Mathematic Modeling and Software for the Solidification Simulation of Centrifugal Castings

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Abstract
The solidification of castings was studied with the help of different software at the Transilvania University of Brasov. To simulate the solidification of the castings with rotational symmetry, 2D software have been made in cylindrical coordinates, especially used to simulate the solidification of centrifugal tubular casting. The two versions of the software are: one for the casting made of eutectic alloys and the other for castings that are made of alloys that solidify in solid solution. This paper presents the mathematic model and the soft designed for the solidification of the rotational symmetry castings from alloys with a solid solution. The possible results obtained with the help of this soft are also shown. The case of a casting from a CuSn12 alloy (alloy with solid solution solidification) is exemplified and the results obtained are compared to those obtained in the case when the casting is made from an alloy with constant temperature solidification (eutectic alloy). The results show great differences regarding the solidification of the two types of alloys. The necessity of having different software is then justified to simulate the solidification of the casting. These have to be adapted to the type of solidification of each type of alloy (that is, alloy with a solidification interval and eutectic alloy).

Key words
casting, centrifugal casting, solidification simulation, cast alloy, mathematic modelling

1. The Purpose of the Paper

Several software have been made at the Transilvania University of Brasov that simulate the solidification of the castings that can reproduce exactly the solidification according to the transformations that accompany it [1, 2, 3, 7, 8, 17, 19]. 3D soft have been designed in Cartesian coordinates to simulate the solidification of the gravitational castings from metals and eutectic alloys, but also from alloys that solidify totally in solid solution (with a solidification interval) [7, 17, 19]. Also, a 3D soft was made in Cartesian coordinates to simulate the solidification of the pieces that were casted continuously and one in 2D cylindrical coordinates to simulate the casting solidification with a revolution symmetry, gravitationally casted from pure metals or eutectic alloys [8]. The latter can also be used to simulate the casting solidification from metals and alloys with constant temperature solidification.

In the case of simple tubular casting that have a constant wall thickness (with straight line generators, cylinders, bushings) and a big length compared to the wall thickness, the solidification simulation around the middle of the castings' length can also be made with enough precision with 1D models in cylindrical coordinates. In this case, the simulation lasts less compared to the 2D one.

A precise simulation of the casting solidification made of eutectic alloys, together with alloys that solidify in a temperature interval, requires mathematic models and therefore, different software. Most of the non-ferrous alloys casted in a centrifugal way solidify in a temperature interval in solid solution: for example bronze (with Sn, Al, Pb), brass, aluminium or titanium based alloys. So far, internationally, a specialized soft to simulate the solidification of the castings in a centrifugal way from such alloys has not been yet designed (solid solution solidification). Therefore, the production of such a software is considered to be necessary. This paper presents the mathematic model that the software is based on, aspects regarding the structure of the soft and the possible outcomes obtained with its help.

2. The Mathematic Modeling Particularities of the Centrifugal Solidification Castings

The centrifugal castings are usually tubular pieces with rotational symmetry. The solidification simulation of such castings, based on mathematic software in Cartesian coordinates, imply the division of the system in cubic volumetric elements. At the edges at the circular boundaries there are differences between the real edge of the casting and that of the simulated one. For a more accurate geometric reproduction (of the boundaries) of the system elements (piece, mould, core) a division of the assembly would be necessary, in very little elements of cubic volume. However, this could lead to a longer simulation process. Moreover, in the case of the software that do not initialize automatically on the basis of the casting drawing, the introduction of the cast-mould geometrical data requires a more considerable volume of work and time [8].

In the case of the rotational symmetry castings (for example, continuously casted tubular pieces, centrifugal or gravitational) there is the possibility to realize mathematic models and software in cylindrical coordinates. This system of coordinates creates more advantages. The annular volume elements can reproduce exactly the circular borders of the cast-mould assembly. The division is simpler to be made, even in the case of a manual initialization of the simulation. In addition, a volumetric solidification simulation with 2D models is also possible. In the case of 2D models (only two position coordinates of the annular elements), the process duration is considerably reduced. The models in cylindrical coordinates can also show the solidification of the pieces with sectional generators (rollers, drums, wheels, cylinders, etc.) using only two variables of space (through 2D modelling). This way, the duration of the simulation can be cut even hundreds of times compared to the 3D simulation in Cartesian coordinates (at the same step of the mesh network). This happens because of the smaller number of volumetric elements in which the 2D modelling assembly is divided. In the use of Cartesian coordinates, such pieces absolutely require 3D models. Moreover, in the case of 2D models, the equations that describe the thermal balance (the heat transfer and the solidification of the castings) are simpler, and the number of the necessary calculus is much smaller. This also contributes to the reduction of the simulation process.

In conclusion, the usage of cylindrical coordinates in the mathematical modelling of the solidification simulation of the rotational symmetry casting has the great advantage of reducing considerably the simulation duration. For non-sectional cylindrical parts (with straight-line generators), as it is the case of centrifugal castings, 1D models can also be used in cylindrical coordinates. In this case, the equations that build the solidification process of the castings are even simpler, because they only contain two independent variables (one of space "r" – the ray, and "t" – the time) compared to 2D models that use three independent variables (r, y – for space, and t – for time) [8 … 11, 21, 23 … 26].

Another particularity in the rotational symmetry solidification of centrifugal castings is the heat transmission inside the cast-mould assembly through heat currents that are concurrent divergent. The cylindrical coordinates have the advantage that they can model this exact type of heat flow. That is why, for the simulation solidification of the centrifugal rotational symmetry castings, some mathematic models and software in cylindrical coordinates need to be made. In the case of classical castings currently used in the industry (gravitational castings), the pieces have a random geometry. This is the reason why, in academic journals, the models in cylindrical coordinates were used less to simulate the solidification of the castings. Cylindrical coordinates were sometimes used to simulate and study the micro-solidification. In such cases the focus was on the reproduction of the crystal growth because the heat transmission is predominantly unidirectional (only in the direction of the cylindrical sample radius). Modelling the structure's building-up ultimately allows the development of software to predict the structure and the mechanical properties of the castings.

The particularities of heat transmission at centrifugal casting for tubular castings are the following:

- the interior surface of the alloy casted in a mould is practically behaving as a black body, because the heat transmission towards the air in the casting's centre is almost null;
- the mould and alloy rotation intensifies the heat transmission inside the liquid alloy and at the interface of the solidification front;
- the mould rotation determines the intensification of heat transmission between the mould and the environment;

- the heat flow inside the casting-mould assembly is predominantly divergent on the radius direction.

These particularities need to be considered when it comes to the solidification of the centrifugal castings.

3. The Principle of Mathematic Modelling in Cylindrical Coordinates for the Solidification of Centrifugal Casting

The solidification of rotational symmetry parts, with sectional generators (flanges, wheels, cylinders, rollers, etc.) casted centrifugally can be mathematically modelled in 2D, in cylindrical coordinates. In Cartesian coordinates the solidification simulation for this type of piece requires 3D models. The software for 2D simulation in cylindrical coordinates of rotational symmetry solidification have the following advantages:

- accurately reproduce the circular boundaries of the castings;
- model the heat transfer through convergent - divergent flows (on the ray direction) and through parallel flows (on axial direction) more precisely;
- the simulation duration is considerably reduced.

Modelling in cylindrical coordinates of the castings' solidification is based on principles, hypothesis and similar equations as the Cartesian coordinate models previously studied at the Transilvania University of Brasov [7, 8, 30]. The differences are seen in the coordination system, in used variables and in equation differentiation that develops the solutions. As a result, the mathematical equations that describe the heat and cooling transfer processes of the castings are different. This leads to different particularizations of files that are part of the soft structure.

The mathematic model in cylindrical coordinates also uses the method of finite differences. This method has the advantage that it is capable of accurately producing the consecution of heat processes and the phase transformations for all types of solidification (eutectic, hypoeutectic, hypereutectic, peritectic, solid solution). Modelling by finite differences allows the consideration of associated processes too, for example the mould filling, the variation of the thermo-physical characteristics in the temperature, the formation of cavities, etc.

A mathematical model has previously been made, together with a cylindrical coordinates soft, intended to simulate the casting solidification from eutectic alloys [8]. This was used in the study of the solidification of some pieces with a rotational symmetry, casted in a gravitational field. The aspects studied were connected to the soft functioning and the experimental validation of the results, together with aspects connected to the particularities and the optimization of such parts [8, 9, 12, 30].

