



Transilvania University of Brasov,  
Romania

13<sup>th</sup> INTERNATIONAL CONFERENCE  
"STANDARDIZATION, PROTOTYPES AND QUALITY:  
A MEANS OF BALKAN COUNTRIES' COLLABORATION"

Brasov, Romania, November 3 -4, 2016

## Increasing the Quality of Superfinishing and Laminate Manufacturing Process by Modelling

ENESCU Ioan

Transilvania University Brasov, Romania, enescu@unitbv.ro

### Abstract

When a metal strip is passed through a rolling mill to produce an appreciable reduction. In thickness, the plastic deformation is generally large compared with the elastic deformation so that the material can be regarded as being rigid plastic. In the first instance, the elastic deformation of the rolls may also be neglected. We tried to answer to the main question: how are the elastic contact stress and deformation between curved face in contact influenced by surface roughness, and plastic deformation.

### Keywords

contact, elastic, laminate, rough, friction

### 1. Introduction

Many processes involve the passage of a strip or sheet of material through the nip between rollers. In this paper we consider the strip to be perfectly elastic and investigate the stress in the strip, the length of the arc of contact with the roller, the maximum indentation of the strip and the precise speed at which it feeds through the nip in relation to the surface speed of the rollers. If the strip is wide and the rollers are long in the axial direction it is reasonable to assume plane deformation [1, 2].

When a metal strip is passed through a rolling mill to produce an appreciable reduction in thickness, the plastic deformation is generally large compared with the elastic deformation so that the material can be regarded as being rigid plastic. In the first instance the elastic deformation of the rolls may also be neglected.

### 2. Modelling of Roughing and Superfinishing Process

#### 2.1. Elastic contact model of roughing surface

The stresses in an elastic strip due to symmetrical bands of pressure acting on opposite faces have been expressed by Sneddon (1951) in terms of Fourier integral transforms. The form of these integrals is particularly awkward and most problems require elaborate numerical computations for their solution. However, when the thickness of the strip  $2b$  is much less than the arc of contact  $2a$  an elementary treatment is sometimes possible. The situation is complicated further by friction between the strip and the rollers. We can analyse the problem assuming (a) no friction ( $\mu = 0$ ) and (b) complete adhesion ( $\mu \rightarrow \infty$ ), but our experience of rolling contact conditions leads us to expect that the arc of contact will, in fact, comprise zones of both "stick" and "slip".

We will look first at a strip whose elastic modulus is of similar magnitude to that of rollers, and write [3, 4]

$$C = \frac{(1 - \nu_1^2)/E_1}{(1 - \nu_2^2)/E_2} = \frac{1 + \alpha}{1 - \alpha} \quad (1)$$

where  $\alpha$  is defined by the equation (1), and (2), refers to the strip and the rollers respectively.

$$\alpha = \frac{[(1 - \nu_1)/G_1] - [(1 - \nu_2)/G_2]}{[(1 - \nu_1)/G_1] + [(1 - \nu_2)/G_2]} \quad (2)$$

If the strip is thick ( $b \geq a$ ) it will deform like an elastic half-space.

At the other extreme, when  $b \leq a$ , the deformation is shown in Figure 1.

The compression of the roller is now much greater than that of the strip so that the pressure distribution again approximates to the Hertz:

$$p(x) = \frac{2P}{\pi a} (1 - x^2/a^2)^{1/2} \quad (3)$$

The strip is assumed to deform with plane sections remaining plane so that the compression at the centre of the strip is given by:

$$d = \frac{b(1 - \nu_1^2)p(0)}{E_1} = \frac{2b(1 - \nu_1^2)P}{\pi a E_1} \quad (4)$$

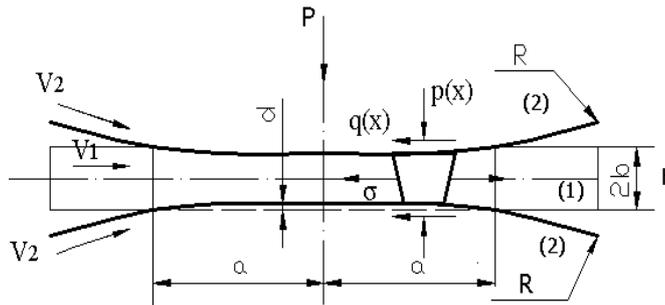


Fig. 1. Elastic contact model of roughing surface

If the deformed surfaces of the strip are more approximated by circular of radius  $R'$ , then:

$$\frac{1}{R'} = \frac{2d}{a^2} = \frac{4b(1 - \nu_1^2)P}{\pi a^2 E_1} \quad (5)$$

The rollers are flattered from a radius  $R$  to  $R'$  so that:

$$a^2 = \frac{4P(1 - \nu_2^2)}{\pi E_2} / \left( \frac{1}{R} - \frac{1}{R'} \right) \quad (6)$$

Eliminating  $R'$  from (5) and (6) gives:

$$\left( \frac{a}{a_0} \right)^2 = 1 + C \frac{b}{a} \quad (7)$$

where  $a_0 = \sqrt{4PR(1 - \nu_2^2)/E_2}$  is the semi-contact width for vanishingly thin-slip.

