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DENAVIT HARTENBERG ANALYSIS OF A REHABILITATION EQUIPMENT

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Abstract. Denavit – Hartenberg method is the most used instrument of robots modeling. It determines the position and the orientation of the end-effector of a robot. In this paper the method is used for kinematic modeling of a rehabilitation equipment, designed for the recovery of the patients with hip and knee affections. It will be determined the displacement of the slide realized during the continuous passive motion of the leg. Studied knee and hip rehabilitation equipment can be modeled as an open loop articulated kinematic chain, with elements connected in series by rotation and translation joints driven by actuators. The kinematic chain of the equipment is composed by two elements connected by three rotation and one translation joints, actuated by a linear actuator. The linear actuator used for the action of the equipment is a FESTO pneumatic muscle. One end of the chain is connected to a support base and the other is free and performs a translational motion that represents the slider displacement of the equipment. The slider displacement is of 300 mm between the positions that correspond to the relaxed and contracted state of the pneumatic muscle.

Keywords: rehabilitation equipment, Denavit Hartenberg analysis, kinematic chain

1. Introduction

The paper presents a rehabilitation equipment modeling, using Denavit-Hartenberg method. The equipment is considered an open loop articulated kinematic chain, with two elements connected in series by three rotation and one translation joints driven by a linear actuator.

The linear actuator used for the action of the equipment is a FESTO pneumatic muscle. The positions of the slide correspond to the relaxed and contracted state of the pneumatic muscle.

Denavit-Hartenberg method proposes the determination of the position and orientation of the slide, with the given values for the joint variables of the equipment.

The results of the paper are homogeneous matrixes for the contracted and relaxed state of the muscle, with the position and orientation of the slide.

A future direction in this research is an in depth analyze of the equipment performances in laboratory.

2. Problem statement

The rehabilitation equipment presented in the figure below allows performing isokinetic exercises in order to offer continue passive motion for recovery of the patients with affections of the hip and knee joints. Because it is actuated by pneumatic muscle, the equipment performs a smooth move of the leg during the isokinetic exercise.

To perform the continuous passive motion, the equipment must realize a specific displacement of the slider.

The objective of the analysis is to determine the displacement realized by the slider, with the use of real values of the components (bar lengths and the rotation angles the hip joint and the knee joint).

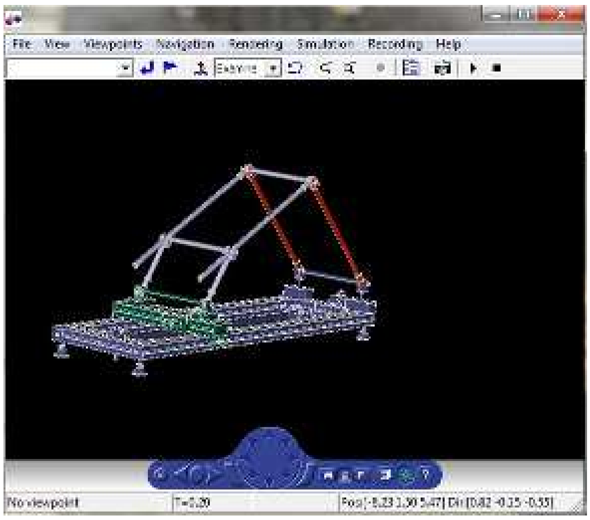

Figure 1 Knee and hip rehabilitation equipment

List of main components of the equipment and their parameters is presented in Table 1.

Table 1.Component description

Components	Characteristics
Base	Al profile 40x40 FESTO
Articulated bar 1	Al bar Ø16, l = 450 mm
Articulated bar 2	Al bar Ø16, l = 400,23 mm
Articulated bar 3	Al bar Ø16, l = 168 mm
Slide	Al profile 80x40
3x rotation joint	Z axis rotation
1x translation joint	X axis translation

The bar 3 is welded with the bar 2 at 150° angle, so, for the kinematic analysis will be considered an articulated bar of length 552.16mm, which connects the slider with the rotation joint 2.

3. Denavit - Hartenberg method

Denavit – Hartenberg method was introduced in 1955, being the most used instrument of robots modeling. Its advantage consists in the reduced number of parameters for switching from one reference system to another.

The D-H model of representation is a very simple way of modeling robot links and joints that can be used for any robot configuration [1].

Denavit-Hartenberg method involves attaching one coordinate axes system to each link of a kinematic chain. After the coordinate axes system attachment can be determined the standard 4x4 homogeneous transformation matrix between two links. [2, 3]

Stages of implementation of the method are:

a) Numbering the elements - from basic to final effectors;

b) Assigning coordinate system;

c) Establishing DH connection parameters;

d) Calculating the transformation matrix from „*i*" element to „*i-1*" element;

Studied rehabilitation equipment can be modeled as an open loop articulated kinematic chain, with elements connected in series by rotation or translation joints driven by actuators. One end of the chain is connected to a support base and the other is free and performs a translational motion that represents the equipment slider displacement.

