KINEMATIC ANALYSIS AND SYNTHESIS OF DIFFERENTIAL SCREW ACTUATOR

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Abstract. The paper considers structural and kinematic studies of differential screw actuators. For a distinct type of kinematic chain named Closed Differential Screw Mechanism, consisting of an ordinary gear train and a crew pair, synthesis by minimal velocity ratio is done. The main dependences and values of the mechanism parameters are obtained so the minimum of velocity of the output link to be achieved.

Keywords: Differential screw mechanism, linear actuator, kinematic synthesis

1. Introduction

Differential power screw drivers have a long history. Such trains could be found in the notes of Leonardo da Vinci although based on different kinematic principles [1]. First mathematical investigations of those kinds of mechanisms had been made by Franz Reuleaux [2]. For the need of precise measurement equipment Reuleaux has created some models of crew mechanisms. Two of them he had named Differential screw measurement mechanism and Differential Screw Mechanism with Two Spur-Gear Pairs. These trains involve the differential power screw actuators for linear motion. The first device consists of two coaxial screws with different pitch trade and the second one has a screw pair connected with an additional spur gear train. The two devices are well known in the modern technique and now they are widely applicable [3, 4, 5].

These mechanisms are used preferably for performing of small precise displacements. Regardless of various constructive possible solutions the mechanism with different pitch trade has a disadvantage concerning its big longitudinal dimension. This is the reason why this kind of mechanisms is considered only of structural point of view here.

This paper discusses the kinematics and the synthesis of differential screw mechanism with additional spur gear train. Some structural and terminological suggestions are given. The synthesis aim is to obtain parameters of a mechanism that combines both gear trains and power screw so a low velocity ratio to be guaranteed.

2. Structural and kinematic outlines

Now let's consider the simple screw mechanism shown in Figure 1. Screw 1 is connected with nut 2 by a helical screw pair.

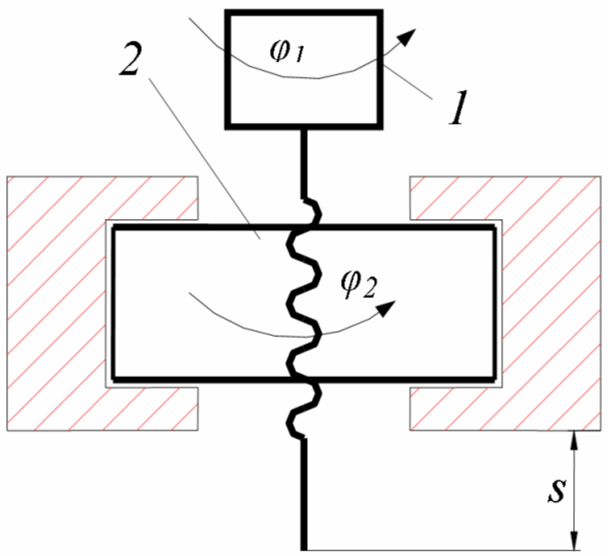

Figure 1. Differential screw mechanism

The nut 2 forms a rotational pair with the fixed link. Let the screw and the nut are turned to some arbitrary angles which are respectively φ_1 and φ_2. The resulting axial motion of the screw in this case can be described by the linear coordinate

$$s = \frac{p}{2\pi} \cdot (\varphi_1 - \varphi_2), \qquad (1)$$

where p is lead of the thread helix.

After differentiating (1) in reference to the velocity of the screw

$$v = \frac{p}{2\pi} \cdot (\omega_1 - \omega_2) \qquad (2)$$

is found. Here ω_1 and ω_2 are angular velocities of the screw and of the nut respectively.

Despite the relatively simplicity of the mechanism, mentioned above, its position and kinematic analysis show that the output motion is a function of the difference of two generalized coordinates. The input parameters are two angles therefore the mechanism has two DOF. It means that it is more correctly the name Differential Screw Mechanism to be given of this kind of structure. As the next mechanisms consist of the mentioned structure but one of the rotations is constrained by additional closing kinematic chain (CKC), it is more correct to be brought in to use the term Closed Differential Screw Mechanism. Kinematic schemes of such kind of mechanisms are shown in Figure 2.

