AN ANALYSIS OF THE DYNAMIC BEHAVIOUR OF THE LINEAR HYDRAULIC MOTOR – NON-LINEAR/LINIARIZED CUTTING FORCE SYSTEM

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Abstract. The work is intended as a numerical simulation analysis of the dynamic behaviour of the linear hydraulic motor under cutting force strain. The case analysed is that of a linear hydraulic motor ensuring the longitudinal advance motion during cutting operation. The analysis focuses on two distinct situations: the non-linear expression of the cutting force and the linear expression of the cutting force, respectively. In conclusion, the conditions for acceptance of the errors resulting from the linear expression of the cutting force are highlighted.

Keywords: dynamics, simulation, hydraulic motor, cutting force

1. Introduction

The analysis by numerical simulation of a system presupposes its mathematical modelling based on the mathematical modelling of its composing subsystems. In most cases, the real systems feature non-linearity, which cannot be neglected. If this non-linearity is considered for mathematical modelling, this leads to the numerical simulation analysis of the real dynamic behaviour of the system. However, such an approach has a disadvantage in as much as the system theory does not offer the optimization methodology for the optimization of the dynamic behaviour of the system. In the case of linear systems, there are theoretical methods of optimization of their dynamic behaviour [4]. Therefore, to optimize the dynamic behaviour of systems as early as in the synthesis stage it is recommended to perform a linearization of their respective mathematical models. The linearization presumes an approximation of the description of the respective system and imposes a clear definition of the applicability domain in accord with the accepted approximation errors.

2. Mathematical modelling of linear hydraulic motor – cutting process system

In case of the linear feeding hydraulic motor, the hydraulic motor and the cutting process, respectively represent the two subsystems.

The input value of the system will be considered as the supply rate of the motor Q_M whereas the output value can be considered the cutting force F_{RM} or the displacement speed of the hydraulic motor v_M, the feeding speed of the lathe cutting tool, respectively.

2.1. Mathematical model of the linear hydraulic motor

The feeding hydraulic motor is considered as being symmetrical with equal bilateral rod to ensure a symmetrical behaviour in both displacement directions. As demonstrated by the specialized literature, the symmetrical, linear hydraulic motor, free of any internal and external leaks, behaves like a **P2** element. The dependency of the output value, motor displacement speed v_M, on the input value, supply rate Q_M to the motor and the disturbing input value, the resisting force at motor F_{RM} is expressed by the relation below [1, 2]:

$$v_M(s) = \frac{K_M}{\frac{s^2}{\omega_M^2} + 2\cdot\frac{\zeta_M}{\omega_M}\cdot s + 1} \times \left[Q_M(s) - T_F \cdot s \cdot F_{RM}(s)\right] \quad (1)$$

and is represented by the block diagram shown in Figure 1.

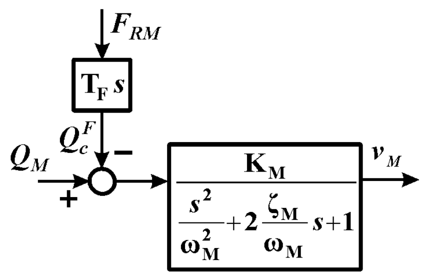

Figure 1. Block diagram of the hydraulic motor

The following notations are used in this relation: K_M – amplification coefficient, T_F – time constant, ω_M – natural pulsation, and ζ_M – damping factor.

2.2. Mathematical model of the cutting process

In this modelling, is considered that the longitudinal turning process is executed with a cutting tool having a setting angle of 90^o (Figure 2).

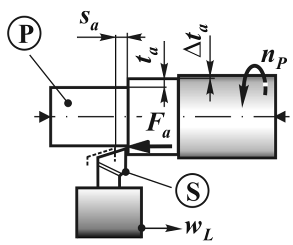

Figure 2. The longitudinal turning

Given these conditions, the cutting process is described by the expression of the advance force during the longitudinal turning [3]:

$$F_{aL} = c_F \cdot t_a^x \cdot s_a^y \cdot HB^n \tag{2}$$

where c_F – cutting force constant, and x, y and n represent the cutting depth t_a, longitudinal advance s_a and HB hardness of the part. Taking into account the relation between the advance speed $w_L = v_M$ and the longitudinal feed, the relation (2) changes as follows:

$$F_{aL} = c_F \cdot t_a^x \cdot \left(\frac{v_M}{n_p} \right)^y \cdot HB^n . \tag{3}$$

This relation singles out the dependency of the cutting force on the motor rpm v_M and part rpm n_p and it is shown in Figure 3 as a simulation diagram drawn in Matlab Simulink programming system.