4. Methodology

4.1 Numbering the elements

Figure 2 present the analyzed kinematic chain and the coordinate systems attached to each link for the rehabilitation of knee and hip equipment.

4.2 Frame assignment

To determine and establish each coordinate system were taken into account three main rules [4]:

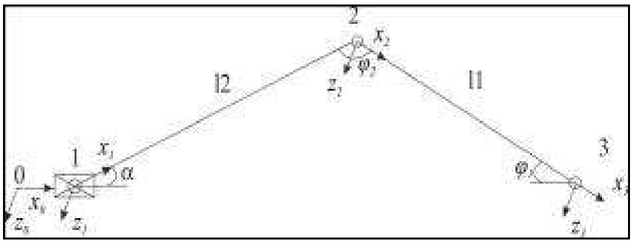

Figure 2 Kinematic chain of the proposed equipment

- z_i axis coincides with the rotation axis of *i* joint,
- x_i axis is perpendicular to the axis z_i and in the same direction as the axis of the base system
- y_i completes the coordinate system on the right hand rule.

Equation 1 is the general form of the translation matrix for a combination of joints where ${}^{i-1}_{i}T$ is the translation of the system from ***i-1*** joint to *i* joint [5].

$$ {}^{i-1}_{i}T = {}^{i-1}_{R}T \cdot {}^{R}_{Q}T \cdot {}^{Q}_{P}T \cdot {}^{P}_{i}T \qquad (1)$$

A set of intermediate transformations P, Q, R is defined for a link between *i-1* and *i* joint. The R system differs from the *i-1* system through a rotation with α_{i-1}. The Q system differs from R system by a translation with a_{i-1}. The P system differs from Q system by a rotation with θ_i and the *i* system differs from P system by a translation with d_i [5]. Matrices corresponding to the four transformations are:

$$ {}^{i-1}_{R}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

$$ {}^{R}_{Q}T = \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

$$ {}^{Q}_{P}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

$$ {}^{P}_{i}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

In all matrixes above or below there is:

$c\theta_i = \cos\theta_i$, $s\theta_i = \sin\theta_i$, $c\alpha_{i-1} = \cos\alpha_{i-1}$, $s\alpha_{i-1} = \sin\alpha_{i-1}$.

Thus, the matrix which describes the general transformation is: [5]

$$
{}^{i-1}_{i}T = \\
= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i \cdot c\alpha_{i-1} & c\theta_i \cdot c\alpha_{i-1} & -s\alpha_{i-1} & -d_i \cdot s\alpha_{i-1} \\ s\theta_i \cdot s\alpha_{i-1} & c\theta_i \cdot s\alpha_{i-1} & c\alpha_{i-1} & d_i \cdot c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)
$$

This transformation can be defined as the concatenation of two matrixes, one of orientation (4x3) and a position vector (4x1) [3].

$$
T = \begin{bmatrix} \text{orientation} & \text{position} \\ \text{matrix} & \text{vector} \\ (4\times 3) & (4\times 1) \end{bmatrix} \quad (7)
$$

4.3 DH Parameters

The four parameters DH - two link parameters (a_i, α_i) and two joint parameters (d_i, θ_i) are defined as follows:

(1) link length (a_i) - The distance measured along the axis x_i of the point of intersection of the x_i axis with z_{i-1} axis to the origin of joint i;

(2) link rotation (α_i) - The angle between z_{i-1} and z_i axes measured about x_i axis and to the right hand sense;

(3) distance between joints (d_i) - The distance measured along z_{i-1} axis from the origin of *i-1* joint to the intersection of x_i axis with z_{i-1} axis;

(4) joint angle (θ_i) - The angle between x_{i-1} axes and x_i axis measured from z_{i-1} axis in the right hand sense.

The four parameters describe the relative pose of a link relative to its predecessor [6].

The connecting parameters for kinematic chain shown in figure 2 are presented in table 2. The values are different for each link between 0-1, 1-2 and 2-3 joints.