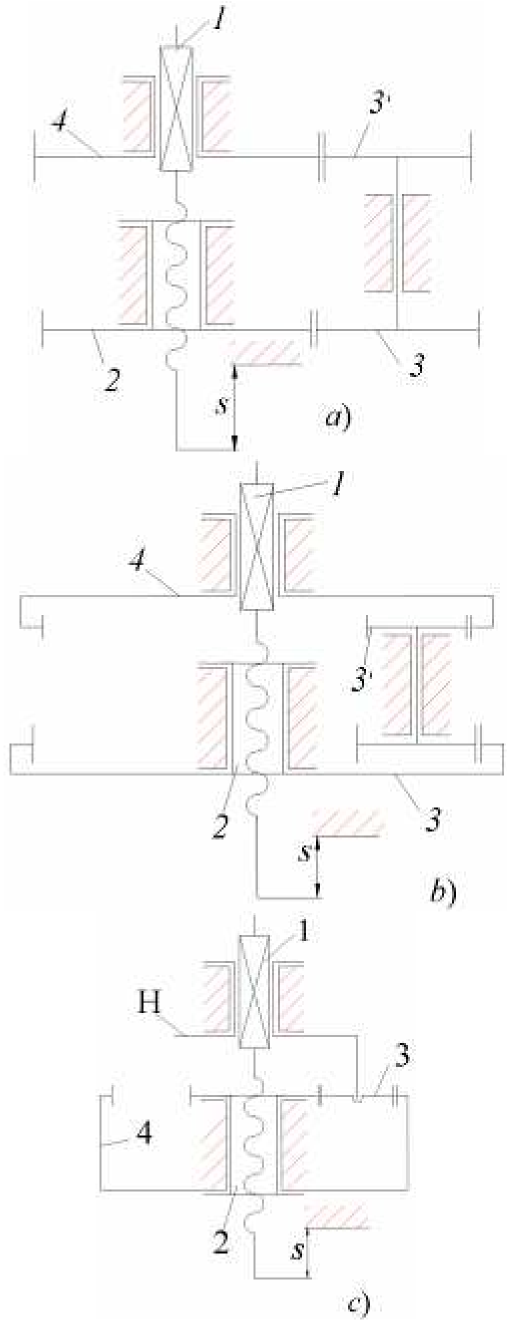

Figure 2. Closed differential crew mechanisms:
a) with external contact gears CKC;
b) internal contact gears CKC;
c) epicyclical CKC

These mechanism structures are the simplest possible ones. In the mechanism shown in Figure 1*a*) the CKC of an ordinary gear train with an external contact gear pairs is used [6]. CKC of ordinary gear train with internal contact gear pairs and epicyclical (planetary) CKC are used in the mechanisms shown in Figure 2, *b*) and *c*) respectively.

Another typical structure also called Differential Screw Mechanism is shown in Figure 3.

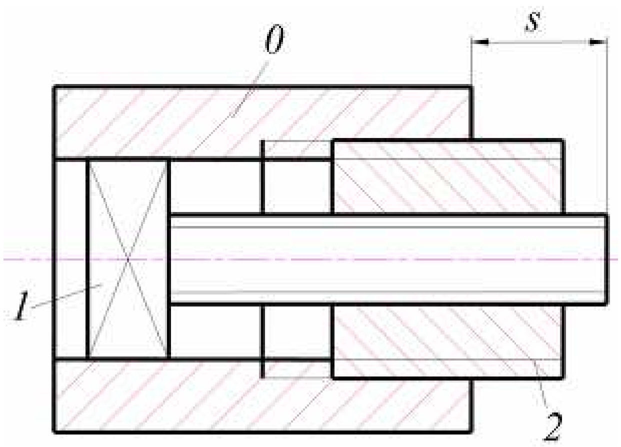

Figure 3. Closed differential screw mechanism with two threads

This mechanism could be presented by different constructive variants [7], but its structure is similar to these shown in Figure 3. Here the screw 1 forms prismatic pair with the fixed link 0 and is connected to nut 2 by tread with lead p_1. The nut can move in fixed link due to second one tread with lead p_2. Since the two threads have different leads the linear position of the screw is

$$s = \frac{p_1 - p_2}{2\pi} \cdot \varphi \qquad (3)$$

where φ is relative rotation between the nut and the fixed link (or screw).