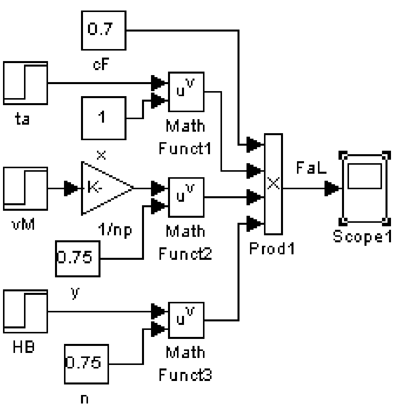

Figure 3. Simulation diagram, non-linear F_{aL}

The linearization of the expression of the cutting force, obtained by developing in Taylor series [2]:

$$\underline{F_{aL}} = (1 - y) \cdot F_{aL0} + K_{Fw} \cdot w_L + \Delta F_{ap} . \tag{4}$$

is represented as a block diagram in Figure 4. In this relation, the following notations were used:

$F_{aL0} = c_F \cdot t_{a0}^x \cdot s_{a0}^y \cdot HB_0^n$ – steady state advance force associated to parameters t_{a0}, s_{a0} and HB_0, $K_{Fw} = y \cdot \frac{F_{aL0}}{w_{L0}}$, while $\Delta F_{ap} = K_{Ft} \cdot \Delta t_a + K_{FH} \cdot \Delta HB$ is the disturbing component of the cutting force, generated by the variation of the cutting depth Δt_{a0} during cutting process and of the hardness ΔHB of the processed material.

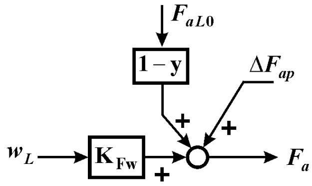

Figure 4. Block diagram, linearized $\underline{F_{aL}}$

The mathematical modelling of the system is obtained by associating the mathematical models of the composing subsystems, in the field of time or complex. The non-linear model is obtained by using relation (3) whereas for the linear model relation (4) shall be used.

3. Numerical simulation of the dynamic behaviour of the linear hydraulic motor – cutting process system

The numerical simulation of the dynamic behaviour of the system is performed in Matlab-Simulink [5] programming system for the following numerical values of the constructive – functional parameters of the two subsystems:

- linear hydraulic motor: cylinder diameter D_M = 50 mm, rod diameter d_M = 30 mm, ω_M = 485 rad/s, ζ_M = 0.034, K_M = 560 m^{-2}, $T_F = 2.5 \cdot 10^{-10}$ s.
- longitudinal turning process: c_F = 0.7, s_{a0} = 0.8 mm/rot, t_{a0} = 2 mm, HB_0 = 180.

The response of the system is analysed for the following work phases:

- movement initialization with no cutting force, cutting start (force stage equal with F_{aL0}= 582 N) after stabilizing motor displacement speed,
- occurrence of a disturbance under the form of cutting force step caused by a cutting depth stage Δt_a = 1 mm, and a stage of the hardness of the processed component part ΔHB = 20 after the previous transient state stops.

3.1. Numerical simulation of the linear hydraulic motor – non-linear cutting force system

Figure 5 shows the response of the system obtained by numerical simulation based on the simulation diagram shown in figure 6 drawn up in compliance with relation (3) and the aforementioned simulation hypotheses. The continuous line represents the evolution in time of the advance speed, of the hydraulic motor, respectively. The starting of the system is accompanied by an oscillating transient state, which lasts for about 0.25 seconds.

