NONLINEAR PATH CONTROL FOR A DIFFERENTIAL DRIVE MOBILE ROBOT

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Abstract. A nonlinear feedback path controller for a differential drive mobile robot is presented in this paper. First, a kinematic model in error coordinates expressed in a moving reference frame partially linked to the robot is developed. The control law is designed using backstepping method yielding exponential stability of the closed-loop system. Stability analysis is performed via Lyapunov stability theory. Simulation results are presented to illustrate the effectiveness of the proposed controller.

Keywords: differential drive mobile robot, path following, nonlinear control, integrator backstepping

1. Introduction

During the last two decades, the wheeled mobile robots have been increasingly presented in industrial and service robotics. In order to perform a task with a mobile robot, one needs to solve many problems from task planning to motion control design. At control level, important results have been established concerning specific control tasks as point stabilization (the parking problem), trajectory tracking and path following. Beyond the relevance in applications, stabilizing a mobile robot at given posture leads to specific control problems. It is known [1] that feedback point stabilization of nonholonomic systems like wheeled mobile robots can not be achieved via smooth time-invariant control law due to limitations imposed by Brockett's necessary condition [2] for feedback stabilization of such a system. Furthermore, the linearization of a nonholonomic system about any equilibrium point is uncontrollable and consequently, linear analysis and design techniques cannot be applied [3]. The alternative approaches can be classified as smooth nonlinear time-varying feedback stabilization [4] and discontinuous nonlinear time-invariant stabilization [5]. For trajectory tracking and path following tasks, standard linear [6] and nonlinear approaches are effective (feedback linearization [7], Lyapunov-based techniques [8, 9, 10]).

The most common way to build a mobile robot is to use two-wheel drive with differential steering and a free balancing wheel (castor). Controlling the two motors independently, such robots have good maneuvering and work well indoors on flat surfaces. Many commercial platforms based on this locomotion scheme exist, such as the mobile robot Pioneer 3-DX [11] from ROBOSOFT.

In this paper, we present a nonlinear feedback path following controller for a differential drive mobile robot and in particular, with application to mobile robot Pioneer 3-DX [11]. The design procedure is based on integrator backstepping method. Stability analysis is performed via Lyapunov techniques. The paper is organized as follows: In Section 2, the kinematic model of the robot is presented. In Section 3, the path following problem in error coordinates expressed in a moving reference frame partially linked to the vehicle is stated. In Section 4, the design of the proposed controller and stability analysis is given. Simulation results are presented in Section 5. Section 6 contains some conclusions and future work.

2. Kinematic model

The mobile robot Pioneer 3-DX considered in this paper is shown in Figure 1. It is an advanced research robot that can has an on-board PC, a range of sensors like a camera and laser rage finder, and communicates via WiFi (Wireless Ethernet).

Figure 1. The mobile robot Pioneer 3-DX

The kinematic scheme of the robot consists of platform with two driving wheels mounted on the same axis with independent actuators and one free wheel (castor). The mobile robot is steered by changing the relative angular velocities of the driving wheels. It is assumed that the wheels are non-deformable and roll without lateral sliding.

A plan view of the robot moving on a horizontal plane is shown in Figure 2.

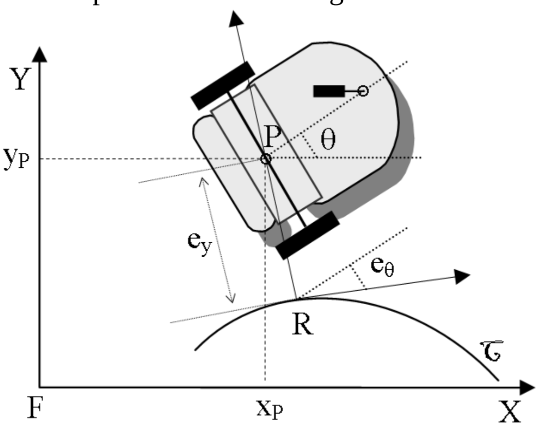

Figure 2. A plan view of the robot

Point P located at the center of the driving wheel axle is used as a reference point of the robot. If the inertia of the wheels with respect to their proper axes is ignored, the configuration of the system can be described by three generalized coordinates

$$q = \begin{bmatrix} x_P \\ y_P \\ \theta \end{bmatrix}, \tag{1}$$

where (x_P, y_P) are the coordinates of point P and θ is the orientation of the robot with respect to an inertial coordinate frame FXY (Figure 2). The system is characterized by the following nonholonomic constraint on the generalized velocities $\dot{q}$