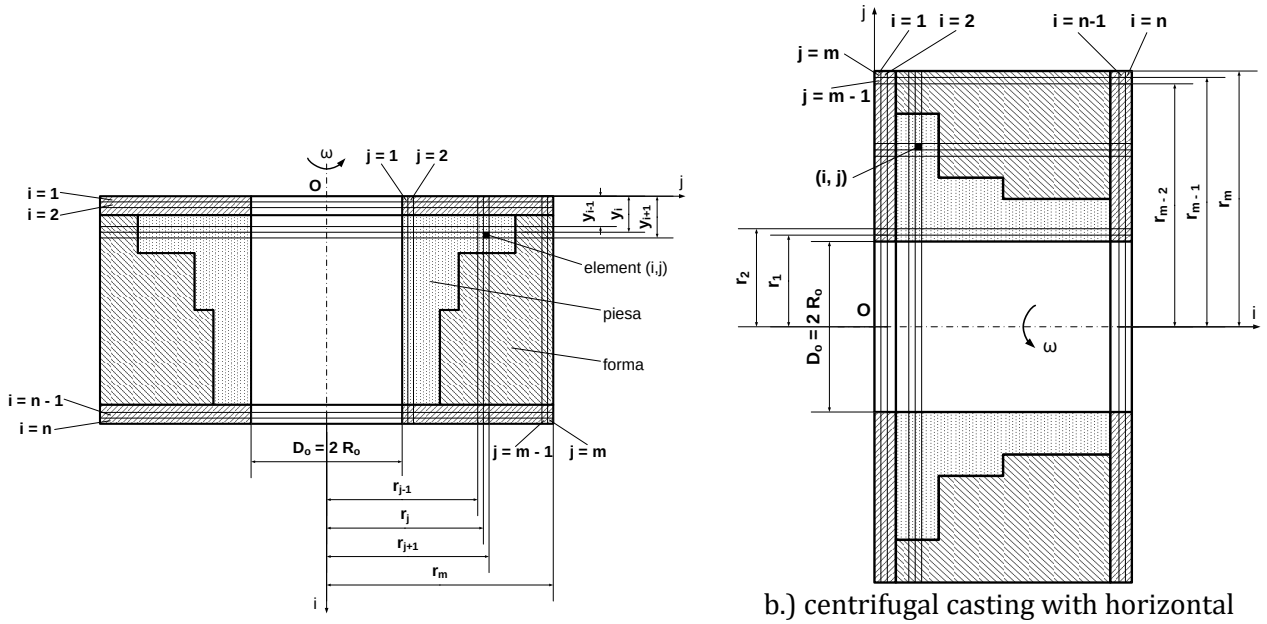
The particularities of centrifugal casting connected to the heat exchange are taken into consideration through adequate values (equivalent) of the thermal conductibility coefficient and that of the heat exchange in the casting - mould - environment assembly. The influence of the rotational movement over the heat transmission inside the liquid alloy and the solid-liquid system is taken into consideration with a coefficient that is equivalent to the thermal conductibility of the liquid alloy.

The cylindrical coordinate's mathematic model is based on the partition of the mould-casting system in annular volumetric elements with a square section having a Δ side. Therefore, the transversal is divided in a 2D system, in square elements. The meshing must be made so that the annular elements are uniform in materials (they should be made of the same material). In Figure 1 you can see the meshing ways in two cases, vertical rotation axis (1.a) and horizontal rotation axis (1.b) [8, 9]. The elements' positioning in the system assembly is given by two numerical coordinates (index): i – axial direction and j – radial direction (Figure 1). The number of elements in axial direction is marked "n", and that in a radial direction is "m" [7, 8, 9, 21 … 26]. The interior ray of the casting is r_0, while the exterior is r_m. Both radii of an annular element (having "i, j" coordinates and Δ thickness) are noted r_{j-1}, respectively r_j. The length of the cylindrical annular elements is Δ. The radii an element are correlated with the relations (Figure 1) [8]:

$$\Delta = r_1 - r_0 = r_j - r_{j-1}. \quad (1)$$

The exterior radius of the assembly is given by the relation (Figure 1)

$$r_j = r_0 + j\cdot\Delta = r_{j-i} + \Delta. \quad (2)$$



a.) centrifugal casting with vertical rotation axis

b.) centrifugal casting with horizontal rotation axis

Fig. 1. Meshing in cylindrical coordinates of centrifugal casting with rotation symmetry
a.) vertical rotation axis; b.) horizontal rotation axis [8, 9]

The mathematical model in cylindrical coordinates takes into consideration the same hypothesis as the models in Cartesian coordinates that refer to the heat transfer in the alloy-mould-environment assembly. The difference is that the heat flow in a radial direction ("*j*" direction) is divergent, while, in axial direction ("*i*" direction) the flow is parallel [8]. Time is divided into finite τ intervals, noted with the numerical coordinate "*k*". The calculus precision is bigger as the space meshing (step Δ) and time (τ time step) is smoother. At a certain moment noted $\tau_k = k \cdot \tau$ (for the time coordinate "*k*"), the state of an annular element (with *i,j* position coordinates) is also given by the T_{ij}^k with the solid fraction ξ_{ij}^k. The annular elements of volume are characterized by the thermal and physical values ρ_{ij}^k – density, c_{ij}^k – specific heat, λ_{ij}^k – thermal conduction coefficient, L_{ij} –latent melting heat. These values depend on the momentary temperature, the element material and the solid fraction.

4. The Particularization and the Resolution of the Mathematical Model

At initial time (time $\tau_0 = 0$, respectively $k = 0$) these parameters are known and noted T_{ij}^0, respectively ξ_{ij}^0. For the annular elements pertaining to the mould assembly (mould, externals cores, mould support) the initial solid fraction is $\xi_{ij}^0 = 1$. For the elements pertaining to the casting, at the initial time ($k = 0$) the solidified fraction is $\xi_{ij}^0 = 0$. For the annular elements pertaining to the cast alloy, in solidification range, at any time ξ_{ij}^k has values between 0 and 1 [7, 8, 17, 19].

In the case of eutectic alloys, the temperature remains constant during solidification. As a result, in the equations that give the internal heat variation (enthalpy), during solidification there is only one unknown quantity- the solid fraction ξ_{ij}^{k+1} of a volumetric element, at the end of a time step. In the case of alloys that solidify in a temperature range (from T_L - liquidus temperature at T_S - solidus temperature) during solidification time, both the temperature and solid fraction of elements are simultaneously changed.

Consequently, the equations that express the internal heat (enthalpy) variation during solidification time contain two unknown quantities - solid fraction ξ_{ij}^{k+1} and temperature T_{ij}^{k+1} (temperature and solid fraction at k+1 time coordinate). The issue of the mathematical model requires that the equations include only one unknown quantity. This is feasible because both unknown quantities – the solid fraction and the temperature of the annular elements – are interdependent. The dependency $\xi = f(T)$ is accurately determined for each alloy composition by applying the law of inverted segments on the equilibrium diagram. This dependency can have different variants according to the chemical

composition of the alloy and the equilibrium diagram's function. The variants of this dependency are shown from Figure 2 to Figure 3, for three alloys in the equilibrium diagram [7]. When the dependency is an equation having degree 2 (or over 2), the answer to this mathematical model is complicated. Moreover, for any type and alloy composition, the analytic expression of the function should be known. In order to obtain an easier way to solve a mathematical model that is applicable to all alloys that solidify as solid solution, it is required that the dependency $\xi = f(T)$ to be given by linear function (degree 1, Figure 3b). On the base of the graph showed in Figure 3b, the equation of this function is [7, 21 ... 26]:

$$\xi = \frac{T_L - T}{T_L - T_S}, \tag{3}$$

where T_L – liquidus temperature, T_S – solidus temperature, T – momentary temperature.

