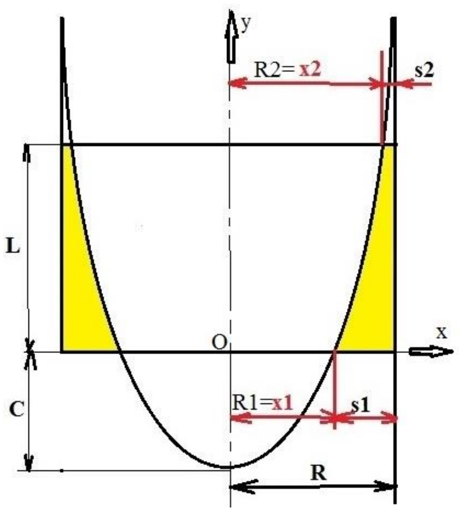

Fig. 2. The interior surface of part cast centrifugally by means of a vertical axis installation (L = length of the part, s_2 = minimum wall thickness)

Important for the machining of the par id the wall thickness at the top (s_2). In this point, the wall thickness is at its smallest, hence the tooling allowance at the interior is also the smallest. In order for the part to be machined, the minimum required tooling allowance has to be ensured. For this reason, the dimensions at the top need be ensured, namely the minimum wall thickness $s_2$and the interior radius at the top R_2. As the casting is thicker at its bottom, the tooling allowance in this area will be greater. Reducing wall thickness at the top under the value of$s_2$could compromise the mechanical machinability of the part.

For this reason, the integration constant in equation (11) has to be determined from the condition of ensuring wall thickness s_2 and radius R_2. This leads to the condition that the parabola of the free surface passes through the point of coordinates ($x_2 = R_2$; $y = L$). The value of the integration constant is obtained by replacing these coordinates in equation (11):

$$C = L - (R_2 \cdot \omega)^2/(2g) \tag{17}$$

The equation of the free surface at the casting of the part becomes:

$$y = (x^2 - R_2^2) \cdot (\omega^2/2g) + L \tag{18}$$

In this equation, the interior radius (R_1) and the wall thickness (s_1) at the bottom of the part are determined for $y = 0$:

$$0 = (R_1^2 - R_2^2) \cdot (\omega^2/2g) + L \tag{19}$$

$$R_1^2 = R_2^2 - 2g \cdot L/\omega^2 \tag{20}$$

The angular speed and rotation speed required for obtaining the desired wall thickness at the bottom of the casting $s_1 = s_2 + \Delta s$ can be determined from this equation:

$$\omega = \sqrt{\frac{2gL}{R_2^2 - R_1^2}} = \sqrt{\frac{gL}{\Delta s \cdot R_{med}}} \tag{21}$$

$$n = \frac{30}{\pi} \cdot \sqrt{\frac{2gL}{R_2^2 - R_1^2}} = \frac{30}{\pi} \cdot \sqrt{\frac{gL}{\Delta s \cdot R_{med}}} \tag{22}$$

The volume of the alloy cast into the mould has to be dosed very accurately in order to obtain a part satisfying the above dimensions (L, R, R_1 R_2).This needs to be equal to the volume of the casting denoted Vp. The volume of the casting is calculated from the schematic in figure 2:

$$Vp = V_1 + V_{\text{int}} = \pi \cdot (R^2 - R_2^2) \cdot L + V_{\text{int}} \tag{23}$$

where V_1 is the volume corresponding to the ring-shaped section of radii R and R_2, and V_{int} is the volume of the liquid alloy beneath the concave interior surface of the casting between R_1 and R_2. This volume is calculated by integration by the methodology shown below.

$$V_{int} = \int_{R_1}^{R_2} \left[\frac{\omega^2}{2g} \cdot (x^2 - R_2^2) + L\right] \cdot 2\pi \cdot x \cdot dx = 2\pi \cdot \left[\frac{\omega^2}{2g} \cdot \left(x^2 - R_2^2 \cdot \frac{x^2}{2}\right) + L \cdot \frac{x^2}{2}\right]\Bigg|_{R_1}^{R_2} \tag{24}$$

