International Conference on Economic Engineering and Manufacturing Systems
Braşov, 25 – 26 October 2007

ACCURATE EVALUATION OF THE ANGLES

Petru CRUCIAT, Flavius Aurelian SÂRBU
"Transilvania" University of Brasov, Romania

Abstract. The precision with which the conical parts that form adjustments are evaluated is unsatisfactory because of the limitations of the evaluation methods and of the measurement units of the plane angles. The second, the minute, the degree, the right angle, the radiant or their divisions are not usable because they are associated with unacceptable discrepancy for the parts that form adjustments.

For using the advantages of the measurement units for lengths, we recommend the association of the conical surfaces with metrical configurations on the basis of some explicitly expressed criteria. We remind that in ancient times, geometricians expressed the angles with the help of the right-angle triangle, what we call today trigonometric functions. Their values are irrational numbers and the correspondence with the triangular configurations is difficult to establish.

Keywords: angle, evaluation methods, measurement units

1. Measuring the angles

The establishment of the circumference of the Earth by Eratostene [1] in year 200 b.C. is an example of the unlimited potential of the human mind. On the account of some simple observations he deduced from the solar clock that the distance between 2 localities, Alexandria and Siene (Assuan) represent the 50th part of the meridian length. He withhold to express in angle units or length this measures, which today would mean 7 degrees and 12 minutes on the solar clock, 800 km between the two localities and respectively 40.000 km the circumference of the Earth, representing an amazing precision. The utterance specific to the ancients communicated to the concerned ones would sound something like: if a swimmer goes at see from Iberia will get to India. Surely, Columbus used this information and the error of not verifying and the quantitative aspect would have cost him the life. Looking on the Earth globe, the distance to India or Asia was very big, we realize that his luck was America.

In the Middle Ages [2], Galileo Galilei gave his contemporaries the most original and engineer advice: measure everything that can be measured and make measurable everything that is not yet!

2. Defining conical surfaces

The cones and pyramids are geometrical forms frequently used in parts for mechanical engineering. The common item is the inclined position with a certain angle of the components. For cones, the generatrix is inclined towards the ax, and at the pyramids, the lateral surfaces are inclined towards the basis.

Further, we will refer only to the resulted conical parts, but the conclusions can be extended and for the pyramidal parts.

The surfaces of the conical parts are, in fact, cone stems and they are defined by the following elements represented in the figure 1:

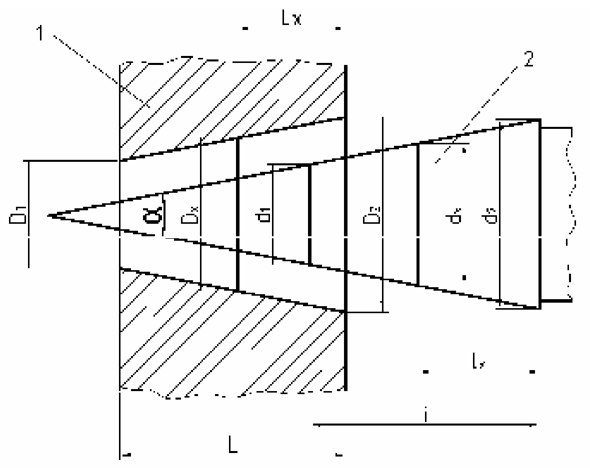


Figure 1. The elements of the conical parts

- nominal diameter of the cone – as one of the following diameters:
- the big diameter of the cone (*D*);
- the small diameter of the cone (*d*);
- the diameter, (*d*) in a normal plan on the axis at the distance, l_x from one of the basis;

- the nominal angle of the cone α;
- the angle of the generatrix of the angle α/2;
- the length of the conical surface, *l*;
- *C* conicalness – defined by the relation;

$$C = \frac{D-d}{l} = 2\ \mathrm{tg}\frac{\alpha}{2} = 1 : \frac{1}{2}\ \mathrm{ctg}\ \frac{\alpha}{2} \qquad (1)$$

The conicalness is notated through a fraction with the unitary denomination: $C = 1{:}x$. Thus, x represents the inverse conicalness. By definition, the conicalness is the axial distance to which the difference of the diameters is 1 mm.

- the inclination, *I* of the cone's generatrix (towards the axis), given by the relation:

$$I = \frac{D-d}{2} = \mathrm{tg}\frac{\alpha}{2} = 1 : \mathrm{ctg}\frac{\alpha}{2} \qquad (2)$$

The inclination is marked by a fraction with the unitary denomination: $I = 1{:}y$; in which y is the inverse of the inclination and is defined as the axial distance on which the range is modified with a millimeter.

The inclination is half the conicalness, and y is double x. The inclination is also called gradient because it is equal the tangent of the α angle.

The inclination is characteristic to the parts with plane inclined surfaces.

