Experimental Results of a Carbon Steel Analyzed Using the Grubbs Test

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Abstract.
The paper presents an example of calculation for the data validation by using the Grubbs tests. The values used of Rockwell hardness (HRC) of C45 medium carbon steel. All specimens were heated an electric baking temperature of 850°C, while keeping time to equalize the temperature in samples being 20 minutes and then the cooling energy is made in water at room temperature, i.e. 20°C. After quenching, samples were subjected to high-tempering operation at 600°C. Solving the Grubbs test was done in seven steps, which have analyzed the results and finally some statistic conclusions were done. The second alternative hypothesis of the Grubbs test that says there is only one abnormal result in the data sets analyzed was checked in this paper so this test was properly applied in this experimental research.

Keywords
C45 steel, heat treatments, hardness, Grubbs test

1. Introduction

Mathematical and statistical methods have gained wide circulation in industrial experimental data analysis and application of different tests for removal of abnormal results of research led to the confirmation of their veracity [1, 2, 3]. One of the tests used in removing abnormal results are the Grubbs test [4]. Grubbs test specific hypotheses are two, namely: one hypothesis asserts that there are no abnormal data and another alternative hypothesis: asserts there that between all experimental data, there is one abnormal result.

2. Research Objectives

This main objective of this paper is to analyze experimental data using Grubbs test, to find abnormalities and remove them from the experimental research. The analyzed results are for the hardness (HRC) of a C45 medium carbon steel, heat treated by quenched and after high-tempering operation.

3. Steps for Applying the Grubbs Test

Identifying data affected by aberrant errors can be accomplished by applying the Grubbs test.

Solving the test is done in the following seven steps [1, 4]:

(1) Grouping of data and determining the minimum ($x_{\min}$) and maximum ($x_{\max}$) of the experimental values.

(2) Calculating the arithmetic average, with the relationships:

$$\frac{1}{n-1}(x_1 + x_2 + \ldots + x_n) = \frac{1}{n-1}\sum_{i=1}^{n-1} x_i \qquad (1)$$

where:

n - the number of experimental determinations;

$\bar{x}$ - the arithmetic average of the experimental data.

(3) Calculating the dispersion, with the relationships:

$$s^2 = \frac{1}{n-1}[(x_1 - \bar{x})^2 + .. + (x_n - \bar{x})^2 = \frac{1}{n-1}\sum_{i=1}^{n-1}(x_i - \bar{x})^2 \qquad (2)$$

where s^2 is the dispersion of the experimental data.

(4) Calculating the average square diverting, with the relationships:

$$s = \sqrt{s^2} \qquad (3)$$

where s is the average square diverting of the experimental data.

(5.a) If we suspect that the maximum value ($x_{\max}$) in the data set may be an outlier with, Grubbs test calculation (G_{calc}) is done with the relationships [5]:

$$G_{calc} = (x_{max} - \bar{x}) / s \tag{4}$$

where:

G_{calc} - the calculated Grubbs test for the maximum value (x_{max}) from the data set;

$\bar{x}$ - the arithmetic average of the experimental data;

s - the average square diverting of the experimental data.

(5.b) If we suspect that the minimum value (x_{min}) in the data set may be an outlier with, Grubbs test calculation (G_{calc}) is done with the relationships [5]:

$$G_{calc} = (\bar{x} - x_{min}) / s \tag{5}$$

where G_{calc} is the calculated Grubbs test for the minimum value (x_{min}) from the data set.

(6) Determining the critical value of test the Grubbs test; for the two-sided test, the hypothesis of no outliers is rejected if:

$$G_{crit} = \frac{(n-1) \cdot t_{crit}}{\sqrt{n \cdot (n - 2 + t_{crit}^2)}} \tag{6}$$

where:

t_{crit} - the critical value for the t distribution with (n – 2) degrees of freedom and a significance level of $\alpha/(2n)$, $\alpha = 0.05$; for one sided tests, using a significance level of level of α/n [1];

n - the number of experimental determinations.

(7) Comparing the G calculated value (G_{calc}) with the G critical value (G_{crit}) and analyzing the results and the conclusions of this test. Two situations can occur [4]:

a) If $G_{calc} < G_{crit}$ the values of the liable data to be affected by aberrant errors (x_{max} or x_{min}) of the experimental values are normal for our experimental research;
b) If $G_{calc} > G_{crit}$ the values of the liable data to be affected by aberrant errors (x_{max} or x_{min}) of the experimental values are outliers for our experimental research, these values are rejected. In both cases when determining abnormal after removing them, the calculations will made using new parameters required analysis.

4. Experimental Procedure and Results

The studied C45 medium - carbon steel, has the following chemical composition (% in weight): 0.47% C; 0.69 % Mn; 0.2% Si; 0.021% S; 0.021% P.

All specimens for testing (20×10×5 mm) were heated in an electric baking temperature of 850°C, while keeping time to equalize the temperature in samples being 20 minutes. Cooling energy has been made in water; the water temperature is at room temperature, i.e. 20°C. After quenching operation, structure that is obtained is martensite hardening, a non-equilibrium structure [6, 7]. The quenched samples were performed such tests Rockwell hardness, resulting in values: 52.6 ÷ 62.6 HRC. The values of the Rockwell hardness (HRC) results after quenching are presented in Table 1.

After quenching, samples were subjected to high-tempering operation. Tempering operation, unlike the annealing and quenching operations, is not a unique stand-alone operation; it applies only after quenching operations and aims to facilitate specific changes only in hardened areas.

The changes are intended to bring the equilibrium structure obtained by quenching (quenching martensite), close to equilibrium structures with distinct properties.

