MATHEMATICAL MODELING AND RESEARCH OF PURPOSE FUNCTION FOR RELATIVE POWER CONSUMPTION PER UNIT PRODUCT IN KID-300 OPERATION

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Abstract. This paper presents the creation of mathematical model of purpose function for relative power consumption per unit product at operation with cone inertial crusher type KID-300. There are presented the choice of governing factors affecting on the purpose function for relative power consumption and formulation of purpose function for relative power consumption. There are experiment and statistical analysis of the experimental results. Results from statistical analysis of relative power consumption purpose function are presented and analyzed with regression models and 3D diagrams for determination of the influence of governing factors. The results will be used for a future work of functional crushing process optimization and multiple factors optimization of the working process in terms of relative power consumption per unit product in cone inertial crushers.

Keywords: KID-300, mathematical modelling, statistical analysis, purpose function, relative power consumption

1. Introduction

The crushers type KID are cone crushers in which the inner cone is driven by unbalanced centrifugal vibrator. Upon rotation of the centrifugal vibrator arises circular centrifugal force, forcing the inner cone to roll along the inner surface of the outer cone being pressed against it according centrifugal force and thus crushed the material located between two cones. This type of cone crushers have no permanent dynamic discharge opening and the magnitude of the eccentricity of the inner cone depends on the thickness of crushed material located in the chamber and the value of the crushing force. The crushers type KID have the opportunity to adjust the static discharge opening width, frequency of rotation and mass static moment of unbalanced vibrator, thereby vary the magnitude of the crushing force. Magnitude of the crushing force at these crushers has considerable influence on the particle size distribution of the production and the productivity of the machine. With crusher type KID-300 can be obtained a product, which is characterized as a crusher for fine crushing and mill for coarse grinding. Changeable rotation frequency of the unbalance vibrator in KID-300 extends the capabilities of the machine to produce outputs with varied particle size distribution [3, 4, 5].

2. Selection of governing factors

In describing the process of crushing in crusher type KID can be defined and researched several purpose functions such as relative power consumption per unit product is one of them. The relative power consumption is the amount of energy consumed to produce a unit of finished product. The power consumption characterizes the energy side of the crushing process and is the most common measure used to assess the effectiveness of crushing process. The relative power consumption per unit output product combines both - the energy side of process and technological side of process.

Mechanical parameters changed during process examination in KID-300 are as following:

- static discharge opening width, as a maximum can reach up to 18 mm;
- mass static moment of unbalanced vibrator, the vibrator is adjustable and has 19 degrees that determine the value of its mass static moment;
- frequency of rotation of unbalanced vibrator, for the purposes of the study was changed by the transmission ratio of the V-belt transmission of the machine.

One of the technological parameters, which can be easily changed during the crushing process is the size of the feed material, there is a limit to the maximum size of 20÷25 mm.

The selected governing factors influencing the particle size distribution of the finished product obtained from KID-300 are as follows:

- Frequency of rotation of unbalanced vibrator *f*, [s^{-1}]. The frequency of rotation depends on the transmission ratio (*i*) of the V-belt transmission and it is changing by different diameters of the V-belt pulleys of the motor. On that way it is settled 0.75; 1; 1.25 and 1.44 for transmission ratio.

- The mass static moment of the unbalanced vibrator S_{ucv}, [kg·m]. Three degrees of the mass static moment are used: minimum (the 1st); average (10th) and maximum (19th). Accordingly S_{usv} accepts the following values: 0.095 kg·m; 0.921 kg·m and 1.295 kg·m.
- The static discharge opening width b, [mm]. Due to the risk of unacceptable increase in the amplitude of oscillation of the inner cone and consequently the dynamics of the machine this parameter accepts the following values: 4 mm, 6 mm and 8 mm.
- The average size of the feed material D_{av}, [mm]. Used is a material which has a different average particle size, a correspondingly fine material having an average particle diameter of 8.575 mm and a coarse material with an average particle diameter of 16 mm.

The selected governing factors are easily determinable, can be measured in absolute units and relatively easy to change. In Table 1 shows the governing factors and their intervals and steps of change.