The standard cones angles and standard conicalness for smooth conical parts (plain) are established by STAS 2285/1-81, and the normal prism angles and inclinations are established by STAS 2285/2 – 81.

For define a conical surface is necessary to know three parameters, among which at least one of them should be a diameter.

The conical assembling is made out of two conical parts, from which one forms the comprehensive cone (conic cylinder bore or interior cone), and the other is the comprehended cone (conic shaft or exterior cone).

The conic assembles have as main goal fixing the relative position of two parts, in radial direction, as well as in axial direction. The conic assembles are used at fix assembles as well as for mobile ones. In the first case, they are used in view of rapid fixing and with foraging of the tools in the main axes of the tool-machineries, foraging assuring friction that oppose the whirl moments. Mobile conic assembles are used in the case of conic bearings, to which during exploitation the clearance can be adjusted during the fraying of the parts.

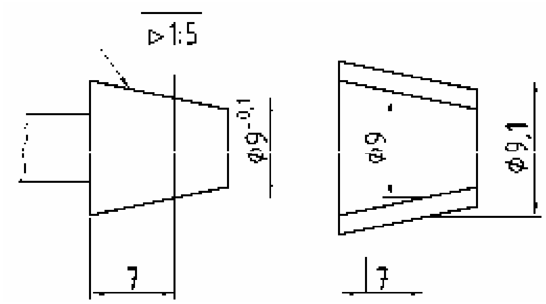
A special category of conic assembles are the ones that assure the sealing up. The conditions of form precision and the quality of the surface are special. They can be easily exceeded by choosing some plastic distortional materials or by paired grinding. All the cocks used for liquids or gases are built in other way. This category of conic parts are nor yet standardized.

3. Tolerances of the conic parts

The limitation of the fields in which the surface of a conic part is compressed assumes the tolerance of the dimensions that define that surface.

The prescription of the accuracy of the conic parts can be made using two methods: the method of nominal conicalness and the method of tolerated conicalness.

By the method of nominal conicalness (of the nominal angle), exemplified in figure 2, we define a tolerance field include between two limit cones, coaxial ones, both having the conicalness equal to the nominal conicalness (respectively the angle at the nominal point), where the bordering of the conic surface is admitted. All the diameters shall be tolerated according to the ISO system for the cylindrical parts (STAS 8100-88). The diameters of the cones are provided with the two limits of the part's diameter tolerance.



a) b)

Figure 2. Method of normal conicalness
a) the drawing; b) the tolerance field

The highlighting of the fact that the two limit cones both have the nominal conicalness is made by framing in the value of the conicalness or of the inclination, respectively of the angle of point or inclination.

The prescription of the size of the tolerance field can be made in two ways:

- by tolerating a diameter in a determined plan, whose position is established by a reference quota and maintaining the other two items as framed dimensions;
- by tolerating the quota that determines the position of the reference plan, maininting the other two items as framed.

The shape deviations (deviations from the rectilinearity, circularity, etc.) are admitted provided that, at no point, the surface of the real cone is beyond the tolerance field determined by the limit cines. In case of functional necessity, we can foresee restrained tolerances for the shape deviations.

By the method of the tolerated conicalness (or of the tolerated angle) we independently establish tolerances for:

- sizes;
- conicalness or the cone's angle.

The value of the dimensional tolerance is applied only in one section, the prescription being made in two ways:

- by tolerating a diameter situated in a determined plan whose position is precisely established (on the grounds of a reference quota);
- be tolerating the quota that determines the position of the plan in which the cone's diameter is, indicated as reference quota.

The tolerances for conicalness or for the cone's angle are prescribed independently of the dimensional tolerances. For these reasons, there can be a great variety of variants for the tolerances field. There were some trials to identify them, but they were abandoned. In most of the cases, the fields are delimited by combined truncated cones; two of them have a common diameter, and the ones in the exterior have and increased angle with the angle deviation and those from the inside have diminished the same deviation.

Choosing the quotation and tolerating method depends on the functional conditions imposed to the conic parts assembling, such as: the relative longitudinal position of the two conjugated elements, the conditions imposed for the contact between the conic surfaces etc.

The method of nominal conicalness is preferably used when, from the functional point of view, tolerating the position of the conic element is necessary, without being necessary to particularly mention the cone angle, for examples for cones that do not form nozzles. Within this method, we shall prefer the tolerance of the diameter, for small conicalnesses that need to ensure a clearance. For the big conicalnesses, we shall prefer the tolerance of the of the position quota of the plan in which the diameter is prescribed.

Quoting two conjugated items of a conical assembly must include:

- the same nominal conicalness;
- a reference quota: either the diameter (figure 2) either the position quota of this diameter.

In STAS 10.120-75 the tolerances system for conicalnress from 1:3 to 1:500 and lengths of the cone from 6 mm to 630 mm is established.

