

QUATERNION ACCURACY MODEL OF INDUSTRIAL ROBOTS'

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Abstract. The article describes existing methods and approaches to industrial robots accuracy simulation based on the matrix methods at multiply industrial robots' motions. The main problems and disadvantages of using these existing approaches based on the analysis of informational sources also shown. Quaternion-based method of industrial robot kinematics and accuracy simulation proposed to decrease these disadvantages for stochastic simulation at multiply motion of industrial robots' grippers. Forward kinematics task and Forward accuracy task solving using the proposed quaternion-based method described also. The Industrial robots' quaternion accuracy model proposed to use as a basis of accuracy simulation for examine their working space.

Keywords: industrial robots, accuracy, quaternions, industrial robots' accuracy model

1. Introduction

Pose accuracy is complex concept. It primarily defined by errors' value which has been defined by errors in industrial robots' (IR) links joints. Forward kinematics task (FKT) solution is prerequisite of successful pose accuracy definition, in particular the position of end-effector coordinate system (local coordinate system) in the Industrial Robot coordinate system (global coordinate system). Industrial robots' grippers (IRGr) are the end-effector for mechanical assembly and technological service of flexible manufacturing cell working position.

To improve understanding of solving problems essence is expedient to divide movements of industrial robots' handling system links and industrial robots generally. It is proposed to distinguish ideal and real movements (Figure 1).

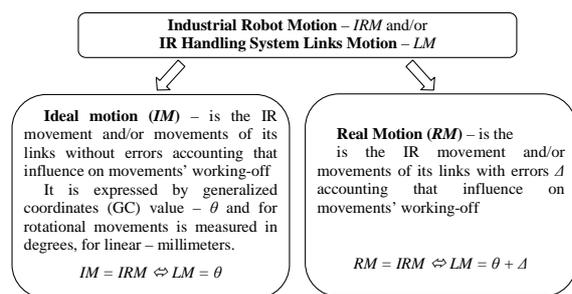


Figure 1. The essence of industrial robots movements

2. Methods of industrial robots' accuracy simulation and problems of their using

FKT can be solved by different methods, for example, Denavit-Hartenberg, using rotation matrix and Euler angles which are extended for accuracy characteristics research of additional variable parameters which are reproduced errors random appearance and also used for accuracy simulation of

worldwide researchers [1 ÷ 21]. For today there are known different descriptions of IR movements' such as matrix which are use homogeneous transformations and Euler angles [1 ÷ 21] and also quaternions [9, 22 ÷ 27].

Detailed analysis of these descriptions denotes that the large majority of researchers use matrix methods for IR kinematics description but quaternions used by fewer researchers and only for IR ideal movements' description.

Using of mentioned classical matrix mathematical apparatus for FKT solving, for description of the local coordinate systems position in the global coordinate systems have some obvious disadvantages which are substantively complicates their usage namely IRGr pose accuracy definition at multiple motion into i -th ($i = \overline{1, I}$, where I – total amount of point of IR working space) point working space which coordinates is given by IR control system.

Especially appreciable complications during probabilistic investigations of IRGr pose errors random components forasmuch including errors value of links joints to homogeneous transformation matrix which are array of the certain random variables and displaying IRGr multiple motion to i -th point of IR working position. Said increases in K times digit capacity of data arrays, where K – number of movements into one i -th of IR working space.

The multiple motion into i -th point working space accounting lead to durable, many operations, requiring the expenditure of much labour calculations under the multidimensional arrays $4 \times 4 \times 100$ and it is necessary to perform near the 16.777216^{18} operations [21 ÷ 24, 28, 29, 30] for one point of Working space (WS) to form the array of homogeneous transformations IR handling system

(HS). The investigations of other points of carrying similar calculations, which in turn leads to the increasing of quantity the calculations in I times, where I is number of IR WS analyzed points.

Moreover classical mathematical apparatus [1, 2, 4, 6, 7, 9, 10] that are used to describe the mutual movements and locations of the IR HS links complicate or don't allow to create a mathematical model for the case when the direction of the link's axis does not coincide with the main axes direction of the accepted coordinate system ($X^{IR}Y^{IR}Z^{IR}$).

As an alternative to the mentioned it is proposed to use of the mathematical apparatus of quaternions, quaternion operations a similar to the vectors and they are less time-consuming and lengthy if to compare with matrix operations.