With friction less rollers the longitudinal stress in the strip  $\sigma_x$  is either zero or equal to any external tension in the strip. Due to the reduction in thickness, the strip extends longitudinally, whilst the roller surface compresses to the Hertz theory, so that in fact frictional tractions  $q(x)$  arise (acting inwards on the strip) whether or not materials of the strip and rollers are the same. For equilibrium of an element of the strip we have:

$$\frac{d\sigma_x}{dx} = \frac{1}{b} q(x) \quad (8)$$

Slip between the rollers and the strip is governed by the equation (9):

$$\frac{s_x}{V} = \xi_x - \frac{\psi y}{c} + \left( \frac{\partial u_{x_1}}{\partial x} - \frac{\partial u_{x_2}}{\partial x} \right) \quad (9)$$

where  $\xi_x \equiv (\delta V_{x_1} - \delta V_{x_2})/V$  and  $\xi_y \equiv (\delta V_{y_1} - \delta V_{y_2})/V$  are the creep ratios,  $\gamma$  is the non-dimensional spin parameter  $(\omega_{x_1} - \omega_{x_2})cV$ , and  $c = \sqrt{ab}$ .

In addition, it is consistent with neglecting second order terms in  $\phi$  to replace  $h$  by the mean thickness.

In a stick region:  $\dot{x}_x = \dot{x}_y = 0$ .

In additional, the resultant tangential traction must not exceed its limiting value:

$$|q(x, y)| < \mu p(x, y) \tag{10}$$

and the direction of  $q$  must oppose the velocity:

$$\frac{q(x, y)}{|q(x, y)|} = \frac{s(x, y)}{|s(x, y)|} \tag{11}$$

If there is no slip, equation (11) reduces to:

$$\frac{\partial u_{x_1}}{\partial x} - \frac{\partial u_{x_2}}{\partial x} = -\xi \tag{12}$$

where  $\xi$  is the creep ratio  $(V_1 - V_2)/V_2$  of the strip relative, to the periphery of the rollers. The longitudinal strain in a roller within the contact arc is given by equation:

$$\frac{\partial u_{x_1}}{\partial x} = \frac{1 - \nu_1^2}{E_1} \left( \sigma_x + \frac{\nu_1}{1 - \nu_1} p(x) \right) \tag{13}$$

The integral equation (10) is satisfied by the traction:

$$q(x) = \left( 1 - \frac{4\beta}{1 + \alpha} \right) \frac{b}{2a} p_0 \frac{x}{\sqrt{(a^2 - x^2)}} \tag{14}$$

where  $\beta$  is defined by the equation:

$$\beta = \frac{1}{2} \left[ \frac{(1 - 2\nu_1)/G_1 - (1 - 2\nu_2)/G_2}{(1 - \nu_1)/G_1 + (1 - \nu_2)/G_2} \right] \tag{15}$$

The distribution of traction and also the stress difference  $(s_x - s_y)$  on the centre plane of the strip are show in Figure 2.

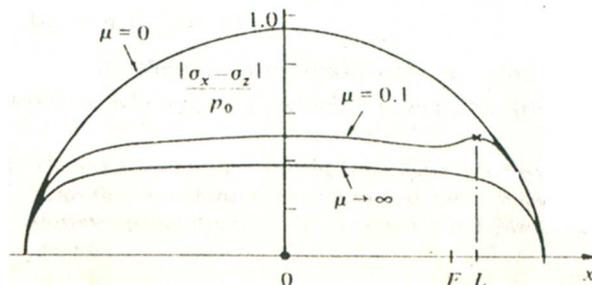


Fig. 2. The distribution of traction and also the stress difference

## 2.2. Plastic rolling of strip

In the first instance the elastic deformation of the rolls may also be neglected. For continuity of flow, the rolled strip emerges from the nip at a velocity greater than it enters, which is in inverse proportion to its thickness if no lateral spread occurs. Clearly the question of sticking and slipping between the rolls and the strip, arises in the metal rolling process. In the hot rolling the absence of lubricant and the lower flow stress of the metal generally mean that the limiting frictional traction at the interface exceeds the yield stress of the strip in shear so that there is no slip in the conventional sense at the surface.

It is for the condition of no slip encountered in hot rolling that the most complete analyses of the process have so far been made. We saw in the previous section that interfacial friction inhibits plastic reduction, so that in cold rolling the strip is deliberately lubricated during its passage through the rolls in order to facilitate slip.

At entry the strip is moving slower than the roll surfaces so that it slip backwards , at exit the strip is moving faster so that slips forwards. At same points in the nip, referred to as the “neutral point” the strip is moving with the same velocity as the rolls. At this point the slip and the frictional traction change direction. In reality, however, we should not expect this change to occur at a point. In the last section, when a thin elastic strip between elastic rollers was being examined, we saw that plastic deformation and slip would initiate at entry and exit ; in between there is a region of no slip and no plastic

deformation. It seems likely there for that a small zone of no slip will continue to exit even when appreciable plastic reduction is taking place in the nip as a whole. Current theories of cold rolling, which are restricted to the idea of a neural point, must be regarded as complete slip solutions.