Table 1. Governing factors

Designation	x_1			x_2			x_3			x_4	
Factor	V-belt pulley			Vibrator			Discharge opening			Feed material size	
Dimensionality	mm			number			mm			mm	
Step	52	52		9	9		2	2		7.425	
Designation	D_{vip}, mm			Degree, pcs.			b, mm			D_{av}, mm	
Value	156	208	260	1	10	19	4	6	8	8.575	16
Interval	104			18			4			7.425	
Level	-1	0	1	-1	0	1	-1	0	1	0	1

3. Formulation of purpose function for relative power consumption

In most real cases the mathematical models defining a purpose function are not known and are sometimes unsuitable because of its complexity and must to use an appropriate approximation. Then arises the difficult task of determining the structure of the model. In the simplest case, the mathematical model may be selected as a linear function [1, 2] of the input parameters:

$$f = \sum_{i=0}^{n} N_i \cdot x_i \qquad (1)$$

For initial study of purpose function for relative power consumption per unit product is assumed linear model with included constant:

$$W = N_0 + N_1 \cdot x_1 + N_2 \cdot x_2 + N_3 \cdot x_3 + N_4 \cdot x_4, \quad kWh/t \qquad (2)$$

where:

x_1 (f, s^{-1}) – frequency of rotation of unbalanced vibrator;

x_2 (S_{ucv}, kg·m) – mass static moment of the unbalanced vibrator;

x_3 (b, mm) – static discharge opening width;

x_4 (D_{av}, mm) – average size of the feed material;

N_0, N_1, N_2, N_3, N_4 – coefficients of the governing factors.

In Table 2 are presented the dimensions of the purpose function and the governing factors and the coefficients of the governing factors in the so defined purpose function.

Table 2. Dimensionality of the purpose function, governing factors and their coefficients

Factor		x_1	x_2	x_3	x_4
Designation		f	S_{ucv}	b	D_{av}
Dimensionality		Hz	kg·m	mm	mm
SI		s^{-1}	kg·m	m·10^{-3}	m·10^{-3}
W	N_0	N_1	N_2	N_3	N_4
kWh/t	kWh/t	kW·s^2/3600·t	kWh·10^3/kg^2	kWh·mm/t	kWh·mm/t

4. Experimental results

Table 3. Experimental results

№	x_1	x_2	x_3	x_4	y	
	f	S_{ucv}	b	D_{av}	W_i	W_e
	s^{-1}	kg·m	mm	mm	kWh/t	kWh/t
1	17.82	1.295	4	16	33.477	28.175
2	17.75	1.295	6	16	12.003	10.104
3	17.70	1.295	8	8.575	7.524	5.612
4	17.80	0.921	4	8.575	42.673	36.314
5	17.73	0.921	8	16	9.918	8.444
6	17.83	0.095	6	8.575	46.081	38.653
7	17.85	0.095	8	16	0.000	0.000
8	24.48	0.921	7	16	15.700	14.451
9	24.50	0.921	6	16	15.358	–
10	24.52	0.921	6	16	15.364	–
11	24.52	0.921	6	16	14.940	–
12	24.52	0.921	6	16	13.828	–
13	24.53	0.921	6	16	13.849	–
14	24.53	0.921	6	16	13.852	–
15	24.55	0.921	6	8.575	12.391	–
16	24.78	0.095	4	8.575	38.361	–
17	24.78	0.095	6	16	32.739	–
18	24.78	0.095	8	16	27.511	–
19	24.38	1.295	6	8.575	12.022	–
20	24.40	1.295	8	8.575	10.222	–
21	24.57	1.295	4	16	32.749	–
22	30.90	0.095	4	8.575	31.758	–
23	30.90	0.095	4	16	33.399	–
24	30.93	0.095	6	16	27.605	–
25	30.92	0.095	8	8.575	17.647	–
26	30.48	0.921	4	16	17.094	–
27	30.37	0.921	6	16	15.941	17.442
28	30.03	0.921	8	8.575	17.300	19.141
29	30.22	1.295	4	8.575	15.249	–
30	30.05	1.295	6	8.575	14.627	–
31	29.97	1.295	6	16	13.560	–
32	29.85	1.295	8	16	14.691	–
33	35.92	0.095	4	8.575	47.149	–
34	35.87	0.095	4	16	76.627	51.441
35	35.95	0.095	6	16	28.604	24.474
36	35.98	0.095	8	8.575	17.722	15.471
37	35.17	0.921	4	16	20.259	15.832
38	34.42	1.290	6	16	17.923	17.069
39	33.72	1.290	8	8.575	16.768	11.754
40	34.35	1.290	4	8.575	20.604	18.808

In order to study the selected purpose function was held incomplete factorial experiment on cone inertial crusher type KID-300. The used material of crushing process is gravel, which is previously washed, dried and classified by screening. A total of 40 tests have been made in "dry" mode of operation of the machine. In Table 3 are presented the governing factors that are changed during the experiment, and obtained experimental results for the purpose function for relative power consumption per unit product (which is obtained based on an installed power W_i power and RMS power W_e).