For example, the operation of multiplication of two quaternions only which intentionally reflects the rotary motion of the IR HS link relative to the specific axis IR CS requires from the CPU 12 multiplications and 35 additions. However, it requires 27 multiplications and 18 additions during the operation using the rotation matrix. Considering that processor spends only 1 cycle on the operation of addition and subtraction and multiplication is performed for 4 cycles, it can be determined that multiplication of two quaternions is for 83 cycles and the multiplication of two rotation matrixes is for the 126 cycles. In consideration of the multidimensionality of data arrays (1 by 4 by 100) that reproduce randomness of the occurrence of errors in IR HS links joints, the shown cycles quantity increases in hundreds times.

One of the possible approaches to the mentioned avoid is to use of the mathematical apparatus of quaternion as an information basis for the IR HS accuracy model. It makes possible to describe changes of the mutual movement of IR HS links in a terms of generalized coordinates in positions that don't coincide with the axes of the basic previously accepted CS and to reduce the operating load on the CPU in the automated processing of IR HS accuracy models that in turn leads to the reduction of costs for the acquiring of modern expensive computers.

Besides this a functional model IR HS [30] using in which angular and linear IR HS links motions are described by angular and linear quaternions respectively from the unified system positions it is appropriate to introduce a description of the errors in the HS links' joints and total pose error of the IR HS characteristic point (the pole P_{Gr} IRGr) using mathematical apparatus of quaternions.

Preliminary analysis of information sources [1, 2, 5, 9, 10, 25, 26, 31] in which a description of the HS link's kinematic features with using of different mathematical apparatus is carried out and also [11 ÷ 15] investigations of the IR pose errors and identified earlier [4, 5, 13 ÷ 16,] that the IR error is a vector which has angular and linear components and allows to replace the classic view of IRGr errors in matrix form (1) by the corresponding quaternion analog (6) in the case of angular motion errors and (7) in the case of errors of linear movements.

$$\Delta_{l-1}^l = \begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & \delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & \delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where:

- $\varepsilon_x, \varepsilon_y, \varepsilon_z$ – the errors of angular movements around the corresponding axes;
- $\delta_x, \delta_y, \delta_z$ – the errors of linear movement along the corresponding axes.

To decrease operational load during computer simulation of kinematical features IR HS links that consider random components of errors in the IR HS links joints and from the unified system positions it is proposed to use of mathematical apparatus of quaternions to describe the errors of IR HS motions. That's allowed to carry out not only deterministic simulations but also stochastic systems and objects that in turn expands the area of IR HS kinematic structures quaternion descriptions using.

3. Quaternion accuracy model

Using the mathematical apparatus of quaternions that is used to describe the objects motions in three-dimensional space in relative to arbitrary vector, that can coincide or not with the IR CS axes, there is proposed the introduction of the so-called Q-model of IR HS accuracy characteristics which is a description of the errors in the IR HS links joints using the specified mathematical apparatus of quaternions.

Q-model of IR HS accuracy characteristics envisages the introduction of the concept of a Generalized Quaternion of active links' movements (QLM) from the number of activated (Figure 2), which is a function from the quaternion of ideal link's movement and the quaternion of errors that occurs during the links move:

$$QLM = f(Q, EQ), \quad (2)$$

where:

- Q – the type and direction of constructive and

certain ideal movements of each l -th link in relative to the previous $(l-1)$ -th link in terms of quaternions and generally is denoted as $Q_{l-1}^l(s; v) \in (LIN_{l-1}^l, ROT_{l-1}^l)$, here is the LIN_{l-1}^l – the quaternion of linear motions of IR HS links; ROT_{l-1}^l – quaternion of angular IR HS links motions;

EQ – quaternion that reflects the error of working-off generalized coordinates (GC) in joints of active IR HS links and similar to the type of motion in joints of links MS PR are generally marked as $EQ_{l-1}^l(s; v) \in (\varepsilon_{l-1}^l, \delta_{l-1}^l)$, here is ε_{l-1}^l – quaternion of angular errors; δ_{l-1}^l – quaternion of linear errors.

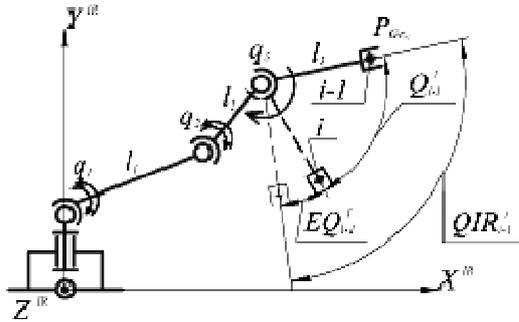


Figure 2. A graphical representation of the essence of quaternion rotational motion during the moving IRGr from $(i-1)$ -th to the i -th point of WS