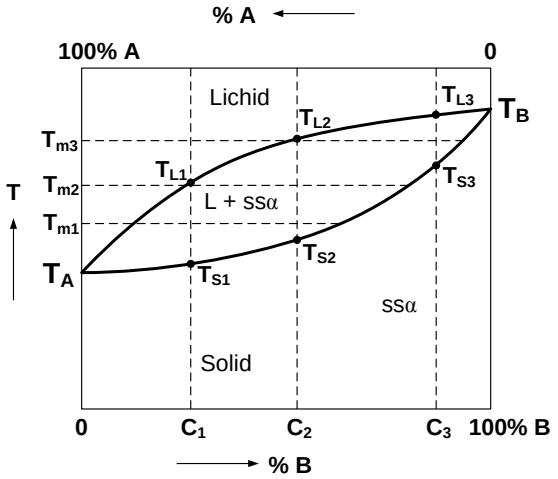
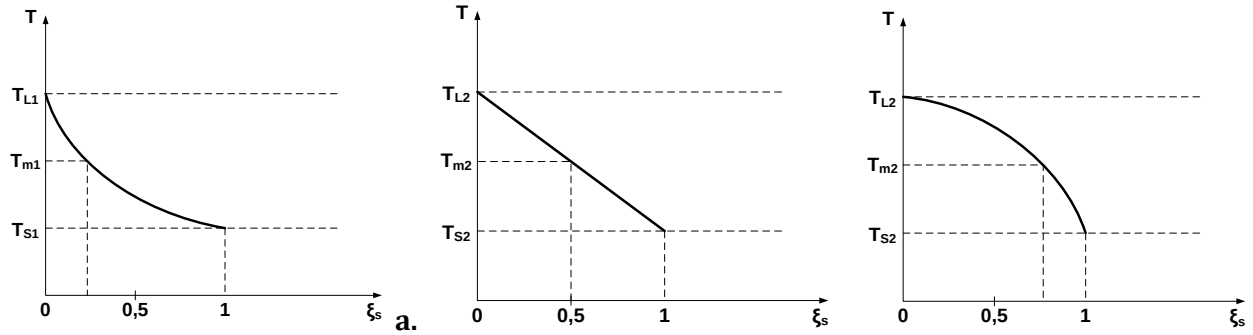


Fig. 2. Binary alloy - stability diagram, solidification in solid solution [7]



Alloy C1 (fig. 2) **b.** Alloy C2 (fig. 2) **c.** Alloy C3 (fig. 2)

Fig. 3. Graph temperature - solid fraction $T = f(\xi)$, alloy with total solubility in solid state [7]

Habitually, this dependency is used to calculate the temperature from solid fraction, given by the equation:

$$T = T_L - \xi \cdot (T_L - T_S) \tag{4}$$

The mathematical model is based on the heat balance equation for each element "*i,j*" from the mould - casting assembly. The equation is written and resolved corresponding to an elementary time interval τ (modelling time step). The heat balance equation for some annular element (having "*i,j*" coordinates) consists in equalizing the internal heat variation (enthalpy) $(\Delta Qm)_{ij}^{k}$ of that element and the changed heat $(\Delta Qs)_{ij}^{k}$ with the four neighbouring elements in a time step. Into all mathematical models, the overall used equation for heat transfer it is [7, 8, 17, 19]:

$$\left(\Delta Qm\right)_{ij}^{k} = \left(\Delta Qs\right)_{ij}^{k}. \tag{5}$$

The transmitted heat from "*i,j*" element to the four bordering elements (situated on inward side, on outside and respectively on upside and downside of this element – Figure 4) is given by the equation [7, 8, 17, 19]:

$$(\Delta Qs)_{ij}^{k} = 2\pi\cdot\Delta\cdot[\alpha i_{ij}^{k}\cdot(T_{ij}^{k}-T_{i,j-1}^{k})\cdot r_{i-1}+\alpha e_{ij}^{k}\cdot(T_{ij}^{k}-T_{i+1,j}^{k})\cdot r_{i}]\cdot\tau+$$
$$+\pi\cdot(r_{i-1}+r_{i})\cdot\Delta\cdot[\alpha h_{ij}^{k}\cdot(T_{ij}^{k}-T_{i-1,j}^{k})+\alpha j_{ij}^{k}\cdot(T_{ij}^{k}-T_{i,j+1}^{k})]\cdot\tau \quad (6)$$

In this equation αh_{ij}^{k}, αj_{ij}^{k}, αi_{ij}^{k}, αe_{ij}^{k} represent the heat transfer coefficients between "*i,j*" element and the neighbouring elements (situated on inward side and on outside). The others symbols are defined above or in Figure 4.

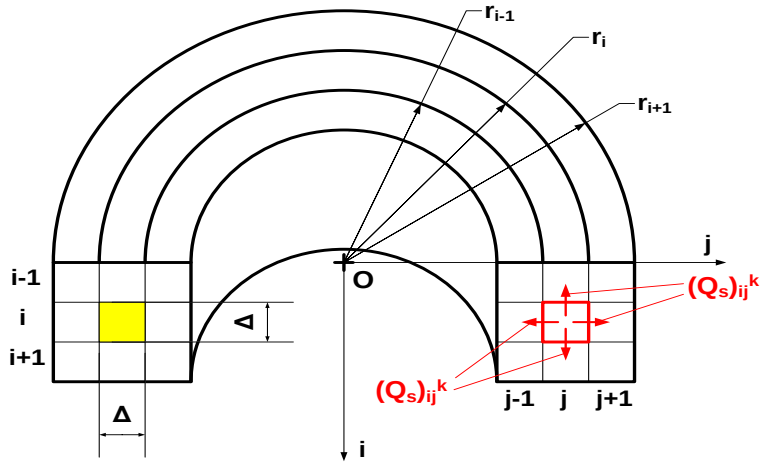


Fig. 4. Heat transfer between annular elements (cylindrical coordinates 2D) [8]

The internal heat (enthalpy) variation of an annular element is calculated according to the temperature and the solid fraction variation [8]. The general equation is:

$$(\Delta Qm)_{ij}^{k} = V_{ij}\cdot\rho_{ij}\cdot\left[c_{ij}^{k}\cdot\left(T_{ij}^{k}-T_{ij}^{k+1}\right)+\left(\xi_{ij}^{k+1}-\xi_{ij}^{k}\right)\cdot L_{ij}\right] \quad (7)$$

where V_{ij} is the volume of the "*i,j*" annular element; c_{ij}^{k} – specific heat of the „*i,j*" element, at „τ_k" time; ρ_{ij} – density of element. By replacing the volume of the annular element according to dimensions (radii and Δ) this equation becomes:

$$(\Delta Qm)_{ij}^{k} = \pi\cdot(r_{i}+r_{i-1})\cdot\Delta^{2}\cdot\rho_{ij}\cdot\left[c_{ij}^{k}\cdot\left(T_{ij}^{k}-T_{ij}^{k+1}\right)+\left(\xi_{ij}^{k+1}-\xi_{ij}^{k}\right)\cdot L_{ij}\right] \quad (8)$$

Within this mathematical model (for alloys that solidify in a temperature range between T_L – liquidus temperature and T_S – solidus temperature), the equation (8) is developed in relation to the placement of (T_{ij}^{k}) and (T_{ij}^{k+1}) temperature relatively to T_L and T_S temperature of "*i,j*" element. (T_{ij}^{k}) and (T_{ij}^{k+1}) are temperature of "*i,j*" element at "*k*", respectively "*k*+1" time coordinates (beginning and end of a time step). Nine particular cases are developed from the equation (8) and respectively of the thermal balance equation (5). These cases are shown in the Figures 5 ... 7. The obtained equations and their resolution are the following.

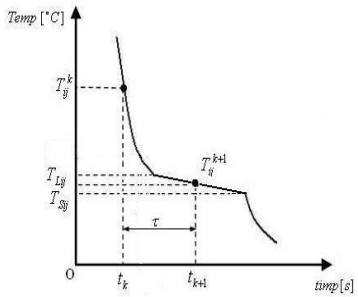
Case 1 - cooling and partial solidification from liquid state (Figure 5a).

If the initial temperature and solid fraction (at "*k*" time) are $T_{ij}^{k} > T_{Lij}$ and $\xi_{ij}^{k} = 0$, and the final state (at "*k*+1" time, after a τ time step) is $T_{ij}^{k+1} \in [T_{Sij}; T_{Lij}]$ and $\xi_{ij}^{k+1} \in [0; 1]$, then the equation (8) becomes:

$$(\Delta Qm)_{ij}^{k} = \pi\cdot(r_{i}+r_{i-1})\cdot\rho_{ij}\cdot\Delta^{2}\cdot\left[\left(T_{ij}^{k}-T_{Lij}\right)\cdot c_{Lij}+\xi_{ij}^{k+1}\cdot L_{ij}+\left(T_{Lij}-T_{ij}^{k+1}\right)\cdot\left(c_{Lij}+c_{Sij}\right)/2\right], \quad (9)$$

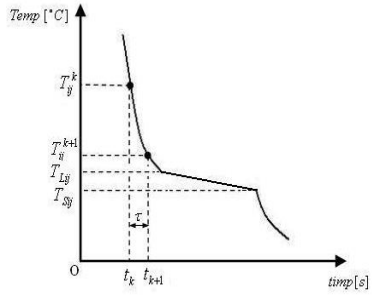
where: L_{ij} – solidification latent heat of "*i,j*" element; c_{Lij}, c_{Sij} – specific heat in liquid and respectively solid state for "*i,j*" element at moment t_k.