5. Statistical analysis of experimental results

Based on experimental data obtained from 40ty tests in working with KID-300 are composed regression mathematical models to study the purpose function for relative power consumption per unit product. They are composed linear models with included constant (and models without constant) and nonlinear models. Searches are such models, which have a high measure of definiteness (square of the coefficient of multiple correlation R^2). The main criteria for selecting an appropriate model are as follows:

- maximum value of multiple correlation coefficient R^2;
- maximum value of Fisher criterion (F-Ratio) for the model;
- minimum values of confidence probability (p-Value) indicator for the model;
- minimum values of confidence probability (p-Value) indicators for insignificance of the regression coefficients.

The purpose function is made and researched with the computer program for statistical analysis STATGRAPHICS Centurion XV [7]. This program allows the study of different function types with one and more than one variables that have or not functional relationship between variables. The program also has options to assess the variables and their coefficients of correlation. For the purposes of practical research of the working process are looked for models and regression coefficients that can be adopted with levels of confidence probability at least 95 %, as it is expected that the levels of engineering error of 5 % is acceptable for this type machines as cone inertial crushers.

The critical measure of models definitive according to which they could be accepted or rejected as significant depends at model degrees of freedom with accepted minimum level of significance. In this case, the models are a total number of degrees of freedom $v = 40$ and level of significance $\alpha = 0.05$. According to [6] at $v = 40$ and $\alpha = 0.05$ the critical coefficient of definiteness is $R_{cr} = 0.304$ (for $\alpha = 0.1$; $R_{cr} = 0.257$). The models which values of measure of definiteness are above a critical value would be accepted as statistically defined and significant. Figure 1 is a block diagram in which regression mathematical models are composed and then accepted or rejected.

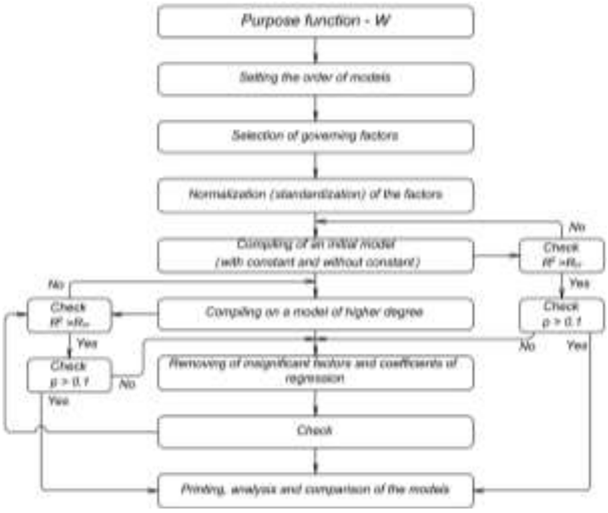

Figure 1. Block diagram of the sequence of actions in theoretical and experimental research of purpose function for relative power consumption per unit product

6. Research models for relative power consumption obtained in base of installed power

The relative power consumption (based on a installed power) is defined as a ratio of the installed power of the drive electric motor (11 kW) at the quantity of finished product obtained during the respective test or:

$$W_i = \frac{P}{Q_m}, \quad kWh/t \tag{3}$$

The best model obtained for the relative power consumption determined based on installed power is a regression model of first and second degree of the variables without included constant and with excluded insignificant variables. This model includes all governing factors and is of the following type:

$$W_i = \sum_{i=1}^{n} N_i \cdot \frac{1}{x_i} + \sum_{i=1}^{n} N_i \cdot \frac{1}{x_i^2}, \quad kWh/t \tag{4}$$

To illustrate the comparison between the model and experimental values is applied comparative graph (Figure 2 and 8). The points on the graph represent the experimental values and the straight line shows the location of the function values obtained from the model. The abscissa and ordinate of this graph are the same dimension and that is the dimension of the researched function. The comparison between experiment and model is a visual basis as the arrangement of the points as close to a straight line shows good similarity between experiment and model.