In turn QLM each link reflects the real IR HS links motion and depends on the amount of movement of a certain kind of motion the l -th link relative to previous $(l-1)$ -th link – Q_{l-1}^l , and the error's value of in the joints of these links – EQ_{l-1}^l at working-off the required for positioning IRGr in the i -th point IR WS GC by the expression (2) and it is also a quaternion product of Q and EQ that is determined by the expression (3)

The specification of the quaternion motion QLM of l -th link relative to the previous $(l-1)$ -th link definition makes it possible to obtain the expression (3) that substantially reflects the consideration of errors in joints of IR HS active links at working-off by them GC and it is the equivalent of the real IR HS links movement which is determined by the expression (4).

$$QLM(v, s)_{l-1}^l = \left(Q_{l-1}^l \cdot EQ_{l-1}^l \Big|_{Q_{l-1}^l = ROT_{l-1}^l} \right) \vee \left(Q_{l-1}^l + EQ_{l-1}^l \Big|_{Q_{l-1}^l = LIN_{l-1}^l} \right), \quad (3)$$

where:

Q_{l-1}^l is a quaternion of the l -th link motion relatively to the $(l-1)$ -th link of IR HS at working-off the given by control system θ_{l-1}^l -th of generalized coordinates;

EQ_{l-1}^l – the quaternion of motion error of the l -th link relatively to the $(l-1)$ -th link IR HS at working-off the given by control system generalized coordinates;

“.” and “+” are mathematical designation of algebraic operations of multiplication and addition respectively;

\vee is the mathematical designation of disjunction logic function [33].

In the context of the mentioned and illustrated (Figure 1) above, the actual real LM movement of the l -th link relatively to the $(l-1)$ -th link of IR HS with the consideration of errors in the links' joints is determined by the expression:

$$LM_{l-1}^l = \theta_{l-1}^l + \Delta_{l-1}^l, \quad (4)$$

where:

θ_{l-1}^l – GC that is worked-off by the l -th link relatively to the $(l-1)$ -th link MS PR for the movement of IRGr to the given by control system point IR WS;

Δ_{l-1}^l – IR HS joint links errors at working-off the θ_{l-1}^l GC.

Forasmuch to the IRGr pose error is a random value so it defines the necessity of investigations by the way of IRGr multiple positioning in the i -th point of its WS. The multiplicity is displayed like multidimensional data array $EQ(v; s)_{l-1}^l$, which is determined by the expression (5) that contains K “pages”, where K is the number of IRGr consecutive movements from $(i-1)$ -th to the i -th point of IR WS.

$$EQ(s, v)_{l-1}^l = EQ_{l-1_k}^l \mid k = \overline{1, K} \rightarrow (EQ_{l-1_1}^l, EQ_{l-1_2}^l, \dots, EQ_{l-1_{k-1}}^l, EQ_{l-1_k}^l), \quad (5)$$

where:

$EQ_{l-1_k}^l$ – the quaternion of motion error of the l -th relatively to the $(l-1)$ -th IR HS link at the k -th working-off by given control system GC, here K – total amount of analyzed worked-off motions;

$(s; v)$ – scalar and vector components of the quaternion;

\rightarrow – means the logical inheritance;

Taking into account mentioned above the quaternion of the links' joints errors $EQ(s; v)_{l-1_k}^l$ which describes l -th link movement relatively to $(l-1)$ -th for rotational degrees of motions takes the form:

$$EQ(s,v)_{l-1k}^l = \left(\cos \frac{\varepsilon_{l-1k}^l}{2} + v \cdot \sin \frac{\varepsilon_{l-1k}^l}{2} \right)_{Q_{l-1}^l = ROT_{l-1}^l \Rightarrow EQ_{l-1}^l = \varepsilon_{l-1}^l} \quad (6)$$

$$EQ(s,v)_{l-1k}^l = \left(\cos \frac{\varepsilon_{l-1k}^l}{2} + X \cdot \sin \frac{\varepsilon_{l-1A_k}^l}{2} + Y \cdot \sin \frac{\varepsilon_{l-1B_k}^l}{2} + Z \cdot \sin \frac{\varepsilon_{l-1C_k}^l}{2} \right)$$

where:

EQ_{l-1k}^l and $(s; v)$ – see expression (5);

ε_{l-1k}^l – error's value in the joints of rotational motion modules l -th and $(l-1)$ -th links of IR HS during the k -th working-off given by control system GC;

v is the unit vector means the axis direction of the relative movement of IR HS links, $v \in (X, Y, Z)$.