Case 1
$T_{ij}^k > T_{Lij}$ and $T_{ij}^{k+1} \in [T_{Sij}; T_{Lij}]$



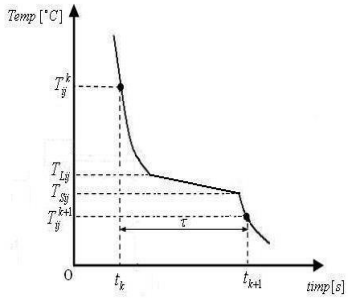
a.) partial solidification

Case 2
$T_{ij}^k > T_{Lij}$ and $T_{ij}^{k+1} > T_{Lij}$



b.) cooling in liquid state

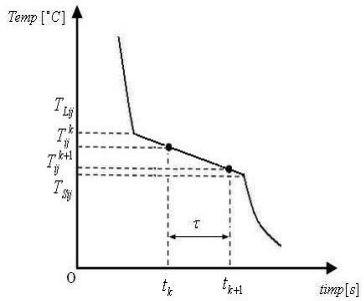
Case 3
$T_{ij}^k > T_{Lij}$ and $T_{ij}^{k+1} < T_{Sij}$



c.) total solidification

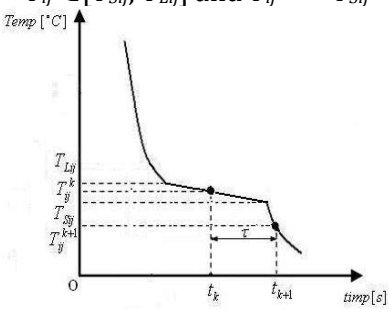
Fig. 5. Change of the element temperature for a time step "τ" (cases $T_{ij}^k > T_{Lij}$) [7]

Case 4
$T_{ij}^k \in [T_{Sij}; T_{Lij}]$ and $T_{ij}^{k+1} \in [T_{Sij}; T_{Lij}]$



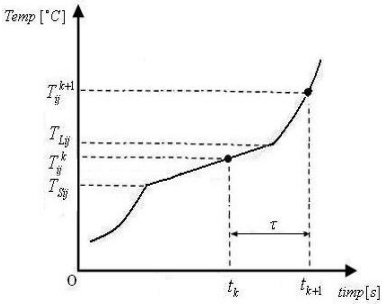
a.) partial solidification inside solidification range

Case 5
$T_{ij}^k \in [T_{Sij}; T_{Lij}]$ and $T_{ij}^{k+1} < T_{Sij}$



b.) total solidification from solidification range

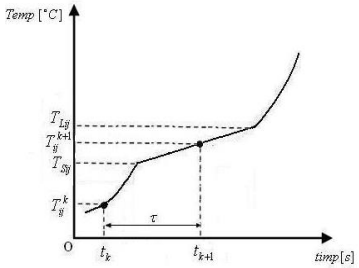
Case 6
$T_{ij}^k > T_{Lij}$ and $T_{ij}^{k+1} > T_{Lij}$



c.) total remelting from solidification range

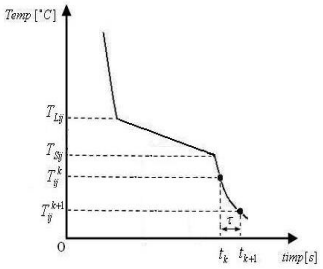
Fig. 6. Change of the element temperature for a time step "τ" (cases $T_{ij}^k \in [T_{Sij}; T_{Lij}]$) [7]

Case 7
$T_{ij}^k < T_{Sij}$ and $T_{ij}^{k+1} \in [T_{Sij} \div T_{Lij}]$



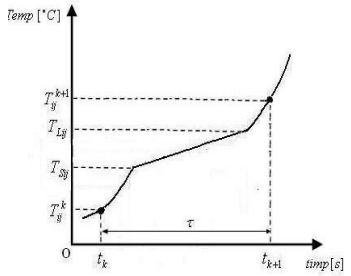
a.) partial remelting from solid state

Case 8
$T_{ij}^k < T_{Sij}$ and $T_{ij}^{k+1} < T_{Sij}$



b.) cooling in solid state

Case 9
$T_{ij}^k < T_{Sij}$ and $T_{ij}^{k+1} > T_{Lij}$



c.) total remelting from solid state

Fig. 7. Change of the element temperature for a time step "τ" (cases $T_{ij}^k < T_{Sij}$) [7]

If $T_{ij}^{k+1} = T_{Lij} - \xi_{ij}^{k+1}(T_{Lij} - T_{Sij})$ (equation (4)) is substituted in this equation (9), the internal heat variation of „*i,j*" element is obtained. The resulted equation has only one unknown quantity (ξ_{ij}^{k+1}).

$$\left(\Delta Qm\right)_{ij}^k = \pi \cdot \left(r_i + r_{i-1}\right) \cdot \rho_{ij} \cdot \Delta^2 \cdot \left[\left(T_{ij}^k - T_{Lij}\right) \cdot c_{Lij} + \xi_{ij}^{k+1} \cdot L_{ij} + \xi_{ij}^{k+1} \cdot \left(T_{Lij} - T_{Sij}\right) \cdot \left(c_{Lij} + c_{Sij}\right) / 2\right] \tag{10}$$

To resolve the model, the equations (6) and (10) are substituted in the thermal balance equation (5). From the obtained equation we can calculate ξ_{ij}^{k+1}. The solution is given by the relation:

$$\xi_{ij}^{k+1} = \left(\frac{\left(\Delta Qs\right)_{ij}^k}{\pi \cdot \left(r_i + r_{i-1}\right) \cdot \rho_{ij} \cdot \Delta^2} - \left(T_{ij}^k - T_{Lij}\right) \cdot c_{Lij} \right) / \left(L_{ij} + \left(T_{Lij} - T_{Sij}\right) \cdot \frac{c_{Lij} + c_{Sij}}{2} \right) \tag{11}$$

The calculated value ξ_{ij}^{k+1} (solid fraction at the following time step "k+1") by this equation, is compatible (is accepted as a solution) if $\xi_{ij}^{k+1} \in [0; 1]$. If not, we get to one of the following cases. If the obtained value by the equation (11) is $\xi_{ij}^{k+1} < 0$, then $\xi_{ij}^{k+1}=0$ and T_{ij}^{k+1} temperature is calculated by case 2. Else, if the calculated value by equation (11) is $\xi_{ij}^{k+1}>1$, then a compatible solution for the solid fraction is $\xi_{ij}^{k+1} = 1$ and T_{ij}^{k+1} temperature is calculated in concordance with Case 3.

Case 2 - cooling and heating only in liquid state (Figure 5b) corresponds with the situation $T_{ij}^{k} > T_{Lij}$, $\xi_{ij}^{k} = 0$ and $T_{ij}^{k+1} > T_{Lij}$, $\xi_{ij}^{k+1} = 0$. If the answer of Case 1 is $\xi_{ij}^{k+1} < 0$, Case 2 is then solved. In this case, the internal heat variation is given by following equation:

$$\left(\Delta Qm\right)_{ij}^{k} = \pi \cdot \left(r_i + r_{i-1}\right) \cdot \rho_{ij} \cdot \Delta^2 \cdot \left(T_{ij}^{k} - T_{ij}^{k+1}\right) \cdot c_{Lij} . \tag{12}$$

By replacing this relationship in the thermal balance equation (5), the answer for T_{ij}^{k+1} temperature is obtained:

$$T_{ij}^{k+1} = T_{ij}^{k} - \frac{\left(\Delta Qs\right)_{ij}^{k}}{\pi \cdot \left(r_i + r_{i-1}\right) \cdot \rho_{ij} \cdot \Delta^2 \cdot c_{Lij}} . \tag{13}$$

The solution calculated is compatible if $T_{ij}^{k+1} > T_{Lij}$. In this case $\xi_{ij}^{k+1} = 0$.

If the issue of case (1) is not compatible as a solution and $\xi_{ij}^{k+1} > 1$, we resolve Case 3.

Case 3 - cooling in liquid state, followed by total solidification and cooling in solid state (Figure 5c).