$$A \cdot \dot{q} = 0, \tag{2}$$

where A is a 1×3 matrix as follows

$$A = [-\sin\theta \quad \cos\theta \quad 0]. \tag{3}$$

The mobile robot has two degrees of freedom in the plane. The constraint equation (2) can be converted in an affine driftless control system

$$\dot{q} = C \cdot \eta, \tag{4}$$

where the columns of the 3×1 matrix $C(q)$

$$C = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

form a basis of the null space of matrix $A(q)$. The control inputs η is a 2×1 vector of independent quasi-velocities of the form

$$\eta = \begin{bmatrix} v_{Px} \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_P \\ \dot{y}_P \\ \omega \end{bmatrix}, \tag{6}$$

where v_{Px} is the velocity of point P and $\omega = \dot{\theta}$ is the angular velocity of the robot. The angular velocities of the driving wheels $\dot{\varphi} = [\dot{\varphi}_1, \dot{\varphi}_2]^T$ are related to the quasi-velocities η trough the following expressions

$$\eta = D \cdot \dot{\varphi}, \tag{7}$$

where

$$D = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ -\frac{r}{2d} & \frac{r}{2d} \end{bmatrix} \tag{8}$$

is 2×2 invertible matrix; r is the wheel radius and d is the lateral semi-base of the robot.

3. Problem formulation

The path following geometry used in this paper is presented in Figure 2. Consider a differential-drive robot moving on a flat surface. It is assumed that the path $\mathcal{C}$ is a smooth planar curve. A moving reference coordinate frame Rx_Ry_R is defined such that the x_R axis is tangent to the path and oriented in the direction of motion to follow, and the y_R axis passes through the reference point P of the robot. It is supposed that the distance between points P and R is smaller than the reference curvature radius at point R and in that way, ensuring that the reference path is uniquely defined [4].

Using the frames Pxy and Rx_Ry_R, the position and orientation of the mobile robot with respect to the moving reference frame, i.e., the error coordinates $e = [e_x, e_y, e_\theta]^T$ can be obtained by geometrical projection transformation as follows

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_P - x_r \\ y_P - y_r \\ \theta - \theta_r \end{bmatrix}, \tag{9}$$

where $[x_P, y_P, \theta]^T$ and $[x_r, y_r, \theta_r]^T$ are the position and orientation of the frames Pxy and Rx_Ry_R with respect to an inertial frame FXY.

Differentiating (9) and taking into account the nonholonomic constraints, after some work [12], the error dynamics of the robot is obtained in the form

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\theta \end{bmatrix} = -\begin{bmatrix} v_{Rx} \\ 0 \\ \omega_R \end{bmatrix} + \begin{bmatrix} \cos e_\theta & -\sin e_\theta & 0 \\ \sin e_\theta & \cos e_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{Px} \\ 0 \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & \omega_r & 0 \\ -\omega_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix}, \quad (10)$$

where $\omega \overset{\text{def}}{\equiv} \dot{\theta}$ and $\omega_r \overset{\text{def}}{\equiv} \dot{\theta}_r$. It is assumed that the robot linear velocity $v_{Px}(t)$ is bounded ($\bar{v}_{PxMax} = cte \geq v_{Px}(t)$) and does not converge to zero.

Since the Ry_r axis of frame Rx_Ry_R (Figure 2) passes through the reference point P of the robot, from the first equation of (10), it follows that

$$e_x(t) \equiv \dot{e}_x \equiv 0. \quad (11)$$

Using (11), from the first equation of (10), the reference angular velocity $\omega_r = v_{Rx}/\rho_r$ of the frame Rx_Ry_R can be expressed as a function of the velocity of the mobile robot as follows

$$\omega_r = \frac{c_r \cdot v_{Px} \cdot \cos e_\theta}{1 - c_r \cdot e_y}, \quad (12)$$

where $c_r = 1/\rho_r$ is the curvature of the reference path at point R.

Finally, using (11), from the second and third equations of (10), an error dynamics which is appropriate for path following applications can be derived in the form

$$\begin{aligned} \dot{e}_y &= v_{Px}(t) \cdot \sin e_\theta, \\ \dot{e}_\theta &= e_\omega, \end{aligned} \quad (13)$$

where

$$e_\omega = \omega - \omega_r. \quad (14)$$

with ω_r obtained from (12).