Table 4. Parameters of the model

					Parameters of the model
		Standard	T		
Parameter	Estimate	Error	Statistic	P-Value	
$1/X_1$	-775.479	288.817	-2.68502	0.0110	
$1/X_3$	147.969	26.2291	5.64139	0.0000	
X_4/X_2	0.138645	0.0230494	6.01514	0.0000	
$1/X_1^2$	12041.0	4578.76	2.62976	0.0126	
					Analysis of the probabilities
Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	24860.5	4	6215.12	92.59	0.0000
Residual	2349.27	35	67.1219		
Total	27209.8	39			
					Statistics of the model
		R^2 =	91.3661	%	
		R^2 (adjusted for d.f.) =	90.626	%	
		Standard Error of Est. =	8.1928		
		Mean absolute error =	5.51968		
		Durbin-Watson statistic =	1.96461		
		Lag 1 residual autocorrelation =	0.00952459		

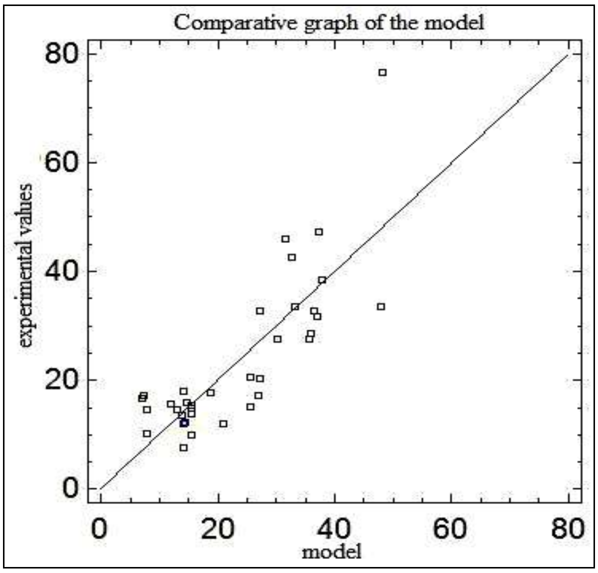

Figure 2. Comparative graph of the model

The equation of the model in the natural variables according Table 4 is:

$$W_i = -\frac{775.479}{f} + \frac{147.969}{b} + 138.645 \cdot 10^{-3} \cdot \frac{D_{av}}{S_{ucv}} + \frac{12041}{f^2}, \quad kWh/t \tag{5}$$

Although the equation of this model is complicated it contains within itself all the governing factors. The coefficients of multiple correlation R^2 and R^2(adj) are with acceptable values over 90 %. This model has the highest value of the Fisher's indicator compared to all received models for relative power consumption. The value of indicator of confidence probability (P-Value) for the model is under critical. All regressors and the model itself can be accepted for adequate with a level of confidence probability more than 90 %.

Based on the resulting regression model were made (using the program Mathematica 5.2) three-dimensional diagrams. From these diagrams can be reported the influence of the four governing factors (f, S_{ucv}, b and D_{av}) on the purpose function for relative power consumption determined on the basis of installed power W_i.