Eventually there results:

$$V_{int} = \left[\frac{\pi\omega^2}{4g} \cdot (x^4 - 2R_2^2 \cdot x^2) + \pi L \cdot x^2\right]\Bigg|_{R_1}^{R_2} \tag{25}$$

$$V_{int} = \pi L \cdot (R_2^2 - R_1^2) - \frac{\pi \cdot \omega^2}{4g} \cdot (R_2^2 - R_1^2)^2 \tag{26}$$

The optimum total volume of liquid required for casting is given by equation (27):

$$Vp = \pi \cdot (R^2 - R_2^2) \cdot L + V_{\text{int}} = \pi \cdot (R^2 - R_1^2) \cdot L - (R_2^2 - R_1^2)^2 \cdot (\pi\omega^2)/(4g) \tag{27}$$

In the case of very small differences Δs in wall thickness, the profile of the interior surface of the casting can be assumed as linear. In this hypothesis, a simpler approximate relationship can be determined for the commutation of the liquid alloy volume necessary for casting the part. For this, the cross section of the casting wall is assumed trapezoidal.

$$Vp = \pi \cdot (R + R_{\text{med}}) \cdot L \cdot (s_1 + s_2) = \pi \cdot (2R + R_1 + R_2) \cdot L \cdot (s_1 + s_2)/2 \tag{28}$$

4. Calculated Results of the Casting Wall Thickness

Equation (22) allows the calculation of Δs (the top-to-bottom difference in wall thickness) in dependence on the mould speed and the dimensions. It is assumed that the liquid alloy rotates (in stationary mode) at the same speed as the mould. The equation is rewritten as follows:

$$\Delta s = 900g \cdot L/(\pi^2 n^2 \cdot R_{med}) = 900g \cdot L/[\pi^2 \cdot n^2 \cdot (R_2 - \Delta s/2)] \quad (29)$$

$$\Delta s^2 - 2R_2 \cdot \Delta s + 1800g \cdot L/(\pi^2 \cdot n^2) = 0 \quad (30)$$

The solution for Δs is:

$$\Delta s = R_2 - \sqrt{R_2^2 - 1800g \cdot L/(n \cdot \pi)^2} \quad (31)$$

The equation reveals that the non-uniformity of the wall thickness Δs depends on the speed, the length and the interior radius of the casting. Tables 1 and 2 features the values of Δs calculated by this equation for the part in Figure 3, for various speeds and dimensions of the casting. Figures 4-6 show the influence of these three parameters on the top-to-bottom difference in wall thickness of the casting.

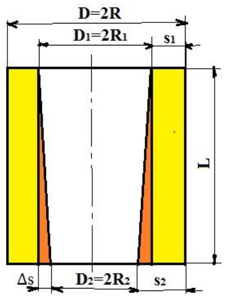

Fig. 3. Casting

Table 1. Non-uniformity of the wall thickness in dependence on the speed and the dimensions of the part

No.	Speed n	Interior radius at the top R_2	Top-to-bottom difference in wall thickness Δs		
			L = 100 mm	L = 200 mm	L = 300 mm
-	n	R_2	Δs	Δs	Δs
	[rpm]	[mm]	[mm]	[mm]	[mm]
1	500	50	7.810	17.442	31.561
2	800	50	2.896	5.983	9.303
3	1000	50	1.833	3.740	5.728
4	1200	50	1.266	2.566	3.902
5	1500	50	0.806	1.626	2.460
6	1700	50	0.627	1.261	1.905
7	2000	50	0.452	0.908	1.369
8	500	100	3.667	7.480	11.456
9	800	100	1.416	2.853	4.312
10	1000	100	0.904	1.816	2.737
11	1200	100	0.627	1.258	1.893
12	1500	100	0.401	0.803	1.207
13	1700	100	0.312	0.625	0.939
14	2000	100	0.225	0.451	0.677
15	500	150	2.419	4.879	7.382
16	800	150	0.940	1.887	2.839
17	1000	150	0.601	1.205	1.811
18	1200	150	0.417	0.836	1.255
19	1500	150	0.267	0.534	0.802
20	1700	150	0.208	0.416	0.624
21	2000	150	0.150	0.300	0.451