In this case, using the parameterization (e_x, e_y, e_θ) and given a path $\mathcal{C}$, under the assumptions that $v_{Px}(t) \geq \bar{v}_{Px} = cte > 0$ and $|c_r \cdot e_y| < 1$, the path following problem consists of finding a feedback control $\omega = \omega(v_{Px}, e_y, e_\theta, c_r)$ for the system (13) such that

$$\begin{aligned} \lim_{t\to 0} e_y(t) &= 0, \\ \lim_{t\to 0} e_\theta(t) &= 0. \end{aligned} \quad (15)$$

4. Nonlinear control design

In this Section, a nonlinear controller based on a backstepping design method [13] is presented. The control objective is to regulate the lateral and orientation errors (e_y, e_θ) to zero. In our case, the design procedure is completed at the second step by finding a control law which makes the time derivative of the constructed Lyapunov function negative definite.

Consider the system (13) and assume that $|e_\theta| \in [0, \pi/2)$. Using the change of variables $(e_y, e_\theta) \Rightarrow (z_1, z_2)$

$$\begin{aligned} z_1 &= e_y \\ z_2 &= \sin e_\theta \end{aligned} \quad (16)$$

with the input $(e_\omega) \Rightarrow (u)$ transformation

$$u = e_\omega \cos e_\theta, \quad (17)$$

in the new coordinates (z_1, z_2) and input (u), the system (13) is expressed as

$$\begin{aligned} \dot{z}_1 &= v_{Px}(t) z_2 \\ \dot{z}_2 &= u \end{aligned} \quad (18)$$

Step 1. Using the quadratic function

$$W_1 = \frac{1}{2} z_1^2 \quad (19)$$

and a virtual control γ for the first equation of (18)

$$\gamma = -k_1 z_1, \quad k_1 = \text{cte} > 0, \quad (20)$$

the derivative of W_1 becomes

$$\dot{W}_1 = -v_{Px} k_1 z_1^2 < 0. \quad (21)$$

Step 2. Consider an augmented quadratic function

$$W_2 = W_1 + \frac{1}{2}(z_2 - \gamma)^2. \quad (22)$$

The control

$$\begin{gathered} u = -v_{Px} z_1 - v_{Px} k_1 z_2 - k_2 (x_2 + k_1 z_1), \\ k_2 = \text{cte} > 0 \end{gathered} \quad (23)$$

renders the derivative of W_2

$$\dot{W}_2 = -v_{Px}(t) k_1 z_1^2 - k_2 (z_2 + k_1 z_1)^2 < 0 \quad (24)$$

negative definite. The resulting closed-loop system is

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & v_{Px}(t) \\ -(v_{Px}(t) + k_1 \cdot k_2) & -(v_{Px}(t) \cdot k_1 + k_2) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (25)$$

Finally, using the inverse transformations of (16), the actual control ω is obtained from (23), (17) and (14).

The designed controller exponentially

stabilizes the system (16). Indeed, using (22) and (24), and choosing $\alpha = \min(2k_1 \cdot \bar{v}_{Px},\ \ 2k_2) > 0$, the following inequality holds

$$\dot{W}_2 + \alpha \cdot W_2 \le 0\ . \qquad (26)$$

Application of the Convergence Lemma [14], indicates the exponential convergence of W_2 to zero, i.e.,

$$W_2(t) \le W_2(0) \cdot e^{-\alpha t} \qquad (27)$$

and from (16), (19) and (20), this in turn implies that e_y and e_θ converge to zero exponentially.

The performance of the closed-loop system depends of the choice of the controller gains k_1 and k_2. In order to determine appropriate values of the gains, a desired constant velocity $\bar{v}_{Px}$ of the mobile robot during the motion along the desired path is set. In this case, the closed-loop system (25) can be written in the form of a second order differential equation with constant coefficients

$$\frac{1}{\bar{v}_{Px}(\bar{v}_{Px} + k_1 k_2)}\ddot{z}_1 + \frac{\bar{v}_{Px}k_1 + k_2}{\bar{v}_{Px}(\bar{v}_{Px} + k_1 k_2)}\dot{z}_1 + z_1 = 0 \qquad (28)$$

Given a desired settling time t_s defined as the time required for the step response to decrease and stay within a 5% of its final value, in the case of two identical real roots (damping ratio $\zeta = 1$) of the characteristic equation, one has

$$t_s \approx 5T\ , \qquad (29)$$

where

$$T = \sqrt{\frac{1}{\bar{v}_{Px} \cdot (\bar{v}_{Px} + k_1 \cdot k_2)}} \qquad (30)$$

is the time-constant of (28).