The samples for testing the resilience of the high-tempering operation of 600°C, took 15 minutes during the maintenance to equalize temperature. Following high-tempering operation, a tempering soorbite structure was obtained. The high-tempering samples have been performed at such tests as Rockwell hardness, resulting in values: 34.1 ÷ 38.4 HRC. The values of the Rockwell hardness (HRC) results after high-tempering operation are presented in Table 2.

5. Applying the Grubbs Test

After calculating the seven specific steps in applying the Grubbs test, the results of the Rockwell hardness (HRC) after quenching heat treatment are presented in Table 3.

The results of the Rockwell hardness (HRC) after high-tempering heat treatment are presented in Table 4.

Table 1. Data analysis of Rockwell hardness (HRC) values after quenching heat treatment

Observations (n)	Rockwell hardness [HRC]
1	57.8
2	57.5
3	53.5
4	54.5
5	55.7
6	55.6
7	62.6
8	53.5
9	54.5
10	54.5
11	46.9
12	57.8
13	63.8
14	59.5
15	60.6
16	52.5

Table 2. Data analysis of Rockwell hardness (HRC) values after high-tempering operation

Observations (n)	Rockwell hardness [HRC]
1	36.8
2	42.3
3	36.5
4	37.4
5	37.9
6	37.0
7	37.0
8	36.4
9	35.1
10	37.5
11	37.5
12	38.4
13	34.2
14	37.7
15	35.1
16	34.1

By studying all the data presented in Table 3, following a general remarkable conclusions:

- the value of x_{max} (63.8) is a normal value for our experimental research, because $G_{calc} < G_{crit}$ (1.710 < 1.865), in this case it noted with a Yes;
- the value x_{min} (49.9) is identified as data affected by aberrant errors for our experimental research and was rejected because $G_{calc} > G_{crit}$ (2.239 > 1.865), in this case it noted with a No;
- the new minimum value of experimental data x_{min} (52.6) is checked with Grubbs test and it is observed that new value is a normal value for our experimental research, because $G_{calc} < G_{crit}$ (1.272 < 1.858), in this case it noted with a Yes.

By studying all the data presented in table 4, following a general remarkable conclusions:

- the value x_{max} (42.3) is identified as data affected by aberrant errors for our experimental research and was rejected because $G_{calc} < G_{crit}$ (2.777>1.865), in this case it noted with a No;

- the value of x_{min} (34.1) is a normal value for our experimental research, because $G_{calc} < G_{crit}$ (1.465 < 1.865), in this case it noted with a Yes;
- the new maximum value of experimental data x_{max} (38.4) is checked with Grubbs test and it is observed that new value is a normal value for our experimental research, because $G_{calc} < G_{crit}$ (1.359 < 1.858), in this case it noted with a Yes.

Table 3. Grubbs test for hardness (HRC) data sets after quenching heat treatment

Experiment	n	x	Measured values	$\bar{x}$	t_{crit}	G_{calc}	G_{crit}	Values remains Yes/No
$E_{63.8}$	16	x_{max}	63.8	56.48	2.14	1.710	1.865	YES
$E_{49.9}$ rejected	16	x_{min}	49.9			2.239		NO
$E_{52.6}$ recalculated	15	x_{min}	52.6	57.12	2.16	1.272	1.858	YES

Table 4. Grubbs test for hardness (HRC) data sets after high-tempering heat treatment

Experiment	n	x	Measured values	$\bar{x}$	t_{crit}	G_{calc}	G_{crit}	Values remains Yes/No
$E_{42.3}$ rejected	16	x_{max}	42.3	36.93	2.14	2.777	1.865	NO
$E_{32.1}$	16	x_{min}	34.1			1.465		YES
$E_{38.4}$ recalculated	15	x_{max}	38.4	36.57	2.16	1.359	1.858	YES

6. Conclusion

Analyzing all data taken into account there can be said the following:

(a) the heat treatments of the studied C45 medium - carbon steel samples were:
- quenching at 850°C and cooling energy in water;
- high-tempering at 600°C operation at were made;

(b) the structure after quenching operation, structure which is obtained is martensite hardening, a non-equilibrium structure

(c) following high-tempering operation, a tempering soorbite structure was obtained;

(d) Rockwell C measurements of hardness after quenched in water and after high-tempering operation were made, with the following results:
- the resulting in values of Rockwell C measurements of hardness after quenched in water are between 52.6 ÷ 62.6 HRC;
- the resulting in values of Rockwell C measurements of hardness after high-tempering operation are between 34.1 ÷ 38.4 HRC;

(e) by studying all the data presented in the table 3 and 4, after calculating the seven specific steps in applying the Grubbs test for Rockwell C measurements of hardness, following a general remarkable conclusions:
- all the normal experimental values of our research work, meet condition: $G_{calc} < G_{crit}$ and the results were noted with Yes;
- all the outliers experimental values of our research work, meet condition: $G_{calc} > G_{crit}$ and the results were noted with No;
- in both cases when determining outliers values (for the experiment $E_{49.9}$ and $E_{42.3}$) after removing them, the calculations will be made using new parameters required by analysis and the new values ($E_{52.6}$ and $E_{38.4}$) are normal experimental values for our research work.

(f) the second alternative hypothesis of Grubbs test that says there is only one abnormal result in the data sets analyzed was checked in this paper:
- the resulting in values of Rockwell C measurements of hardness after quenched in water contain only one abnormal experimental value ($E_{49.9}$) which was rejected;
- the resulting in values of Rockwell C measurements of hardness after high-tempering operation contain only one abnormal experimental value ($E_{42.3}$) which was rejected;

(g) Grubbs test was properly applied in this experimental research.

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