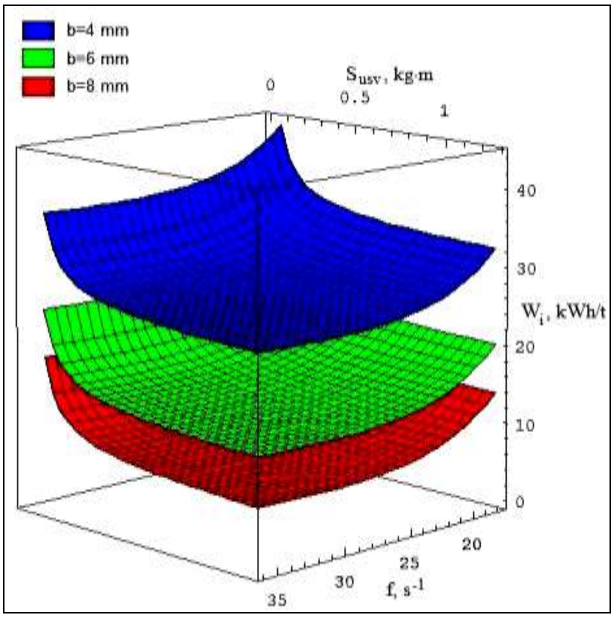

Figure 3. Relative power consumption W_i, kWh/t at D_{av} = 8.575 mm

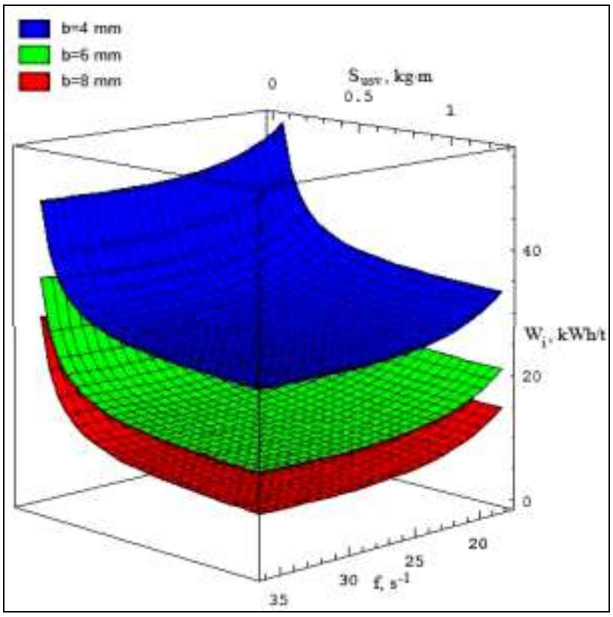

Figure 4. Relative power consumption W_i, kWh/t at D_{av} = 16 mm

In Figure 3 at D_{av} = 8.574 mm and in Figure 4 at D_{av} = 16 mm are presented obtained three-dimensional diagrams of variation of the purpose function depending on mass static moment of the unbalanced vibrator, static discharge opening width and frequency of rotation of unbalanced vibrator.

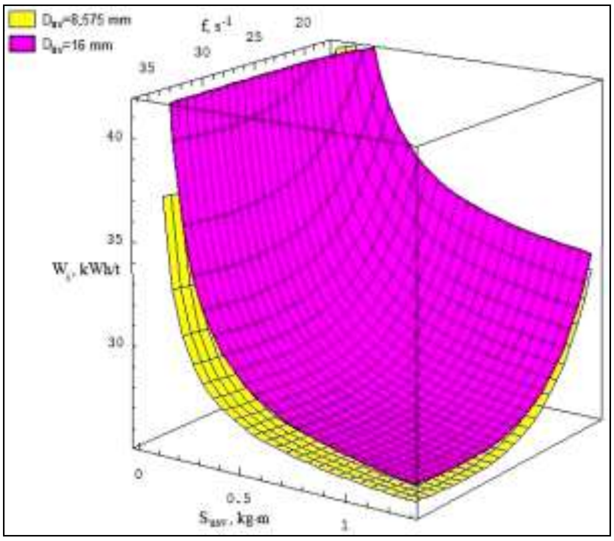

Figure 5. Relative power consumption W_i, kWh/t at b = 4 mm

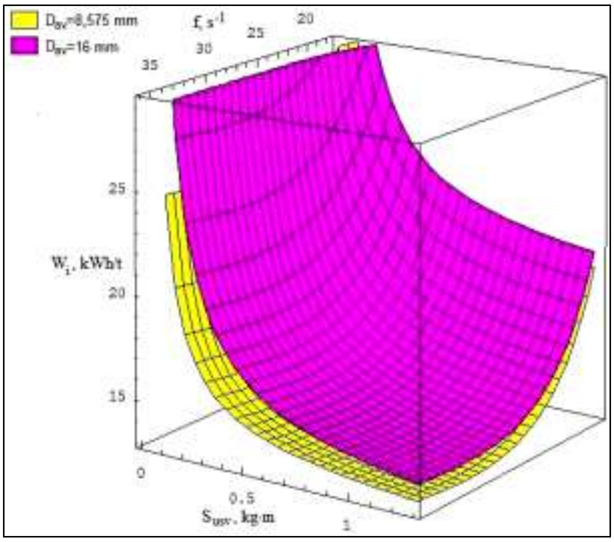

Figure 6. Relative power consumption W_i, kWh/t at b = 6 mm

In Figure 5 at b = 4 mm, in Figure 6 at b = 6 mm and in Figure 7 at b = 8 mm are presented obtained three-dimensional diagrams of variation of the purpose function depending on mass static moment of the unbalanced vibrator, static discharge opening width and frequency of rotation of unbalanced vibrator.