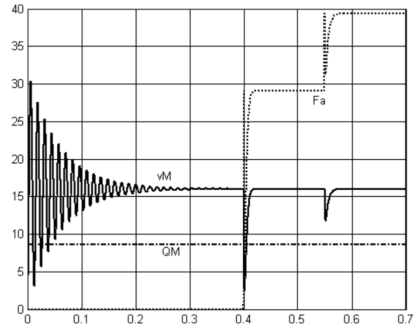

Figure 5. Response of the system, non-linear F_{aL}

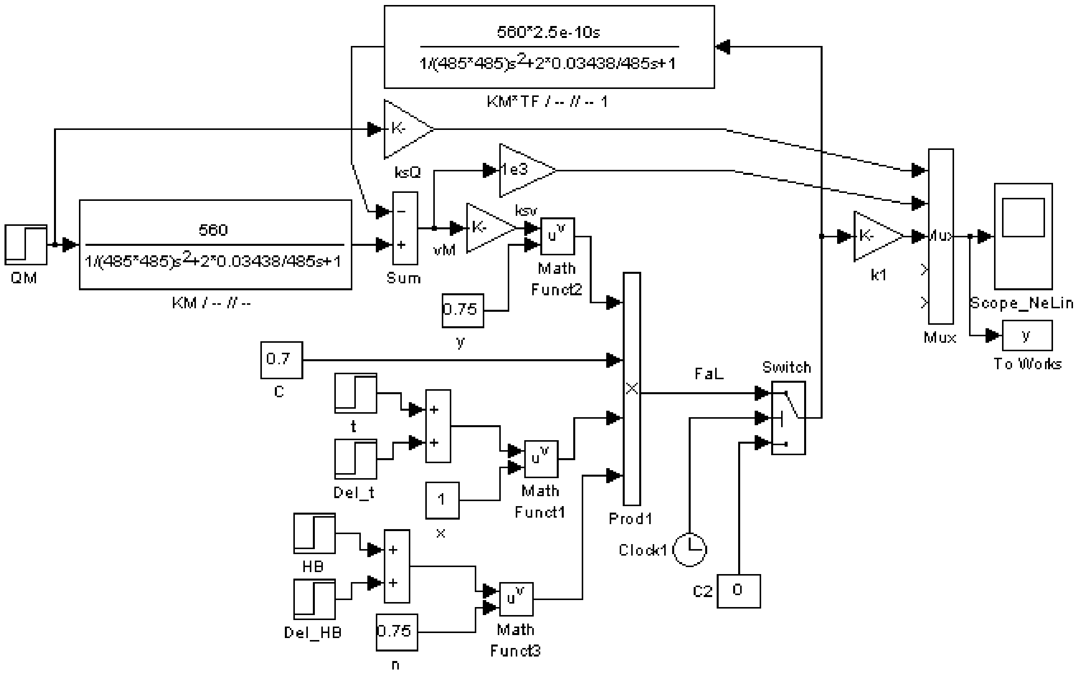

Figure 6.Simulation diagram of the system, for non-linear

On initializing the cutting process and on the occurrence of the force disturbing stage the rpm suddenly drops only to immediately resume its steady state value associated to the supply rate Q_M. The duration of each of these transient states is of percentiles of a second.

The evolution in time of the cutting force is represented by dots. The system is idle started and is determined by the constant supply flow rate of the hydraulic motor (flow rate stage Q_M represented by dash-dot line). The transient states associated to the cutting input, the occurrence of variations in the cutting depth and hardness of the processed part are of a very short duration. The reason for this is the fact that the cutting force has a strong damping effect. The instantaneous increase of the cutting force is immediately followed by a sudden drop in the force triggered by the drop in the rpm of the advance motor. In compliance with the constant flow rate supply, the rpm rapidly resumes the steady state value.

Therefore, the force resumes its steady state values corresponding to values t_{a0}, s_{a0} and HB_0, of the cutting depth, advance and hardness and, subsequently corresponding to the stage disturbances of the cutting depth and hardness disturbances.

3.2. Numerical simulation of the linear hydraulic motor – linearized cutting force

Figure 7 shows the simulation diagram associated to the system where the block diagram of the linearized cutting force corresponding to relation (4) can be seen.

This relation explicitly singles out the cutting force viscous component $K_{Fw} \cdot v_M$. This composing element of the cutting force contributes in a decisive manner to the damping of the system, to the reduction of the transient state duration, respectively (figure 8).