Table 2. Non-uniformity of the wall thickness in dependence on the casting dimensions at speed n = 1500 rpm

No.	n	L	R_2	Δs
m.u.	[rpm]	[mm]	[mm]	[mm]
1	1500	100	50	0.806
2	1500	150	50	1.215
3	1500	200	50	1.626
4	1500	250	50	2.042
5	1500	300	50	2.460
6	1500	350	50	2.883
7	1500	400	50	3.309
8	1500	200	30	2.797
9	1500	200	50	1.626
10	1500	200	100	0.803
11	1500	200	150	0.534
12	1500	200	200	0.400
13	1500	200	250	0.320
14	1500	200	300	0.267

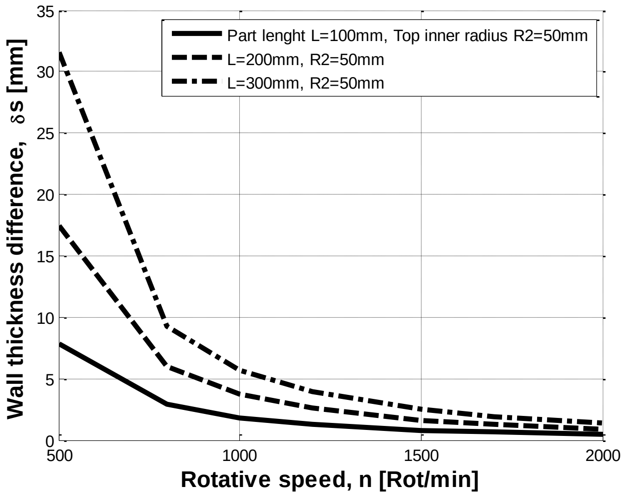

Fig. 4. Non-uniformity of the wall thickness, Δs, versus speed (for parts of length L = 100-300 mm and interior radius R_2 = 50 mm)

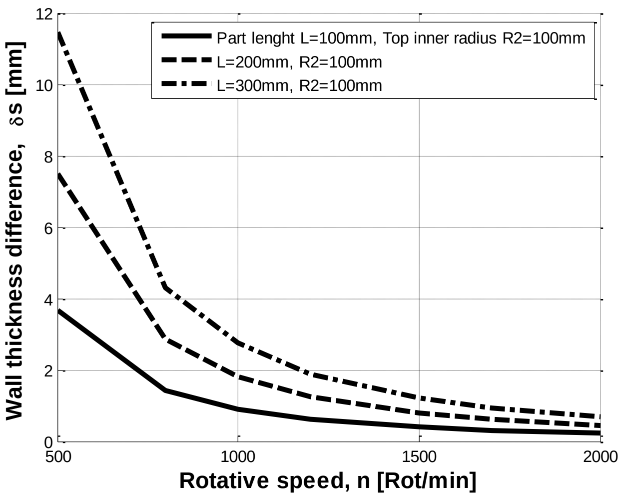

Fig. 5. Non-uniformity of the wall thickness, Δs, versus speed (for parts of length L = 100-300 mm and interior radius R_2 = 100 mm)