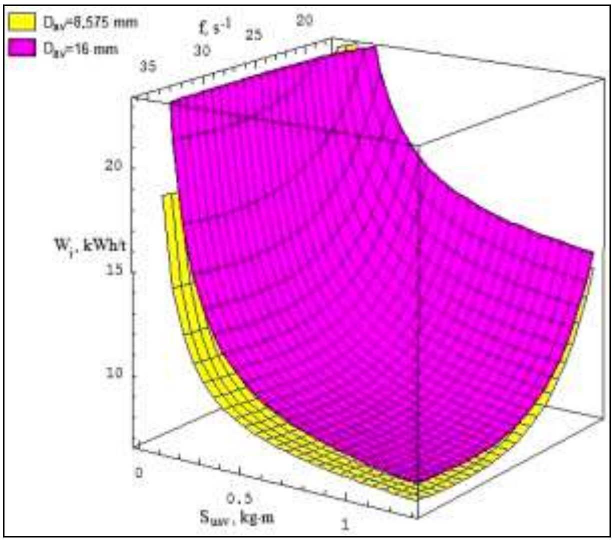

Figure 7. Relative power consumption W_i, kWh/t at b = 8 mm

7. Research models for relative power consumption obtained in base of RMS power

The relative power consumption (based on a RMS power) is defined as a ratio of the RMS power of the drive electric motor for the respective test at the quantity of finished product obtained during the same test or:

$$W_e = \frac{\widetilde{P}}{Q_m}, \quad kWh/t \tag{6}$$

The best model obtained for the relative power consumption determined based on RMS power is a regression model of first and second degree of the variables without included constant and with excluded insignificant variables. This model includes all governing factors and is of the following type:

$$N_e = \sum_{i=1}^{n} N_i \cdot x_i + \sum_{i=1}^{n} N_i \cdot x_i^2, \quad kWh/t \tag{7}$$

Table 5. Parameters of the model

					Parameters of the model
		Standard	T		
Parameter	Estimate	Error	Statistic	P-Value	
X_4	9.95611	1.467	6.7867	0.0000	
$X_1 \cdot X_2$	-0.37194	0.14232	-2.61341	0.0215	
X_1^2	-0.498837	0.107849	-4.62531	0.0005	
X_4^2	-0.448342	0.0779395	-5.75244	0.0001	
					Analysis of the probabilities
Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	8303.71	4	2075.93	29.79	0.0000
Residual	905.77	13	69.6746		
Total	9209.48	17			
					Statistics of the model
		R^2 =	90.1648	%	
		R^2 (adjusted for d.f.) =	87.8951	%	
		Standard Error of Est. =	8.34713		
		Mean absolute error =	5.95327		
		Durbin-Watson statistic =	1.72867		
		Lag 1 residual autocorrelation =	0.0895907		

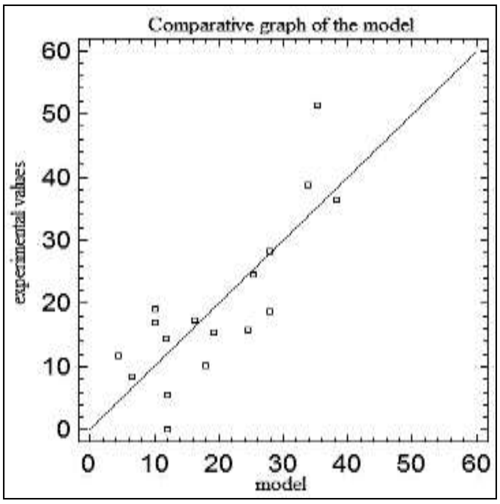

Figure 8. Comparative graph of the model

The equation of the model in the natural variables according Table 5 is:

$$W_e = 9.95611\cdot D_{av} - 371.94\cdot 10^{-3}\cdot f\cdot S_{ucv} - 498.837\cdot 10^{-3}\cdot b^2 - 448.342\cdot 10^{-3}\cdot D_{av}^2,\quad kWh/t \tag{8}$$

This model contains within itself all the governing factors. The values of coefficients of multiple correlation R^2 and R^2(adj) and the value of the Fisher's indicator are low. The value of indicator of confidence probability (P-Value) for the model is under critical. All regressors and the model itself can be accepted for adequate with a level of confidence probability more than 90 %.