**2.3. An elastic model of the process**

The complete solution of a problem involving the plane deformation of a rigid perfectly plastic material calls for the construction of a slip field. So far this has been achieved only for the condition of no slip line field. So far this has been achieved only for the condition of a slip line field. So far this has been achieved only for the condition of on slip, which applies to lot rolling. Before looking at these solutions we shall examine the elementary theories, with and without slip, which derive from von Karman (1925).

The geometry of the drafting roller, neglecting elastic deformation, is shown in Figure 3. The mean longitudinal (compressive) stress in the strip is denoted by  $\bar{\sigma}_x$  and the transverse stress at the surface by  $\bar{\sigma}_z$ .

Equilibrium of the element gives:

$$\bar{\sigma}_x dx = (p \cos \phi + q \sin \phi) R d\phi \tag{16}$$

and

$$d(h\bar{\sigma}_x) = (p \sin \phi - q \cos \phi) 2R d\phi \tag{17}$$

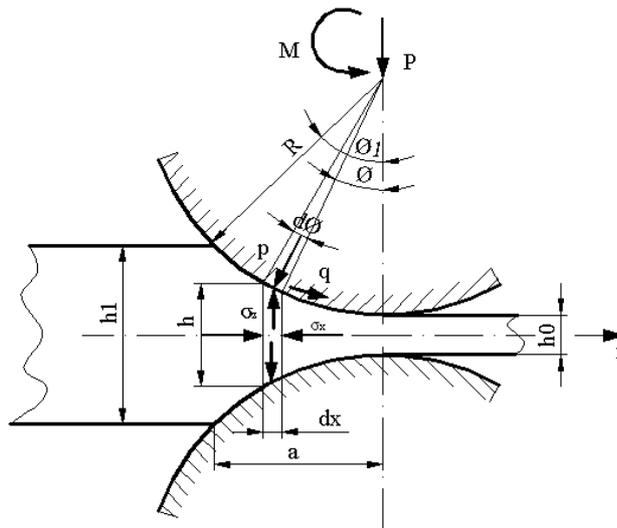


Fig. 3. The geometry of the drafting roller

In this simple treatment, it is assumed that in the plastic zone  $\bar{\sigma}_x$  and  $\bar{\sigma}_z$  are related by field criterion:

$$\bar{\sigma}_x - \bar{\sigma}_z = 2k \tag{18}$$

This simplification implies a homogeneous state of stress in the element that is clearly not true at the surface of the strip where the functional traction acts. Nevertheless, by combining equations (16), (17), and (18) we obtain:

$$\frac{d}{dt} [h(p + q \tan \phi - 2k)] = 2R(p \sin \phi - q \cos \phi) \tag{19}$$

which is von Karman equation, it is perfectly straightforward to integrate this equation numerically to find the variation in contact pressure  $p(\phi)$  once the frictional conditions at the interface are specified. Before electronic computer were available, however, various simplifications of von Karman equation were proposed to facilitate integration. For relatively large rolls it is reasonable to put  $\sin \phi \approx \phi$ ,  $\cos \phi \approx 1$  etc. and to retain only first order terms in  $\phi$ . The roll profile is then approximated by

$$h \approx h_0 + R\phi^2 \approx h_0 + x^2/R \quad (20)$$

Making these approximations in (19), neglecting the term  $q \cdot \tan\phi$  compared with  $p$ , and changing the position variable from  $\phi$  to  $x$  give:

$$h \frac{dp}{dx} = 4k \frac{x}{R} + 2q \quad (21)$$

In addition, it is consistent with neglecting second order terms in  $\phi$  to replace  $h$  by the mean thickness addition, it is second order terms in  $\phi$  to replace  $h$  by the mean thickness  $\bar{h} = \frac{1}{2}(h_1 + h_0)$ . To proceed, the frictional traction  $q$  must be specified.

### 3. Conclusion

The paper explains through the theory of contact mechanics how are the elastic contact stress and deformation between rigid plastic in contact, influence by surface deformation and stress in laminate and the roughing process, and how are the elastic contact stress and deformation between rigid plastic in contact, influence by surface deformation and stress in laminate process.

### References

1. Johnson, L. (1985): *Contact mechanics*. Cambridge University Press, ISBN 0-521-25576-7, Cambridge, London
2. Pavelescu, D., Muşat, M., Tudor, A. (1977): *Tribologie (Tribology)*. Editura Didactică și Pedagogică, ISBN 973-9187-42-0, Bucharest, Romania (in Romanian)
3. Enescu, I., Lepădateșcu, B., Enescu, D. (2010): *Simulation of laminate process*. Proceedings of Advanced Composite Materials Engineering and International Conferences Research & Innovation in Engineering, ISSN 1844-9336, p. 273-276, Brasov, Romania
4. Enescu, I., Buzatu C., Lepădateșcu, B., Enescu, D. (2008): *Modelling of the superfinishing process*. Proceedings of Advanced Composite Materials Engineering and International Conferences Research & Innovation in Engineering, ISSN 1844-9336, p. 140-144, Brasov, Romania