Table 2 Connecting parameters for kinematic chain

	R_x	T_x	T_z	R_z
i	α_{i-1} [degrees]	a_{i-1} [mm]	d_i [mm]	θ_i [degrees]
0-1	0	0	0	360-α
1-2	0	l2	0	180-φ_2
2-3	0	l1	0	180-φ_1

4.4 Kinematic model

The transformation matrix between element *i* and element *i-1*can be found with equation (8), where ${}^0_1T$, ${}^1_2T$ and ${}^2_3T$ are successive transformations between element 0 and element 3.

$$
{}^0_3T = {}^0_1T \times {}^1_2T \times {}^2_3T \quad (8)
$$

where

$$
{}^0_1T = \begin{bmatrix} c(360-\alpha) & -s(360-\alpha) & 0 & 0 \\ s(360-\alpha) & c(360-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} 1 & 0 & 0 & l2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \\
\cdot \begin{bmatrix} c(180-\varphi_2) & -s(180-\varphi_2) & 0 & 0 \\ s(180-\varphi_2) & c(180-\varphi_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)
$$

$$
{}^2_3T = \begin{bmatrix} 1 & 0 & 0 & l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \\
\cdot \begin{bmatrix} c(180-\varphi_1) & -s(180-\varphi_1) & 0 & 0 \\ s(180-\varphi_1) & c(180-\varphi_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

5. Results

Developed model is used to determine position and orientation of the equipment slide.

For values l_1 = 450 mm, l_2 =552.16 mm, $\varphi_1 = 29.12...61.51^\circ$, $\varphi_2 = 72.7644...127.5528^\circ$, $\alpha = 23.3272...4.7256^\circ$ $(\alpha = 180 - (\varphi_1 + \varphi_2))$, the obtained results is shown below:

Transformation matrix for link 1, α = 23.3272º:

$$
{}^0_1T = \begin{bmatrix} c(336.6728) & -s(336.6728) & 0 & 0 \\ s(336.6728) & c(336.6728) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^0_1T = \begin{bmatrix} 0.9182 & 0.3959 & 0 & 0 \\ -0.3959 & 0.9182 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)
$$

Transformation matrix for link 1, α = 45,7256º:

$$
{}^0_1T = \begin{bmatrix} c(314.2744) & -s(314.2744) & 0 & 0 \\ s(314.2744) & c(314.2744) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^0_1T = \begin{bmatrix} 0.698 & 0.716 & 0 & 0 \\ -0.716 & 0.698 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)
$$

Transformation matrix for link 2, $\varphi_2 = 72.7644^\circ$:

$$ {}_2^1T=\begin{bmatrix}1&0&0&552.16\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}c(107.23)&-s(107.23)&0&0\\s(107.23)&c(107.23)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{12}$$

$$ {}_2^1T=\begin{bmatrix}-0.2963&-0.955&0&552.16\\0.955&-0.296&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Transformation matrix for link 2, $\varphi_2=127.5528^\circ$:

$$ {}_2^1T=\begin{bmatrix}1&0&0&552.16\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}c(52.44)&-s(52.44)&0&0\\s(52.44)&c(52.44)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{13}$$

$$ {}_2^1T=\begin{bmatrix}0.6094&-0.7927&0&552.16\\0.7927&0.6094&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Transformation matrix for link 3, $\varphi_1=29.12^\circ$:

$$ {}_3^2T=\begin{bmatrix}1&0&0&450\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}c(150.88)&-s(150.88)&0&0\\s(150.88)&c(150.88)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{14}$$

$$ {}_3^2T=\begin{bmatrix}0.8736&-0.4866&0&450\\0.4866&0.8736&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Transformation matrix for link 3, $\varphi_1=61.51^\circ$:

$$ {}_3^2T=\begin{bmatrix}1&0&0&450\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}c(118.49)&-s(118.49)&0&0\\s(118.49)&c(118.49)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{15}$$

$$ {}_3^2T=\begin{bmatrix}0.477&-0.8789&0&450\\0.8789&0.477&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Homogeneous matrix ${}_3^0T$ becomes:

- For contracted state

($\varphi_1=61.51^\circ$, $\varphi_2=72.7644^\circ$, α = 45,7256º):

$$ {}_3^0T=\begin{bmatrix}-0.5449&-0.8384&0&600.1128\\0.8384&-0.5449&0&0.156\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{16}$$

- For relaxed state

($\varphi_1=29.12^o$, $\varphi_2=127.5528^o$, α = 23,3272º):

$$ {}_3^0T=\begin{bmatrix}-0.5263&-0.8502&0&900.1467\\0.8502&-0.5263&0&0.3430\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{17}$$

6. Conclusion

Rotation matrix (the first 3 x 3 positions) of the transformation matrix contains the orientation of coordinate system 3 relative to the first system, the base system.

The first three entries in the last column of the transformation matrix represents *x*, *y* and *z* components of the origin O_3 to the base coordinate system.

Position obtained is (600.1128, 0.12560, 0) for the contracted state and (900.1467, 0.3430, 0) for the relaxed state, in a spatial coordinate system with origin in the translation joint.

Thus, it results the displacement of the joint 0 to joint 3(slider displacement) over a distance of 300.0339 mm, between 600.1128 mm in contracted state and 900.1467 mm in relaxed state (with an error of 0.0339 mm).

On the y_0 direction the position is of 0.1256 mm in contracted state and of 0.3430 mm in relaxed state.

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