Based on the resulting regression model were made three-dimensional diagrams. From these diagrams can be reported the influence of the four governing factors (f, S_{ucv}, b and D_{av}) on the purpose function for relative power consumption determined on the basis of RMS power W_e.

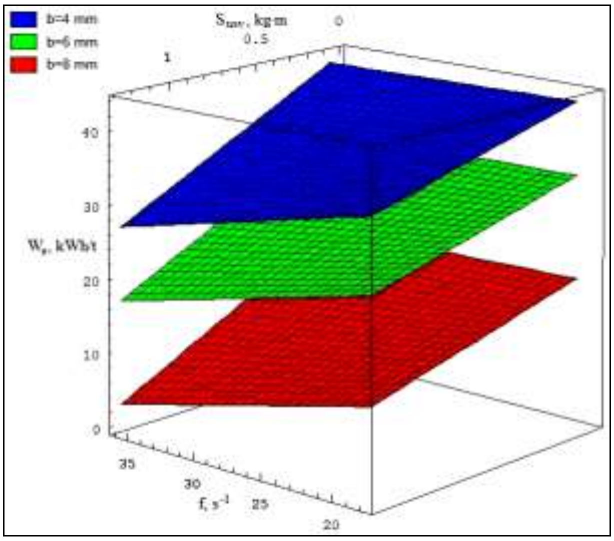

Figure 9. Relative power consumption W_e, kWh/t at D_{av} = 8.575 mm

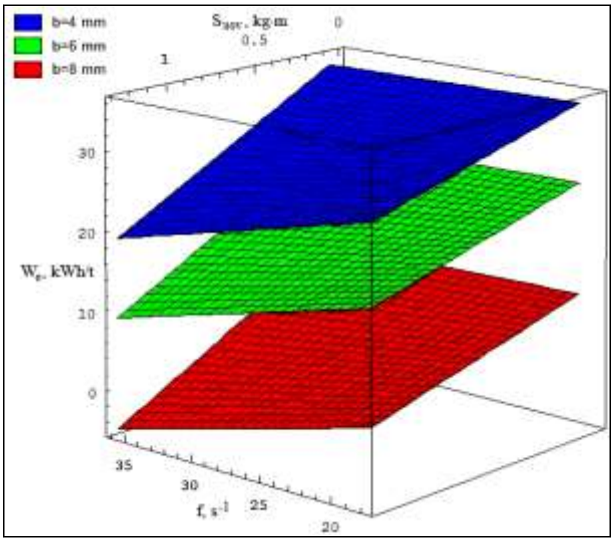

Figure 10. Relative power consumption W_e, kWh/t at D_{av} = 16 mm

In Figure 9 at D_{av} = 8.574 mm and in Figure 10 at D_{av} = 16 mm are presented obtained three-dimensional diagrams of variation of the purpose function depending on mass static moment of the unbalanced vibrator, static discharge opening width and frequency of rotation of unbalanced vibrator.