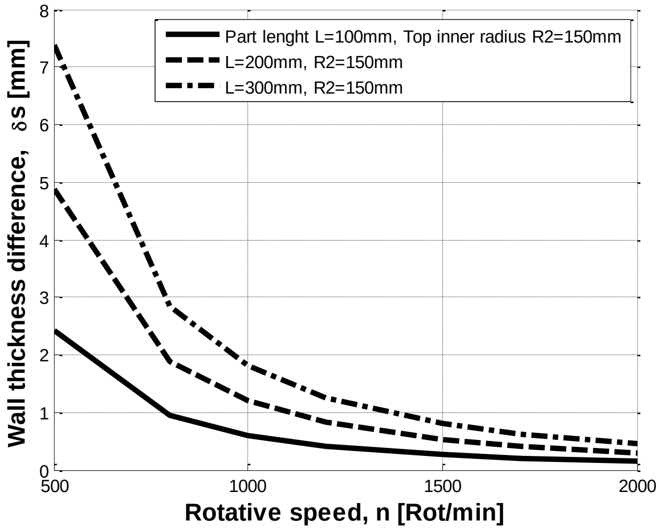

Fig. 6. Non-uniformity of the wall thickness, Δs, versus speed
(for parts of length L = 100-300 mm and interior radius R_2 =150 mm)

The obtained theoretical results reveal the significant influence of the rotational speed on the uniformity of the wall thickness. In the case of the studied parts, at speeds below 1000 rpm the top-to-bottom difference in wall thickness is very large. For this reason, in vertical axis centrifugal casting higher speeds have to be applied compared to horizontal axis centrifugal casting. The speeds recommended for vertical axis casting are of about 1500 rpm. The dimensions of the part (length and interior radius) also influence the non-uniformity of the wall thickness. At a speed of 1500 rpm, the influence of the length and the interior radius (in the case of the studied part) on the non-uniformity of the wall thickness is relatively small (of 2-3 mm). Figures 7 and 8 show that the non-uniformity of the wall thickness Δs, increases linearly with the length of the part (L) and decreases hyperbolically with the increase of interior radius (R_2).

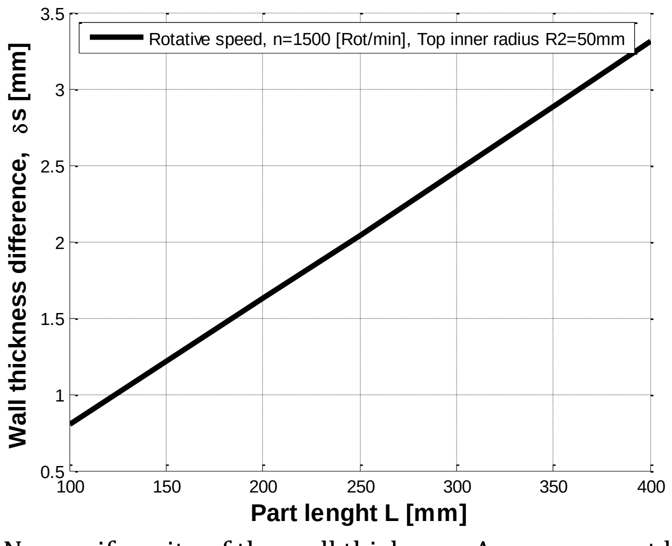

Fig.7. Non-uniformity of the wall thickness, Δs, versus part length
(at speed n = 1500 rpm, interior radius of the part R_2 = 50 mm)

5. Experiment – Calculation Comparison

In paper [11] Singh J.K. and Ojha S.N. presented research results on the wall thickness of parts obtained by vertical centrifugal casting. They cast parts from an aluminium-silicon-lead alloy of exterior diameter $D = 2R$ = 100 mm, length L = 150 mm and top interior radius $R_2 \approx$ 40 mm. The casting temperature of the alloy was of T = 700 °C. The wall thickness was measured at the bottom (s_1) and top (s_2) of the parts cast at various speeds [11]. Based upon the results of these measurements, Table 3 features the calculated difference in wall thickness (obtained experimentally Δs_exp) for various speeds.