Similar to the expression (6) the quaternion of errors in links' joints $EQ(s;v)_{l-1k}^l$ during the motion of the l -th link relatively to the $(l-1)$ -th link for linear degrees of motion takes the form:

$$EQ(s,v)_{l-1k}^l = \left(0 + v \cdot \delta_{l-1k}^l \right)_{Q_{l-1}^l = LIN_{l-1}^l \Rightarrow EQ_{l-1}^l = \delta_{l-1}^l} \quad (7)$$

$$EQ(s,v)_{l-1k}^l = \left(0 + X \cdot \delta_{l-1k}^l + Y \cdot \delta_{l-1k}^l + Z \cdot \delta_{l-1k}^l \right)$$

where δ_{l-1k}^l – the errors value in the joints of the linear modules of motion of the l -th and $(l-1)$ -th IR HS links at the k -th working-off given by the control system GC.

The product of the quaternion motions QLM of the IR HS active links (QIR_{i-1}^i) during the IRGr pole moving from the i -th to the $(i-1)$ -th WS point and it is the matrix equivalent of homogeneous transformations MS PR and is defined as follows:

$$QIR_{i-1}^i = (Q_0^1 \cdot EQ_0^1) \cdot (Q_1^2 \cdot EQ_1^2) \cdot \dots \cdot (Q_{i-1}^i \cdot EQ_{i-1}^i), \quad (8)$$

where:

QIR_{i-1}^i – the quaternion of IR HS real motion;

Q_{l-1}^l and EQ_{l-1}^l – see expression (3).

Taking into account the expressions (2) and (3) the quaternion QIR_{i-1}^i of IR HS real motion (8) of moving from the i -th to the $(i-1)$ -th IR WS point takes the form:

$$QIR_{i-1}^i = QLM_0^1 \cdot QLM_1^2 \cdot \dots \cdot QLM_{i-1}^i \Big|_{l=\overline{1,L}}, \quad (9)$$

where QLM_{l-1}^l – see expressions (2) and (3).

Assuming that each IR HS link is a vector l with coordinates (X^l, Y^l, Z^l) that determine the linear size value (length – $|l|$) of the link which is calculated by (11), description of the relative movements of IR HS links using of Q-model of IR HS accuracy will be in the vector rotation l on the quaternion QLM_{l-1}^l when $Q_{l-1}^l = ROT_{l-1}^l$ and $EQ_{l-1}^l = \varepsilon_{l-1}^l$ which is analytically described by the expression (10) and substantially is a multiplying of the vector to the left on the quaternion and to the right on the conjugate quaternion or in linear movement of the link l on the quaternion QLM_{l-1}^l when $Q_{l-1}^l = LIN_{l-1}^l$ and $EQ_{l-1}^l = \delta_{l-1}^l$.

Taking into account mentioned above, the actual position coordinates' determination of the l -th link in CS $(l-1)$ -th takes the form of the expression (10), as a result of conducted operations is a multidimensional array p_l of the l -th link actual coordinates:

$$p_{l-1}^l = \left(QLM_{l-1}^l \cdot l \cdot \overline{QLM_{l-1}^l} \Big|_{QLM_{l-1}^l = ROT_{l-1}^l} \right) \vee \left(l + QLM_{l-1}^l \Big|_{QLM_{l-1}^l = LIN_{l-1}^l} \right) \vee (p_l = (0, l)), \quad (10)$$

where:

l is the vector with the coordinates (X^l, Y^l, Z^l) ;

$$|l| = \sqrt{(X^l)^2 + (Y^l)^2 + (Z^l)^2} \quad (11)$$

where X^l, Y^l, Z^l – the coordinates of the l -th link position in the own CS.

The actual coordinates' definition of the IRGr pole position at the real motion of active links from the number of activated is in FKT solving using the Q-model of IR HS accuracy characteristics. In the result it is obtained the multidimensional array p_{Cx_i} of the IRGr pole position in the i -th IR WS point with the using of K positioning iterations and is formed as specified in (12):

$$p_{Cx_i} = \left(\left(\left(p_{L-1}^L + p_{L-2}^{L-1} \right) + p_{L-3}^{L-2} \right) + \dots + p_{L-i}^{L-i+1} \right) \Big|_{l=\overline{1,L}} \quad (12)$$

where:

p_l is the vector of coordinates' of the l -th link in CS of the $(l-1)$ -th link position, see expression (10); L is the total number of the IR HS links.

4. Conclusion

This approach allows providing the the IR WS accuracy examination with the perspective of the appropriate methods developing reducing the operational load on the CPU and increasing of the accuracy analysis efficiency by reducing the

duration of the performed calculations.

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