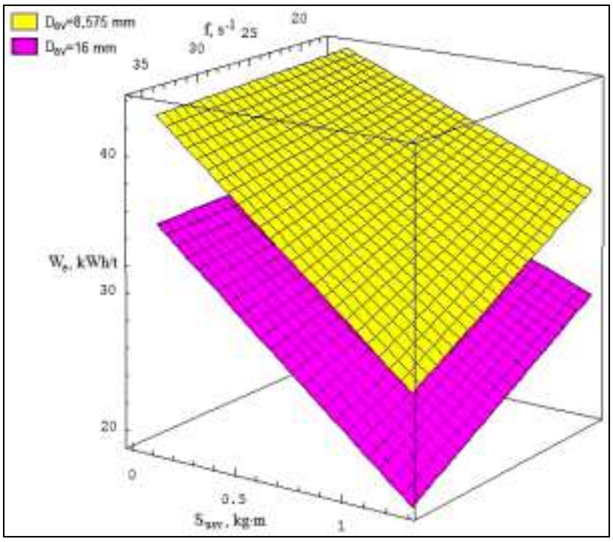

Figure 11. Relative power consumption W_e, kWh/t at b = 4 mm

In Figure 11 at b = 4 mm, in Figure 12 at b = 6 mm and in Figure 13 at b = 8 mm are presented obtained three-dimensional diagrams of variation of the purpose function depending on mass static moment of the unbalanced vibrator, static discharge opening width and frequency of rotation of unbalanced vibrator.

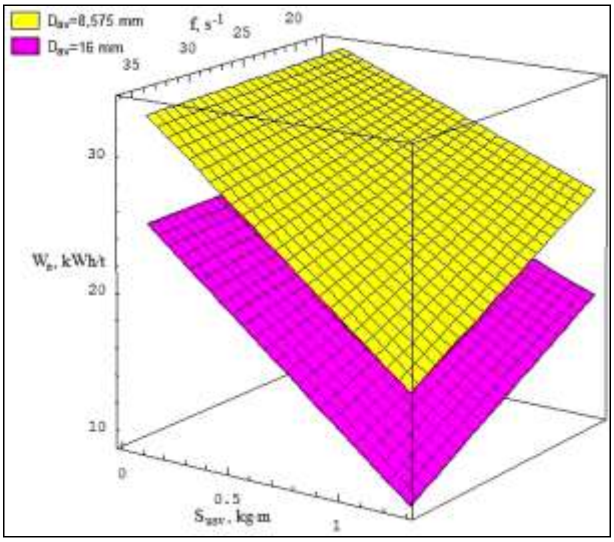

Figure 12. Relative power consumption W_e, kWh/t at b = 6 mm

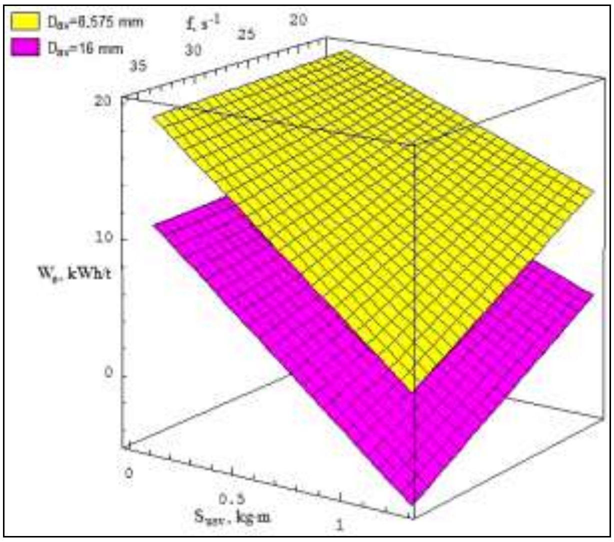

Figure 13. Relative power consumption W_e, kWh/t at b = 8 mm

8. Conclusions

The conclusions made from the results obtained from the statistical analysis about the research of the purpose function for relative power consumption per unit product may be summarized as follows:

- The analysis of models of regression dependence on the relative power consumption per unit product shows that models have only informational nature and may be used for future research. For practical purposes, can be used the second model (with admissible error of 10 %), and to improve it is necessary to make additional measurements in order to obtain a sufficient data for the relative power consumption. Created models contain all governing factors and their coefficient of definiteness is over the critical. The coefficients of definiteness of regressors in both models are high enough for engineering practice.

- Received are respective analytical equations for relative power consumption determined on the basis of installed power Equation 5 and relative power consumption determined on the basis of RMS power Equation 8.

- Based on the resulting regression models were made three-dimensional diagrams for reporting of the influence of the four governing factors (f, S_{ucv}, b and D_{av}) on the purpose function. These diagrams can serve as a basis for optimization of the crushing process in KID-300 in terms of the relative power consumption per unit product.

All performances in the analysis of the results of the theoretical and experimental research of the purpose function for relative power consumption per unit product can be considered reliable within specified limits of change of the governing factors.

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