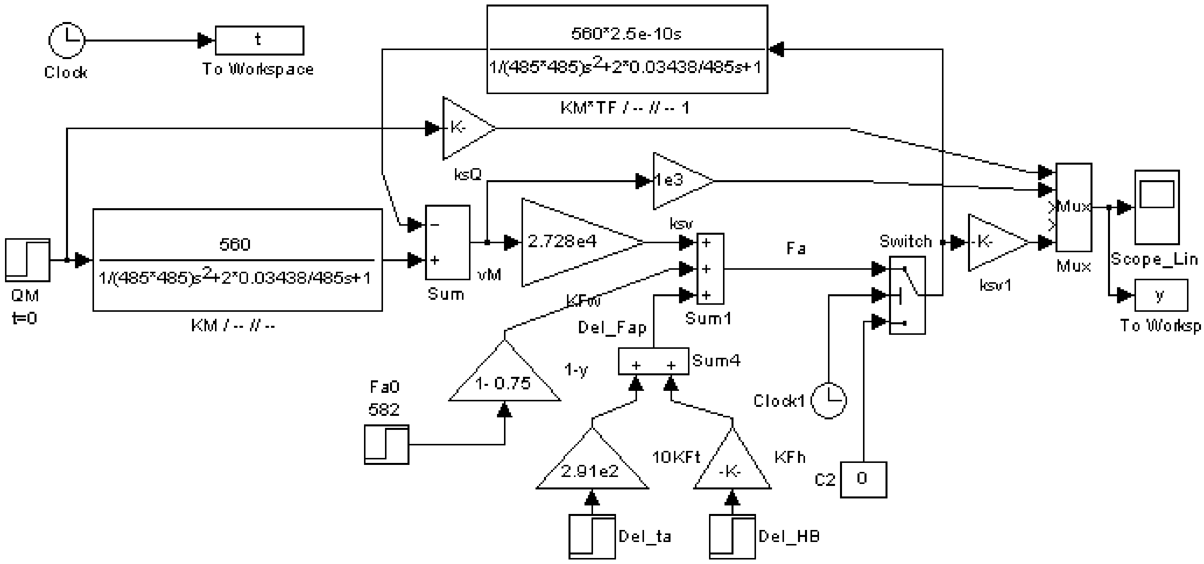

Figure 7. Linearized system simulation diagram

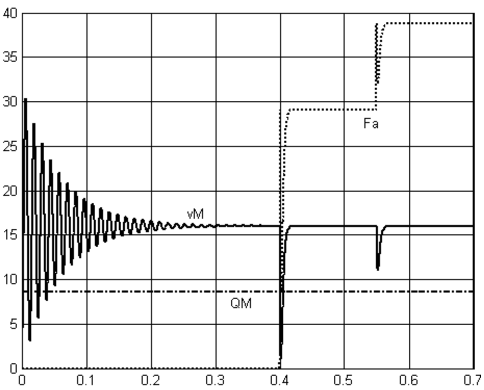

Figure 8. Linearized system response

The comparative analysis of the two responses allows highlighting the following differences as compared with the case of the non-linear expression of the cutting force:

– Somewhat shorter duration of the transient states,

– Higher speed overshoot and lower force overshoot.

4. Conclusions

The numerical simulation singled out a series of relatively small differences between the behaviour of the system, including the non-linear expression of the cutting force and the linearized cutting force, respectively. The transient states are fairly comparable, with bigger differences being recorded between the steady state force values. These differences get bigger as the functioning point is farther from point zero around which the linearization was performed.

To conclude, in the system synthesis phase and in case the operating domain is close to the reference point it is recommended to use the linearized mathematical model. This model is much more convenient to use and allows applying the system optimization methodology. On the contrary, it is recommended to use the non-linear model in the analysis phase as it depicts the real dynamic behaviour of the system.

References

1. Ogata, K.: *Modern Control Engineering*. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, USA, 1990

1. Bălăşoiu, V., Cristian, I., Bordeaşu, Il.: *Echipamente şi sisteme hidraulice de acţionare şi automatizare* (*Hydraulic equipments and systems for command and automation*). Vol. 2. Ed. Orizonturi Universitare, Timişoara, 2008, ISBN 978-973-638-349-6 (in Romanian)

2. Cristian, I.: *Modelarea şi simularea sistemelor mecanice* (*Modelling and simulation of mechanical systems*). Ed. Universităţii Tansilvania din Braşov, 2004 ISBN 973-635-160-2 (in Romanian)

3 Deliu, M.: *Prelucrări mecanice* (*Mechanical processing*). Vol. II: *Prelucrări prin aşchiere* (*Metal cutting*). Ed. Universităţii Transilvania din Braşov, 2007 ISBN 978-973-598-020-7 (in Romanian)

5. ***: *MATLAB 6.5 - The Language of Technical Computing*. MathWorks, Natick, Massachusetts, USA

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