This case corresponds to the conditions $T_{ij}^{k} > T_{Lij}$, $\xi_{ij}^{k} = 0$ and $T_{ij}^{k+1} < T_{Sij}$, $\xi_{ij}^{k+1} = 1$. In this case, the internal heat variation is given by the equation:

$$\left(\Delta Qm\right)_{ij}^{k} = \pi \cdot \left(r_i + r_{i-1}\right) \cdot \Delta^2 \cdot \rho_{ij} \cdot \left[\left(T_{ij}^{k} - T_{Lij}\right) \cdot c_{Lij} + L_{ij} + \frac{\left(T_{Lij} - T_{Sij}\right) \cdot \left(c_{Sij} + c_{Lij}\right)}{2} + \left(T_{Sij} - T_{ij}^{k+1}\right) \cdot c_{Sij} \right]. \tag{14}$$

The relation is compacted as follows:

$$\left(\Delta Qm\right)_{ij}^{k} = \pi \cdot \left(r_i + r_{i-1}\right) \cdot \Delta^2 \cdot \rho_{ij} \cdot \left[T_{ij}^{k} \cdot c_{Lij} - T_{ij}^{k+1} \cdot c_{Sij} + L_{ij} + \frac{\left(T_{Lij} + T_{Sij}\right) \cdot \left(c_{Sij} - c_{Lij}\right)}{2} \right]. \tag{15}$$

The final temperature T_{ij}^{k+1} is calculated by replacing the thermal balance equation (5). The answer is:

$$T_{ij}^{k+1} = T_{Sij} + \frac{L_{ij}}{c_{Sij}} + \frac{\left(T_{ij}^{k} - T_{Lij}\right) \cdot c_{Lij}}{c_{Sij}} + \frac{\left(T_{Lij} - T_{Sij}\right) \cdot \left(c_{Sij} + c_{Lij}\right)}{2c_{Sij}} - \frac{\left(\Delta Qs\right)_{ij}^{k}}{\pi \cdot \left(r_{i-1} + r_i\right) \cdot \Delta^2 \cdot \rho_{ij} \cdot c_{Sij}} . \tag{16}$$

By compaction we obtain:

$$T_{ij}^{k+1} = \frac{L_{ij}}{c_{Sij}} + \frac{T_{ij}^{k} \cdot c_{Lij}}{c_{Sij}} + \frac{\left(T_{Lij} + T_{Sij}\right) \cdot \left(c_{Sij} - c_{Lij}\right)}{2 \cdot c_{Sij}} - \frac{\left(\Delta Qs\right)_{ij}^{k}}{\pi \cdot \left(r_{i-1} + r_i\right) \cdot \Delta^2 \cdot \rho_{ij} \cdot c_{Sij}} . \tag{17}$$

The issue so calculated is compatible (it is accepted) if $T_{ij}^{k+1} < T_{Sij}$. In this case, $\xi_{ij}^{k+1} = 1$.

Case 4 - partial melting or solidification only in solidification range (Figure 6.a).

It corresponds to conditions $T_{ij}^{k} \in [T_{Sij}; T_{Lij}]$, $\xi_{ij}^{k} \in [0; 1]$ and $T_{ij}^{k+1} \in [T_{Sij}; T_{Lij}]$, $\xi_{ij}^{k+1} \in [0; 1]$. The internal heat variation of an annular element is calculated by the equation:

$$\left(\Delta Qm\right)_{ij}^{k} = \pi \cdot \left(r_i + r_{i-1}\right) \cdot \Delta^2 \cdot \rho_{ij} \cdot \left[\left(\xi_{ij}^{k+1} - \xi_{ij}^{k}\right) \cdot L_{ij} + \left(T_{ij}^{k} - T_{ij}^{k+1}\right) \cdot \left(c_{Lij} + c_{Sij}\right) / 2 \right]. \tag{18}$$

By replacing $T_{ij}^{k+1} = T_{Lij} - \xi_{ij}^{k+1} \cdot (T_{Lij} - T_{Sij})$ based on equation (4) it is obtained:

$$\left(\Delta Qm\right)_{ij}^{k} = \pi \cdot \left(r_i + r_{i-1}\right) \cdot \Delta^2 \cdot \rho_{ij} \cdot \left\{ \left(\xi_{ij}^{k+1} - \xi_{ij}^{k}\right) \cdot L_{ij} + \left[T_{ij}^{k} - T_{Lij} + \xi_{ij}^{k+1} \cdot \left(T_{Lij} - T_{Sij}\right)\right] \cdot \frac{c_{Lij} + c_{Sij}}{2} \right\}. \tag{19}$$

The solution for the solid fraction is obtained from the thermal balance equation:

$$\xi_{ij}^{k+1}=\left[\frac{(\Delta Qs)_{ij}^{k}}{\pi\cdot(r_i+r_{i-1})\cdot\rho_{ij}\cdot\Delta^2}+\xi_{ij}^{k}\cdot L_{ij}+\left(T_{Lij}-T_{ij}^{k}\right)\cdot\frac{c_{Lij}+c_{Sij}}{2}\right]\Bigg/\left[L_{ij}+\cdot\left(T_{Lij}-T_{Sij}\right)\cdot\frac{c_{Lij}+c_{Sij}}{2}\right]. \quad (20)$$

This answer is compatible if $\xi_{ij}^{k+1}\in[0;\ 1]$. Then T_{ij}^{k+1} is calculated by equation (4). If the calculated value by the equation (20) is $\xi_{ij}^{k+1} > 1$, the issue is not compatible and case (5) will further be completed.

Case 5 - total solidification and cooling from the liquidus - solidus range (Figure 6b).

This corresponds to conditions $T_{ij}^{k}\in[T_{Sij};\ T_{Lij}]$, $\xi_{ij}^{k}\in[0;\ 1]$ and $T_{ij}^{k+1} < T_{Sij}$, $\xi_{ij}^{k+1} = 1$. The equation to calculate the internal heat variation is:

$$(\Delta Qm)_{ij}^{k}=\pi\cdot(r_i+r_{i-1})\cdot\Delta^2\cdot\rho_{ij}\cdot\left[\left(1-\xi_{ij}^{k}\right)\cdot L_{ij}+\left(T_{ij}^{k}-T_{Sij}\right)\cdot\left(c_{Lij}+c_{Sij}\right)/2+\left(T_{Sij}-T_{ij}^{k+1}\right)\cdot c_{Sij}\right]. \quad (21)$$

From the thermal balance equation the final temperature T_{ij}^{k+1} is calculated by the equation:

$$T_{ij}^{k+1}=\frac{\left(1-\xi_{ij}^{k}\right)\cdot L_{ij}}{c_{Sij}}+\frac{\left(T_{ij}^{k}-T_{Sij}\right)\cdot\left(c_{Lij}+c_{Sij}\right)}{2\cdot c_{Sij}}+T_{Sij}-\frac{(\Delta Qs)_{ij}^{k}}{\pi\cdot(r_i+r_{i-1})\cdot\rho_{ij}\cdot\Delta^2\cdot c_{Sij}}. \quad (22)$$

This value is compatible if $T_{ij}^{k+1} < T_{Sij}$. Then, $\xi_{ij}^{k+1} = 1$.

If ξ_{ij}^{k+1}calculated by the equation (20) is $\xi_{ij}^{k+1} < 0$, then the alloy is completely molten and it is heated in liquid state. In this case, Case 6 is further analyzed.

Case 6 - total melting and heating in liquid state (Figure 6c) corresponds to conditions $T_{ij}^{k}\in[T_{Sij};\ T_{Lij}]$ and $\xi_{ij}^{k}\in[0;\ 1]$, and $T_{ij}^{k+1} > T_{Lij}$ and $\xi_{ij}^{k+1} = 0$. The internal heat variation is calculated by the equation:

$$(\Delta Qm)_{ij}^{k}=\pi\cdot(r_i+r_{i-1})\cdot\Delta^2\cdot\rho_{ij}\cdot\left[-\xi_{ij}^{k}\cdot L_{ij}+\left(T_{ij}^{k}-T_{Lij}\right)\cdot\left(c_{Lij}+c_{Sij}\right)/2+\left(T_{Lij}-T_{ij}^{k+1}\right)\cdot c_{Lij}\right]. \quad (23)$$

From the thermal balance equation, the temperature T_{ij}^{k+1} is calculated by the equation:

$$T_{ij}^{k+1}=T_{Lij}-\frac{(\Delta Qs)_{ij}^{k}}{\pi\cdot(r_i+r_{i-1})\cdot\rho_{ij}\cdot\Delta^2\cdot c_{Lij}}-\frac{\xi_{ij}^{k}\cdot L_{ij}}{c_{Lij}}+\frac{\left(T_{ij}^{k}-T_{Lij}\right)\cdot\left(c_{Lij}+c_{Sij}\right)}{2\cdot c_{Lij}}. \quad (24)$$

This value is compatible if $T_{ij}^{k+1} > T_{Sij}$. Then, $\xi_{ij}^{k+1} = 0$.

Case 7 - heating from solid state and partial melting (Figure 7a).