The sign between p_1 and p_1 depends on direction of the tread helix. Although in equation (3) the used mathematical operation is a "difference", the mechanism could not be referred to differential one.

Another reason for this statement is the mechanism's one DOF. So as more correct term that must be used in this case it is Closed Differential Screw Mechanism. The nature of the CKC here is different from the mechanisms shown in Figure 2. This scheme seems to be the simplest one, but it possesses very large length which could be an important disadvantage in the most cases [8].

3. Synthesis of closed differential crew mechanism by condition of minimum velocity ratio

A linear actuator based on a closed differential screw mechanism is shown in Figure 4.

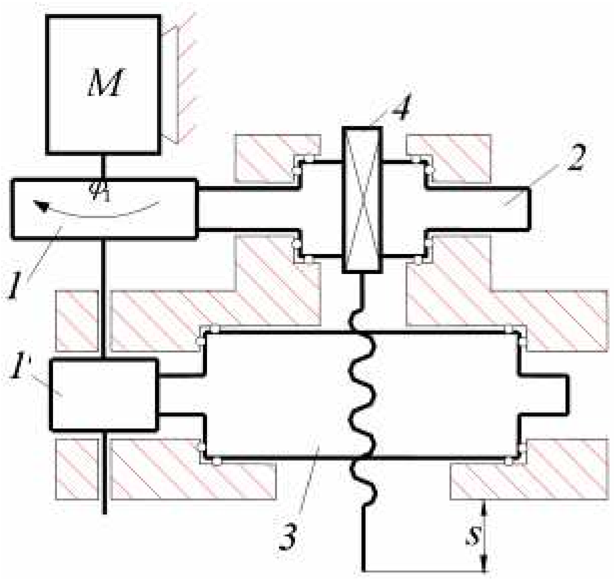

Figure 4. Closed differential screw drive

CKC is the same as this in Figure 2*a*. The input link consisting of spurs gears 1 and 1' is driven by the electrical motor *M*. The output linear coordinate is

$$s=\frac{p}{2\pi}\cdot(\varphi_2-\varphi_3)=\frac{p}{2\pi}\cdot\varphi_{23} \quad (4)$$

where φ_2 and φ_3 are the angular coordinates of spur gears 2 and 3, $\varphi_{23}=\varphi_2-\varphi_3$ is the relative angle of the spur gear 2 with respect to the spur gear 3.

From gear trains follow the expressions

$$\varphi_2=\frac{\varphi_1}{i_{12}} \quad \text{and} \quad \varphi_3=\frac{\varphi_1}{i_{1'3}}, \quad (5)$$

where

$$i_{12}=-\frac{z_2}{z_1} \quad \text{and} \quad i_{1'3}=-\frac{z_3}{z_{1'}} \quad (6)$$

are the gear ratios of the trains, z_1, $z_{1'}$, z_2 and z_3 are respectively the number of teeth gears 1, 1', 2 and 3.

Upon replacing relationships (5) in Eq. (4) it is found

$$s=\frac{p}{2\pi}\cdot\left(\frac{i_{1'3}-i_{12}}{i_{12}\cdot i_{1'3}}\right)\cdot\varphi_1. \quad (7)$$

Taking into account Eqs. (6), realtionship (7) can be rewritten as:

$$s=\frac{p}{2\pi}\cdot\left(\frac{z_{1'}\cdot z_2-z_1\cdot z_3}{z_2\cdot z_3}\right)\cdot\varphi_1. \quad (8)$$

On differentiating (8) with respect to time the linear velocity

$$v=\frac{p}{2\pi}\cdot\left(\frac{z_{1'}\cdot z_2-z_1\cdot z_3}{z_2\cdot z_3}\right)\cdot\omega_1 \quad (9)$$

of the screw is obtained. So the velocity ratio is

$$s'=\frac{s}{\omega_1}=\frac{p}{2\pi}\cdot\left(\frac{z_{1'}\cdot z_2-z_1\cdot z_3}{z_2\cdot z_3}\right). \quad (10)$$

In order to obtain small velocity ratio it is seen from Eq. (7) that the necessary condition is

$$i_{1'3}\approx i_{23}, \quad (11)$$

but $i_{1'3}\neq i_{12}$, because if $i_{1'3}=i_{12}$ than $s=0$.