We provide definitions related to the cone, the sizes of the cones and the tolerances to conicalness, afterwards we approach the problem of the conic parts tolerance.

The tolerances system for conicalnesses is based on the following types of tolerances:

- the tolerance of the cone's diameter - T_D – valid for all the diameters of the cone, the entire length of the cone (*l*);
- the tolerance of the cone's angle AT – provided in angle values (AT_{α}) or symmetric linear values (AT_D), with plus and minus;
- the shape tolerance - T_F (the tolerance to rectilinearity of the generatrix and the tolerance to circularity of the normal section);
- the tolerance of the cone's diameter - T_{DS} – for a given section, valid only for the diameter of the cone in this section.

The tolerance of the cone's diameter - T_D – and the tolerance of the cone's diameter for a given section - T_{DS} – are chosen function of the cone's nominal diameter (*D*, *d* or d_x) respectively the diameter of the given section, among the fundamental IT tolerances in the ISO tolerances system.

The tolerance of the cone's angle - AT – is chosen among the fundamental tolerances stipulated in STAS 10.120-75 (there are established 12 steps of precision, decreasingly noted from the point of view of the precision with numbers from 1 to 12; AT1 ... AT12, reported to 10 intervals of cones' length, with values for AT_{α} and AT_D).

The existing deviations at the conic parts' diameters or the angle deviations of their generatrix shall determine the relative axial position of the two conic parts of the assembly.

For exemplification, we take into consideration the following case: if the exterior and interior cones are executed at the nominal size, (figure 3), the position of the parts is given by the *L* quota.

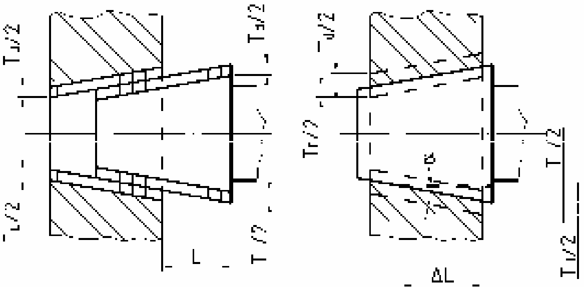


Figure 3. Axial movement of the conic parts

If the interior cone is executed at the superior limit of the D_{max} diameter, and the exterior cone at the inferior d_{min} diameter, there is an axial movement, AL.

The movement between the two cones (figure 2 b) has the value:

$$\Delta L = \frac{1}{2}\cdot\left(T_D + T_d\right)\cdot \mathrm{ctg}\frac{\alpha}{2} = \frac{1}{C}\left(T_D + T_d\right) \qquad (3)$$

The axial position of the conic parts can be used to measure the diameter of a conic part.

4. Sinus ruler

The sinus ruler (figure 4) is a device with a special construction using which we indirectly measure the angle of the α conicalness of the exterior and interior conic surfaces.

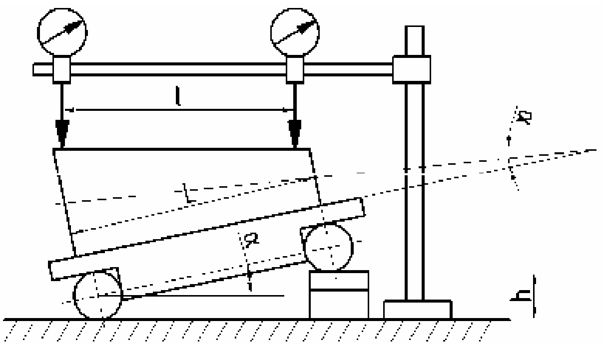


Figure 4. Sinus ruler

$$\alpha = \arcsin\frac{h}{L} \qquad (4)$$

5. The method of the calibrated balls

The calibrated balls are used to measure the interior conicalness, as a helping instrument, a depth micrometer also being necessary (figure 5).

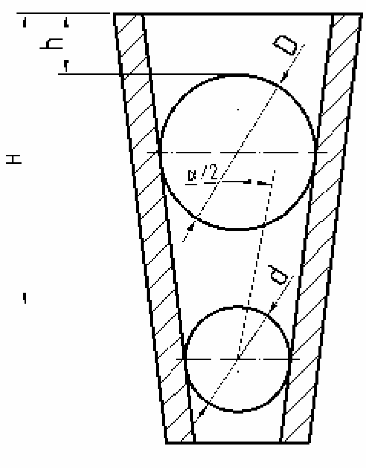


Figure 5. The method of the calibrated balls

Of the measuring scheme, we find that the semi angle α/2 is determined with the relation:

$$\frac{\alpha}{2} = \arcsin\frac{D-d}{2(H-h)-(D-d)} \qquad (5)$$

where D and d represent the diameter of the balls and H and h – the depths measured from the frontal surface of the cone to the two balls.