This case corresponds to the following conditions $T_{ij}^{k} < T_{Sij}$, $\xi_{ij}^{k} = 1$ and $T_{ij}^{k+1}\in[T_{Sij} \div T_{Lij}]$, $\xi_{ij}^{k+1}\in[0;\ 1]$. The internal heat variation is calculated by the equation:

$$(\Delta Qm)_{ij}^{k}=\pi\cdot(r_i+r_{i-1})\cdot\Delta^2\cdot\rho_{ij}\cdot\left[\left(T_{ij}^{k}-T_{Sij}\right)\cdot c_{Sij}-\left(1-\xi_{ij}^{k+1}\right)\cdot L_{ij}+\left(T_{Sij}-T_{ij}^{k+1}\right)\cdot\left(c_{Lij}+c_{Sij}\right)/2\right]. \quad (25)$$

By substituting T_{ij}^{k+1} according to ξ_{ij}^{k+1} (based on (4)), $T_{ij}^{k+1} = T_{Lij} - \xi_{ij}^{k+1}\cdot(T_{Lij} - T_{Sij})$ it is obtained:

$$(\Delta Qm)_{ij}^{k}=\pi\cdot(r_i+r_{i-1})\cdot\Delta^2\cdot\rho_{ij}\cdot\left\{\left(T_{ij}^{k}-T_{Sij}\right)\cdot c_{Sij}+\xi_{ij}^{k+1}\cdot L_{ij}-L_{ij}+\left[T_{Sij}-T_{Lij}+\xi_{ij}^{k+1}\left(T_{Lij}-T_{Sij}\right)\right]\cdot\frac{c_{Lij}+c_{Sij}}{2}\right\}. \quad (26)$$

By replacing in the thermal balance equation, the solid fraction for following time step, ξ_{ij}^{k+1} is detailed:

$$\xi_{ij}^{k+1}=\left[\frac{(\Delta Qs)_{ij}^{k}}{\pi\cdot(r_i+r_{i-1})\cdot\rho_{ij}\cdot\Delta^2}-\left(T_{ij}^{k}-T_{Sij}\right)\cdot c_{Sij}+L_{ij}-\left(T_{Sij}-T_{Lij}\right)\cdot\frac{c_{Lij}+c_{Sij}}{2}\right]\Bigg/\left[L_{ij}+\left(T_{Lij}-T_{Sij}\right)\cdot\frac{c_{Lij}+c_{Sij}}{2}\right]. \quad (27)$$

This value is compatible if $\xi_{ij}^{k+1}\in[0;\ 1]$. Then $T_{ij}^{k+1}$is calculated by equation (4). If the calculated value is $\xi_{ij}^{k+1} > 1$ Case 8 will further be analysed. If $\xi_{ij}^{k+1} < 1$, then Case 9 will be further completed.

Case 8 - heating or cooling only in solid state (Figure 7b) corresponds to conditions $T_{ij}^k < T_{Sij}$ and $\xi_{ij}^k = 1$, while $T_{ij}^{k+1} < T_{Sij}$, while $\xi_{ij}^{k+1} = 1$. The equation to calculate the internal heat variation is:

$$(\Delta Qm)_{ij}^k = \pi \cdot (r_i + r_{i-1}) \cdot \Delta^2 \cdot \rho_{ij} \cdot \left(T_{ij}^k - T_{ij}^{k+1}\right) \cdot c_{Sij}. \quad (28)$$

The thermal balance equation (5) is replaced and thus the temperature T_{ij}^{k+1} is calculated:

$$T_{ij}^{k+1} = T_{ij}^k - \frac{(\Delta Qs)_{ij}^k}{\pi \cdot (r_i + r_{i-1}) \cdot \rho_{ij} \cdot \Delta^2 \cdot c_{Sij}}. \quad (29)$$

The obtained value is a compatible solution if it verifies the condition $T_{ij}^{k+1} < T_{Sij}$. In this case $\xi_{ij}^{k+1} = 1$.

Case 9 - Heating from solid state followed by total melting and heating in liquid state (Figure 7c).

In this case, the conditions $T_{ij}^k < T_{Sij}$ are verified, $\xi_{ij}^k = 1$ and $T_{ij}^{k+1} > T_{Lij}$, $\xi_{ij}^{k+1} = 0$. The internal heat variation is calculated by the equation:

$$(\Delta Qm)_{ij}^k = \pi \cdot (r_i + r_{i-1}) \cdot \Delta^2 \cdot \rho_{ij} \cdot \left[\left(T_{ij}^k - T_{Sij}\right) \cdot c_{Sij} - L_{ij} + \frac{\left(T_{Sij} - T_{Lij}\right) \cdot \left(c_{Sij} + c_{Lij}\right)}{2} + \left(T_{Lij} - T_{ij}^{k+1}\right) \cdot c_{Lij} \right]. \quad (30)$$

The temperature T_{ij}^{k+1} is calculated by the following thermal balance equation

$$T_{ij}^{k+1} = T_{ij}^k \cdot \frac{c_{Sij}}{c_{Lij}} - \frac{L_{ij}}{c_{Lij}} + \frac{\left(T_{Lij} + T_{Sij}\right) \cdot \left(c_{Lij} - c_{Sij}\right)}{2 \cdot c_{Lij}} - \frac{(\Delta Qs)_{ij}^k}{\pi \cdot (r_i + r_{i-1}) \cdot \rho_{ij} \cdot \Delta^2 \cdot c_{Lij}}. \quad (31)$$

This value is an accepted solution if the calculated value verifies the compatibility condition $T_{ij}^{k+1} > T_{Lij}$. In this case, $\xi_{ij}^{k+1} = 0$.

The resolution of this mathematical model needs to admit only one compatible solution for each time step.

5. Elements Regarding the Software Structure

The mathematic model and the solving scheme were transposed in the soft mode. This applies to the solidification simulation of the castings with a centrifugal rotational symmetry but also gravitationally, from alloys with a temperature solidification interval (solid solution type). The designed soft is called **SIM-2D-CIL-SOLSOL**. With a considerable approximation this can also be applied to alloys of type hipo- or hiper- eutectic or with peritectic transformation. This approximation is currently practiced in all the software used in foundries internationally. The approximation consists in the fact that the eutectic landing (or peritectic) at constant temperature that appears in these alloys, is replaced with a solidification in a very small temperature interval, of 3-5°C. Like in the case of other softs previously designed at the Transilvania University, MATLAB software was used in this case too. This choice considers the compatibility of all soft modules made at the Transilvania University of Brasov and also the possibility that these could be further assembled in a general soft applicable to all types of alloys [7, 8, 17, 19].

The soft is realized under the hypothesis that the simulation assembly is made of at most four components (from different materials), that is: the mould, the casted alloy, cooler, core (or layer of thermal insulator paint).

The soft works with four different types of files are: types "_.m", "_.in", "_.out" and "_.jpg".

The files of "_.m" type are files that work with functions and MATLAB operators. They contain the functions that calculate, based on the mathematic model, at every time step, the values of the thermo-physical characteristics and the parameters of the elements in which the system is divided. Moreover, it generates the files that store the numerical and graphical results [7, 8, 17, 19]. According to the role they have inside the soft, the files ("_.m") are of four types [7, 8, 17, 19]:

1. Files that initialize the simulation ("form.m"). Here, values and data are introduced regarding the geometry and the component dimensions from the system, casting, mould, core, cooler and values of the thermo-physical parameters of the materials.

2. Files that calculate the values of the momentary thermo-physical characteristics of the volumetric cells (temperature and solid gravity). These files generate the matrices with the parameter values at each time step.
3. Files that calculate and store in the matrices the parameters' evolution (temperature and solid fraction) of the cells in the division network at every time step. Also, they calculate other parameters that characterize the thermal field and the solidification of the casting (momentary cooling speed, the molten quantity in the system).
4. Files that generate the graphical representation of the results.

In the files of the type "_.in", at every run, the user introduces data regarding the results that the run should give at the casting solidification. For example, the numerical coordinates of the points to retain the evolution of the parameters, the line, column and time coordinates to retain the temperature distribution in the casting-mould system are introduced, together with the coordinates necessary to draw the contour of the casting at the temperature field representations. At the end of the program running, the numerical results are shown in a table having the extension "_.out". The graphic results are given in dependency curves or in colour maps and level curves in "_.jpg" files.

6. Results Related to the Solidification of the Centrifugal Castings Obtained with the Soft SIM-2D-CIL-SOLSOL

As mentioned above, the soft presented in this paper applies to the solidification simulation of rotational symmetry castings centrifugally casted from alloys that form a solid solution in the solidification process. The gravitational casting solidification can also be simulated, as it is a particular case of centrifugal casting (when the speed is zero). It renders similar results with software previously made at the Transilvania University of Brasov. In addition, compared to the software designed to simulate the solidification of castings made from eutectic alloys, these also supply information regarding the biphasic zone (solid + liquid) from the wall of the castings during solidification. The results are shown in tables or in graphs. The graphic representations are functional curves (related to time and space) or in maps in colour fields (related to temperature distribution, cooling speed, etc.). Unlike the internationally commercialized software, widely used in the industry, on the colour maps there are level curves. These allow the precise delimitation of certain areas and processes that happen in the casting. Therefore, the starting points of the cavity formation, the sensitive area positioning to macro and micro cavities are easily highlighted.