For simplicity and better design conditions it could be assumed that the two pinions has equal number of teeth, namely

$$z_{1'}=z_1. \quad (12)$$

In this case the condition (11) can be fulfilled if the difference of teeth number of spur gears 2 and 3 is the smallest possible. This minimal difference is one tooth so it means that

$$z_3=z_2+1. \quad (13)$$

On substituting (12) and (13) in (10) it is found

$$s'=\frac{s}{\omega_1}=\frac{p}{2\pi}\cdot\frac{-z_1}{z_2\cdot(z_2+1)}. \quad (14)$$

Taking in account that $z_2=-i_{12}\cdot z_1$ formula (14) can be rewritten in the view

$$s'=\frac{s}{\omega_1}=\frac{p}{2\pi}\cdot\frac{1}{i_{12}\cdot(1-i_{12}\cdot z_1)}=s_0'\cdot f_{iz}. \quad (15)$$

where $s_0'=\frac{p}{2\pi}$ is the velocity ratio of ordinary power screw mechanism, and $f_{iz}=\frac{1}{i_{12}\cdot(1-i_{12}\cdot z_1)}$. The term in the square brackets denoted as f_{iz} depends on two parameters i_{12} and z_1 and shows how the velocity ratio of ordinary power screw is reduced. A 3D graph of the function $|f_{iz}|$ is shown in Figure 5. The parameters vary as follows $i_{12}\in[-6;-1]$ and z1 $\in$ [6; 30].

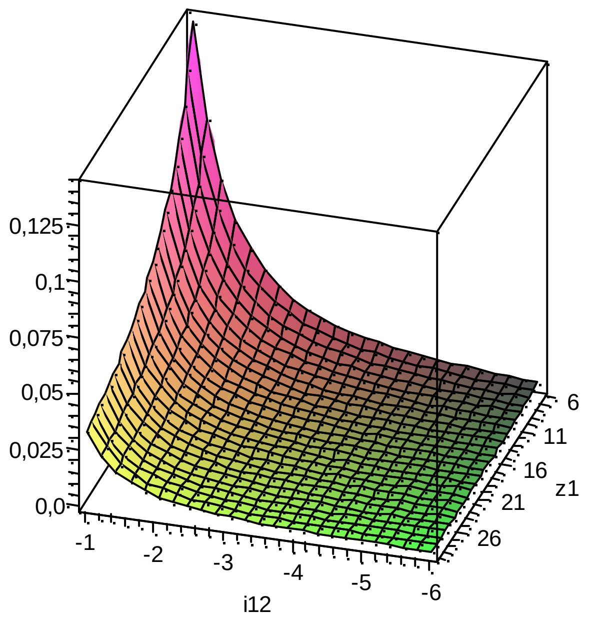

Figure 5. 3D graph of function f_{iz}

It is seen from the graph that function f_{iz} is smooth surface and it obtains its minimal value when the gear ratio i_{12} and number of the teeth z_1 are possibly the largest.

The minimum value of function $f_{iz_{\min}} = 0.00092$ is obtained for $i_{12} = -6$ and $z_1 = 30$. Maximum $f_{iz_{\max}} = 0.142857$ is in point $i_{12} = -1$ and $z_1 = 6$. It is interesting to note that for the given area and the so assumed parameters the variable term f_{iz} in the velocity ratio changes $f_{iz_{\max}} / f_{iz_{\min}} = 155.14$ times.

4. Acknowledgements

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5. Conclusion

The Closed Differential Screw Mechanisms are with relatively simple structure which gives good possibilities for transforming input rotation in linear motion of output link. These mechanisms can be used both for decreasing and increasing output velocity.

The mechanism considered in Figure 4 can be used for low velocity ratios if the gear ratios of the two trains are approximately equal, the pinions are with possible big number of teeth and gear ratios are possibly high. When the gear ratios are small maximal reduction can be obtained if the pinion teeth number is as big as possible. The velocity ratio dependence of pinions teeth number is weak when the gear ratios are high and vice versa.

From formula (15) and Figure 5 it follows that the output velocity of closed differential mechanism can be decreased 1000 times with regard to the output velocity of an ordinary power screw mechanism [9] with the same pitch.

Further investigations of this kind mechanisms show that the mechanical loses are small and mechanical efficiency is relatively high, which is an irrefutable advantage.

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