6. The method of the cylindrical rollers

For the evaluation of the incline angle of the swallowtail guide, we use two pairs of cylindrical rollers consecutively measuring the two lengths; L and l. According to mathematical model shown in figure 6, we express the alpha arch with cotangent.

$$\alpha = 2\,\mathrm{arcctg}\left(\frac{L-l}{D-d} - 1\right) \qquad (6)$$

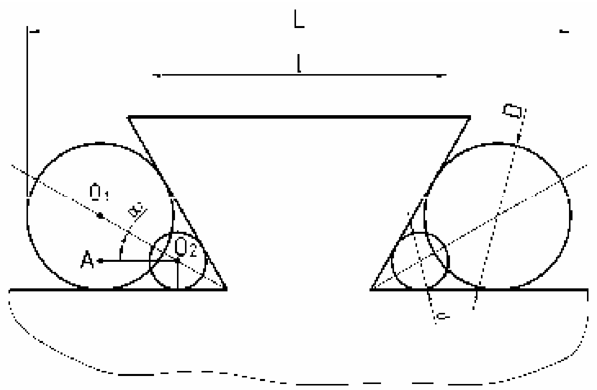


Figure 6. The method of the cylindrical rollers

In order to use the advantages of the means of measuring lengths, we propose the association of the inclined surfaces with metric configurations on the grounds of certain criteria explicitly expressed. In this case, we use the criterion (method) of the cylindrical rollers. The comparison of the inclinations of the similar parts shall be made on the grounds of the difference between the lengths L and l.

The correct way of expression is: the angle opening is determined by the difference between the lengths $L - l$ associated with the method of the cylindrical rollers of D and d diameters.

7. The method of the tangent ruler

The tangent ruler (figure 7) resides in an assembly made up of plate 1, rollers 2 and 3 with different diameters (d and respectively D) and plate 4.

In order to measure the α angle, the part is located on rollers 2 and 3 and the later ones stand on plate 4. The rollers shall be, one to the other, at an H distance (determined by the plan-parallels way block 5), so that plate 1 with the part to be measured gains the necessary inclination (the superior generatrix to be parallel with the surface of plan 4).

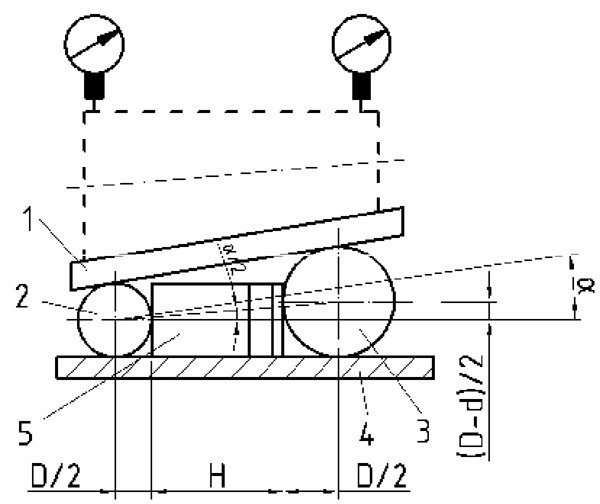


Figure 7. The tangent ruler

The angle value is determined with the relation:

$$\alpha = 2\,\text{arctg}\frac{D-d}{2H+D+d} \qquad (7)$$

8. Conclusions

The accuracy with which are evaluated the conic parts forming nozzles is unsatisfactory because of the limits of the evaluation methods and of the measuring units of the plan angles. The second, the minute, the degree, the right angle, the radian or their divisions are not usable because they associate with the to deviation unacceptably big for the parts forming nozzles.

In order to use the advantages of the means of measuring lengths, we propose the association of the conic surfaces with the metric configurations on the grounds of the criteria specifically expressed.

Expressing the angles using the rectangular triangle, what we call trigonometric functions, leads to difficulties hard to overcome. Their values are irrational numbers and the correspondence with the triangular configurations is hard to establish.

The correct thing is to mention the criterion together with the measured lengths.

References

1. Fradkin, N.: *Of the history of the map*. "Ştiinţifică" Publishing House, Bucharest, 1961 (in Romanian)
2. Iacobescu, F., Ilioiu, N.: *The metrology, etalon of the civilizations*. "Academiei Române" Publishing House, Bucharest, 2004, ISBN 973-27-1104-3 (in Romanian)
3. Cruciat, P.: *Tolerances and adjustments*. "Lux Libris" Publishing House, ISBN 973-9428-68-1, Braşov, 2002 (in Romanian)
4. Ivan, M. et al.: *Tolerances and dimensional control*. "Transilvania" University Press, Braşov, 1993 (in Romanian)