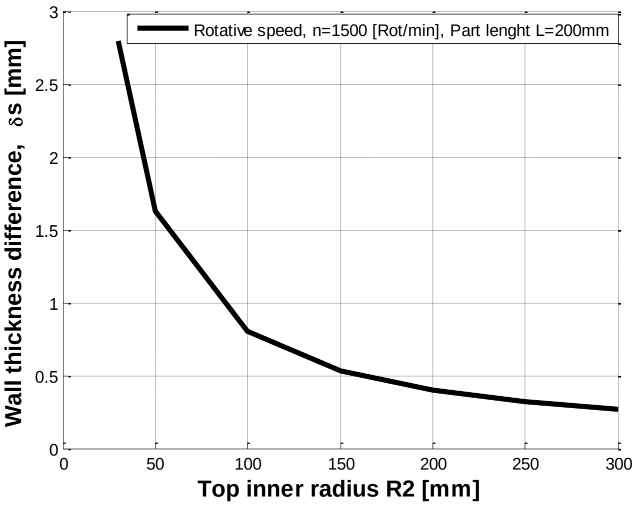

Fig. 8. Non-uniformity of the wall thickness, Δs, versus the interior radius (at speed n = 300 rpm, part length L = 50 mm)

Table 3. Experimental results

No.	n	L	R	s_1 exp	s_2 exp	Δs exp
m.u.	[rpm]	[mm]	[mm]	[mm]	[mm]	[mm]
1	1200	150	40	15.8	9.1	6.7
2	1300	150	40	15.1	9.8	5.3
3	1400	150	40	14.2	10.8	3.4
4	1500	150	40	13.5	11.3	2.2
5	1600	150	40	10.4	10.0	0.4
6	1700	150	40	11.2	12.4	-1.2

The analysis of the results in Table 3 reveals the following:
- at speeds below 1600 rpm, the wall is thicker at the bottom, and the top-to-bottom difference in wall thickness decreases with the increase of speed;
- at speeds over 1600 rpm, the wall is thicker at the top, and the difference in wall thickness Δs_exp = $s_1 - s_1$ becomes negative;
- for these dimensions of the part (and this particular alloy and casting temperature) the optimum speed yielding a uniform wall thickness is of 1600 rpm.

Table 4 features the values for the differences in wall thickness (Δs_calculat) calculated by equation (31) for the same part and speeds (L = 150 mm and R_2 = 40 mm). The comparison of the calculated and experimental values of Δs yields the following remarks:
- the calculated and experimental values are of the same order of magnitude (millimetres);
- the experiment reveals a significantly greater influence of the speed on the non-uniformity of the wall thickness than resulting from the calculations.

The theoretical results (the computational relationship) are valid in the hypothesis of an ideal liquid that rotates at the same speed as the mould and that flows easily (negligible internal viscosity = the free surface deforms easily). In reality, the speed of the liquid alloy is smaller than that of the mould. In addition, close to the solidification temperature (point), the viscosity of the alloy increases considerably. This influences the geometry of the free surface of the liquid alloy undergoing solidification (and thus also the non-uniformity of the wall thickness). The experimental results of the same authors [11] revealed that also the casting temperature has a significant influence on the non-uniformity of the wall thickness in this casting procedure.

Table 4. Non-uniformity of the wall thickness in dependence on the casting dimensions at speed n = 1500 rpm

No.	n	L	R_2	Δs
m.u.	[rpm]	[mm]	[mm]	[mm]
1	1500	100	50	0.806
2	1500	150	50	1.215
3	1500	200	50	1.626
4	1500	250	50	2.042
5	1500	300	50	2.460
6	1500	350	50	2.883
7	1500	400	50	3.309
8	1500	200	30	2.797
9	1500	200	50	1.626
10	1500	200	100	0.803
11	1500	200	150	0.534
12	1500	200	200	0.400
13	1500	200	250	0.320
14	1500	200	300	0.267

5. Conclusions

The utilisation of equation (31) for establishing the optimum speed for vertical axis centrifugal casting needs to consider the hypothesis underlying this equation (ideal liquid). The optimum speed adopted for the mould needs to be greater than the calculated one, in order to consider the viscosity of the alloy. It is, however, not recommended to adopt a very high speed, as this carries the risk of a negative non-uniformity of the wall thickness (a wall thicker at the top). Experimental research needs to be extended to other types of centrifugally cast alloys (iron, bronze, brass, etc.) in order to verify the real effect of viscosity and density on wall thickness in each case.