Using (28) and (29), the gains k_1 and k_2 can be expressed as functions of $\bar{v}_{Px}$ and t_s as follows

$$k_1 = \frac{5}{\bar{v}_{Px} t_s} \pm 1 \qquad k_1, k_2 > 0. \qquad (31)$$
$$k_2 = \frac{5}{t_s} \pm \bar{v}_{Px}$$

5. Simulation results

Simulation results were performed to illustrate the effectiveness of the proposed controller. The control law was implemented using MATLAB.

A circular reference path with radius 2.5 m was chosen for the simulations. The velocity of the vehicle was $v_{Px}(t) = (1 + 0.5 \cdot \sin(0.1t))$, in m/s. The wheel base was 0.5 m, as that of mobile robot Pioneer 3-DX. According to (30), the gains were chosen to be $k_1 = k_2 = 2.25$, ($\bar{v}_{Px} = 1$ m/s, $t_s = 4$s).

The simulation results of the planar path and time plots of the error coordinates (e_y, e_θ) with initial conditions $e_y(0) = 1$ m and $e_\theta(0) = 0$ rad are depicted in Figure 3 and Figure 4, respectively.

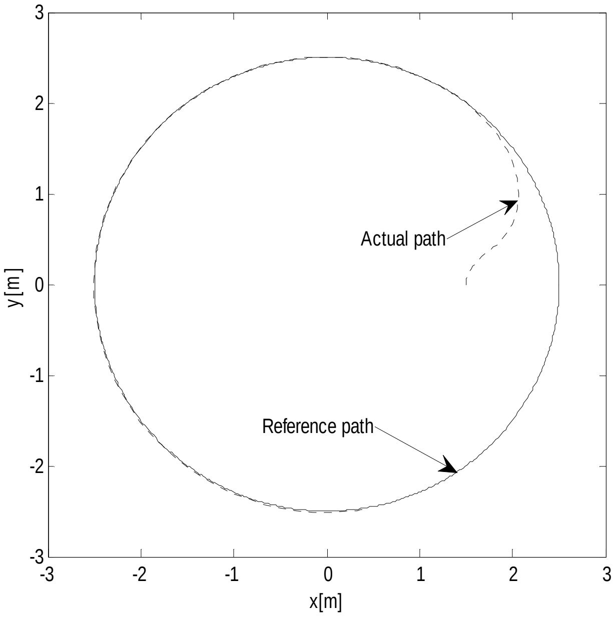

Figure. 3. Following a circular path

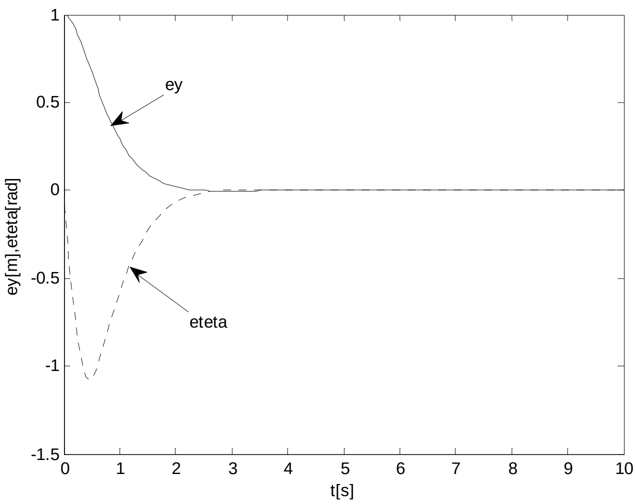

Figure. 4. Evolution in time of the error coordinates (e_y, e_θ)

6. Conclusion and future work

In this paper, a kinematic nonlinear feedback path controller for a differential-drive mobile robot has been presented. The control design procedure is based on backstepping techniques and involves error coordinates in a moving reference frame partially linked to the robot.

An exponential convergence to zero for the error coordinates is established.

The simulation results have shown the effectiveness of the proposed control law.

Future work will address the problems associated with the dynamic extension of the proposed controller in the presence of uncertainty due to unmodeled dynamics of the robot.

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