The main results regarding the solidification of the castings provided by this soft are [4, 5, 7, 8]:

- the numerical coordinates of the points where the solidification of the casted alloy ends;
- the initial and end moments of the alloy solidification casted in a mould;
- the matrix of the elements temperature found in the system at the end of the liquid alloy solidification and at any moment required by the user at reset;
- the matrix of the "*k*" time steps and the "tsol" solidification time where the solidification of each volumetric element ends in the assembly division system;
- the temperature, solid fraction and cooling speed evolution in a time report, for any point required by user at **initialization;**
- the evolution, according to time, of the molten quantity existent in the system;
- the temperature distribution on different lines of the assembly mould at moments ("*k*" time steps) specified by the user at simulation initialization;
- the temperature distribution on different columns, at moments ("*k*" time steps) specified by the user at simulation initialization;
- the evolution, according to time, of the molten quantity in different areas of the casting.

In addition, by data editing, data can be obtained regarding the initial moment of alloy solidification, medium cooling speed in liquid state (until the beginning of the solidification process), the actual time of solidification of any volumetric element in the system, the position and biphasic zone area, the temperature gradient in any point in the system, the presence and the position of secondary thermal loops, the total volume of cavities or of micro-cavities according to the alloy percentage contraction, etc. Based on the numerical results, the soft can deliver (at user's request) the following graphic representations [4, 5, 7, 8, 17, 19]:

- the temperature field and the isothermal lines in the casting-mould assembly at different moments (having an imprinted temperature distribution);
- the temperature, solid fraction and cooling speed evolution, according to time, in specified points at the beginning of the simulation;
- the temperature distribution on lines and at specified moments at initialization;
- the temperature distribution on columns and at specified moments at initialization;
- the solidification front movement according to time in the casting section;
- the positions of the front (*Ts*) solidus and (T_L) liquidus and the zones solid, solid+ liquid (biphasic zone), liquid, at any moment required by initialization.

The exemplification of the way this soft (SIM-2D-CIL-SOLSOL) shows the graphic results in the case of a centrifugal casting from alloy with solid solution solidification. For this exemplification the casting solidification in Figure 8 was simulated. The casting is made of bronze CuSn12, an alloy with a solid solution solidification type. The simulation was made with initialization data from Table 1. The way of establishing the numerical coordinates of the elements inside the matrix equation used to define the system geometry and the initialization of this file, is shown in Figure 9.

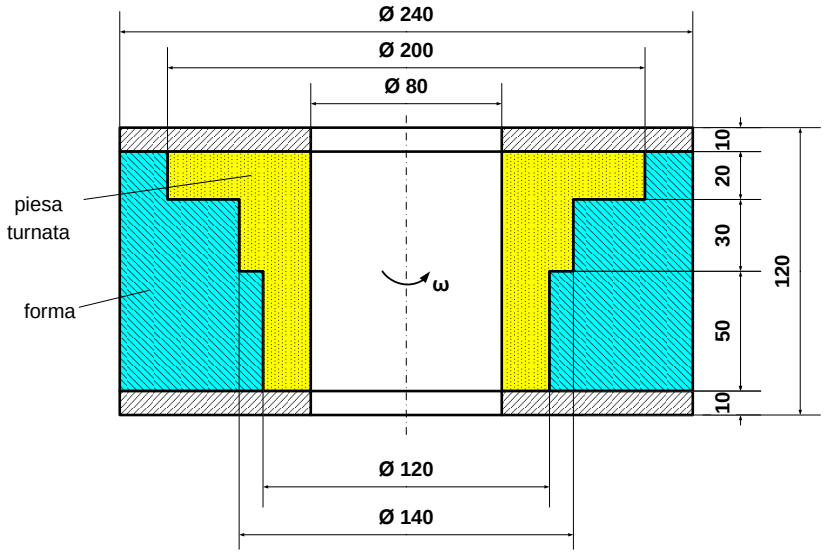


Fig. 8. CuSn12 centrifugal casting studied by solidification simulation with SIM-2D-CIL-SOLSOL software

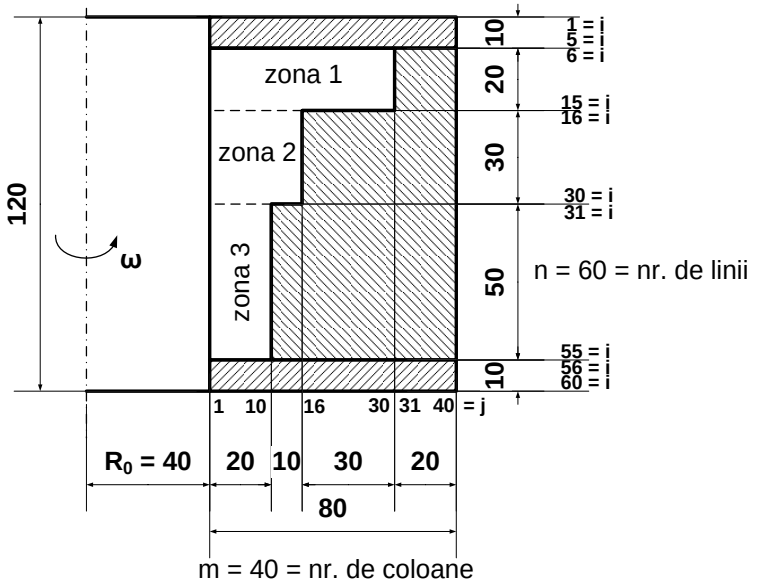


Fig. 9. Numerical coordinates (*i,j*) for the mould - casting system meshing, studied by simulation [4, 5]

Table 1. Geometrical and thermophysical characteristics used in the solidification simulation of CuSn12 centrifugal casting (Figure 8)

No.	Parameter name	Symbol	Measure unit	Value
1	Mesh step	Δ	m	0.002
2	Time step	τ	s	0.008
3	Environment temperature	Tex	^{0}C	20
4	Environment temperature inside mould (in central area of casting)	Tin	^{0}C	200
5	Coefficient of heat exchange mould -environment	αin	W/m^2/K	2
6	Coefficient of heat exchange alloy - environment inside casting	αex	W/m^2/K	50
7	Liquidus temperature of cast alloy	T_{Lme}	^{0}C	790
8	Solidus temperature of cast alloy	Tsme	^{0}C	980
9	Coefficient of thermal conductivity of the mould	λsfo	W/m/K	32
10	Coefficient of thermal conductivity of the cast solid alloy	λsme	W/m/K	80
11	Coefficient of thermal conductivity of the cast liquid alloy	λlme	W/m/K	320
12	Specific heat of the mould	Csfo	J/kg/K	750
13	Specific heat of the liquid alloy	Clme	J/kg/K	500
14	Specific heat of the solid alloy	Csme	J/kg/K	400
15	Mould density	ρfo	Kg/m^3	7600
16	Liquid alloy density	ρme	Kg/m^3	8000
17	Latent heat of solidification of the cast alloy	Lme	J/kg	190000
18	Initial temperature of the mould	T0fo	^{0}C	200
19	Initial temperature of the cast alloy	T0me	^{0}C	1150

In Figures 10-17 there is a graphic representation of the main results supplied by the soft. In Table 2 there are the numerical results of the centrifugal casting solidification from alloys CuSn12. Also, there is data and results regarding the solidification of a centrifugal casting made of eutectic alloy, with similar thermos- physical characteristics (with constant temperature solidification) [4, 5].

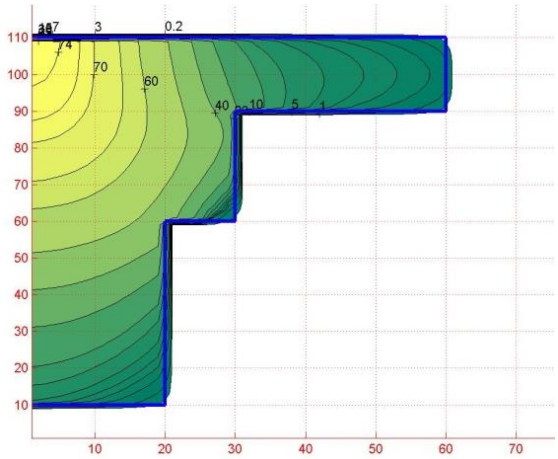


Fig. 10. Movement of solidification line inside CuSn12 centrifugal casting [4, 5]