References

1. Chira, I., Sofroni, L., Brabie, V. (1980): *Procedee speciale de turnare* (*Special Casting Procedures*). Editura Didactică și Pedagogică, Bucureşti, Romania (in Romanian)
2. Chiriță, G.M. (2011): *Mechanical and Fatigue Properties of Functionally Graded Aluminium Silicon Alloys*. PhD Thesis, Universidade do Minho, Escola de Engenharia, Portugal
3. Guliaev, B.B., et al. (1972): *Procedee speciale de turnare* (*Special Casting Procedures*). Editura Tehnică, Bucureşti, Romania (in Romanian)
4. Carcea, I., Gherghe, M. (2009): *Aliaje neferoase de turnătorie* (*Non-Ferrous Foundry Alloys*). Editura Performantica, ISBN 978-973-730-578-7, Iaşi, Romania (in Romanian)
5. Diaconescu, Fl. (2006): *Cercetări și contribuții privind influența parametrilor tehnologici de turnare centrifugă asupra calității pieselor turnate din unele aliaje neferoase* (*Research and Contributions concerning the Influence of Centrifugal casting Technological Parameters on the Quality of Parts Cast from certain Non-Ferrous Alloys*). PhD Thesis, Universitatea Tehnică „Gh. Asachi” din Iaşi, Romania (in Romanian)
6. Diaconescu, Fl. (2011): *Turnarea centrifugală a aliajelor neferoase* (*Centrifugal Casting of Non-Ferrous Alloys*). Editura Tehnopress, ISBN 978-973-702-797-9, Iaşi, Romania (in Romanian)
7. Diaconescu, Fl. (2003): *Cercetări privind influența unor parametri tehnologici de turnare centrifugă asupra cineticii de solidificare a bronzului CuSn5Pb25* (*Research on the Influence of Centrifugal Casting Parameters on the Solidification Kinetics of CuSn5Pb25 Bronze*). Proceedings of the International Conference “Tehnologii şi materiale avansate”, Editura Fundației Universitare “Dunărea de Jos” Galați, ISBN 973-627-066-1 (in Romanian)
8. Diaconescu, Fl., Alexandru, I., Sandu, I.G. (2005): *Studiu privind influența turației cochilei asupra macrostructurii pieselor turnate centrifugal din bronz CuPb10Sn10* (*Study on the Influence of the Shell Mould Speed on the Macrostructure of Centrifugally Cast CuPb10Sn10 Bronze Parts*). Proceedings of the International Conference

Tehnomus XIII, ISBN 973-666-154-7, p. 286-291 (in Romanian)
9. Iudin, S.B., Levin, M.M., Rozenfeld, S.E. (1973): *Turnarea centrifugă* (*Centrifugal Casting*). Editura Tehnică, Bucureşti, Romania (in Romanian)
10. Trejo, E. (2011): *Centrifugal Casting of an Aluminium Alloy*. PhD Thesis, Metallurgy and Materials School of Engineering, University of Birmingham, United Kingdom
11. Singh, J.K., Ojha, S.N. (2013): *Preparation of Leaded Aluminum-Silicon Alloys by Modified Vertical Centrifugal Casting Machine*. International Journal of Engineering and Innovative Technology (IJEIT), ISSN 2277-3754, Vol. 2, is. 9, p. 230-234

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