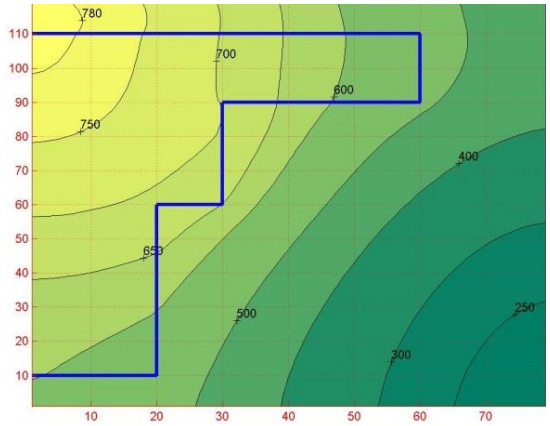


Fig. 11. Isothermal map inside casting - mould assembly at the solidification end of CuSn12 centrifugal casting [4, 5]

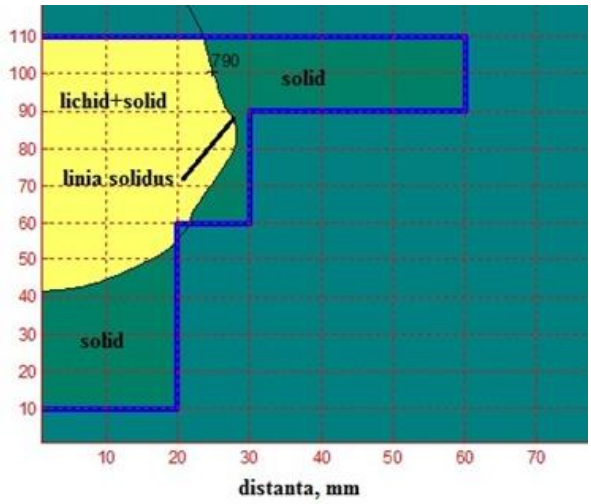


Fig. 12. Solidus line emplacement for a time τ = 40s (CuSn12 casting) [4, 5]

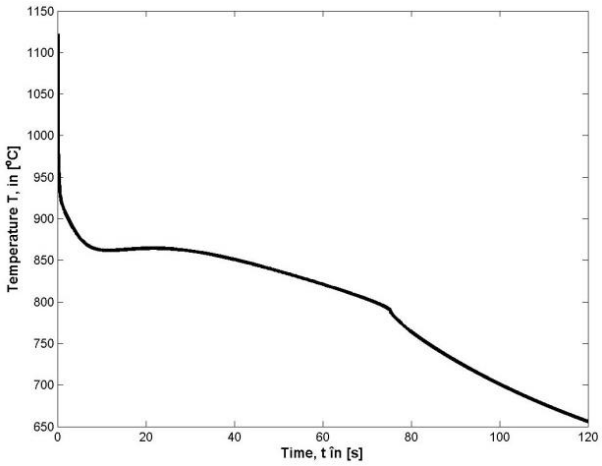


Fig. 13. Temperature variation relative to time inside the casting hot spot (hot spot coordinates (x,y)=(0,110), alloy CuSn12) [4, 5]

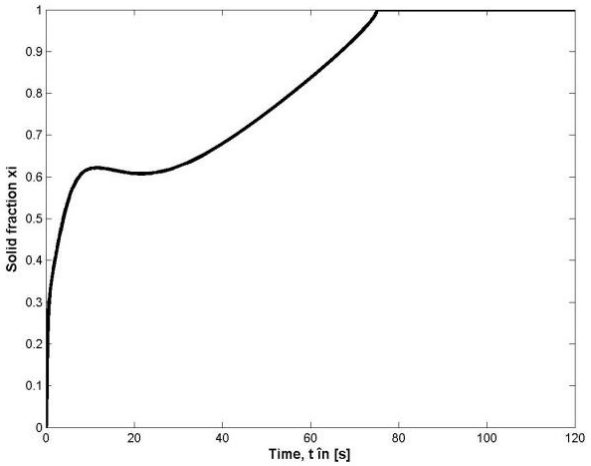


Fig. 14. The variation of solid fraction relative to time in the casting hot spot (hot spot coordinates (x,y)=(0,110), alloy CuSn12) [4, 5]

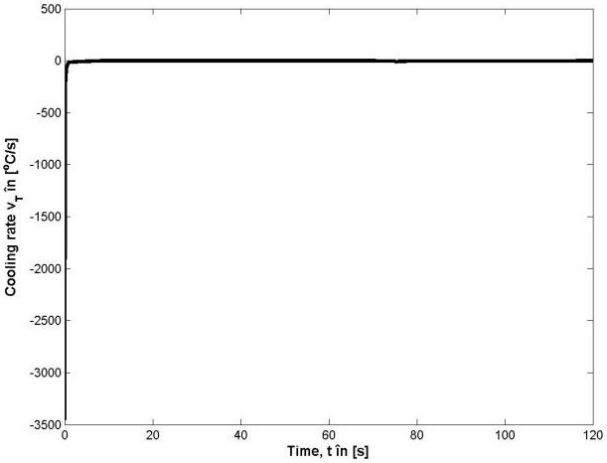


Fig. 15. Cooling rate relative to time inside the casting hot spot (hot spot coordinates (x,y)=(0,110), alloy CuSn12) [4, 5]

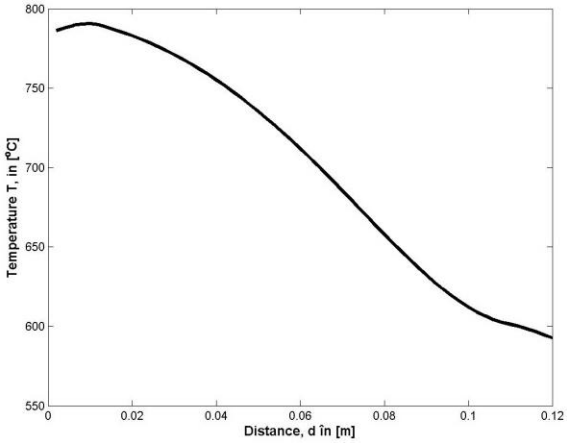


Fig. 16. Temperature distribution on inward surface of centrifugal casting (hot spot column) at the solidification end (CuSn12 alloy, solidification time τ=75.216s) [4, 5]

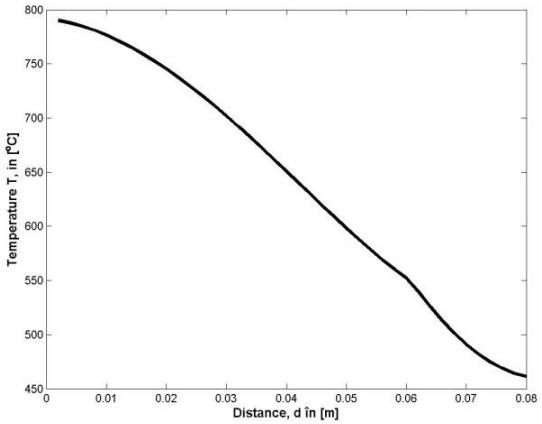


Fig. 17. Temperature on higher surface of centrifugal casting at the solidification end (CuSn12 alloy, solidification time τ=75.216s) [4, 5]

Table 2. Numerical results concerning the centrifugal casting solidification showed in Figure 8 (CuSn12 alloy and eutectic alloy)

No.	Parameter analyzed	Symbol	Simulation for CuSn12 alloy with software 2D-CIL- SOLSOL	Simulation for eutectic alloy with software 2D-CIL-EUT
1	Time of solidification beginning	τ_inc (s)	0.352	0.520
2	Time of solidification end	τ_inc (s)	75.216	104.528
3	Abscissa of hot spot	x (mm)	0	0
4	Ordinate of hot spot	y (mm)	110	99
5	Distance from hot spot to inner surface of casting	x_i (mm)	0	0
6	Distance from hot spot to higher border of casting	y_h (mm)	0	21

Observation: origin of coordinates (x,y) in the left - bottom angle of casting - mould assembly section (Figures 9, 10, 11)

7. Conclusions

The results in Tables 2 and Figures 10 ÷ 17 were compared with results regarding the solidification of an identical casting made of an alloy with an eutectic solidification type (solidification at constant temperature) having similar thermos-physical characteristics [4, 5]. The solidification of the eutectic alloy casting was simulated with an adequate soft (for alloys with constant temperature solidification SIM-2D-CIL-EUT) [4, 5]. The results (like in the casting in Figure 8) showed that the solidification obtained with the two softs (temperature interval and constant temperature solidification) are visibly different. There is a considerable difference between the movements of the solidification front, the hot point emplacement, the solidification duration of the casting, the distribution of the isotherms inside the casting, the temperature variation, the solid fraction and the cooling rate inside the hot point, together with other solidification parameters (Figures 10-17). These differences show that software must be used for the simulation of the castings made of alloys, adapted to the solidification way of each alloy (with constant temperature solidification or with a temperature interval). Therefore, the necessity of designing the soft SIM-2D-CIL-SOLSOL, specialized for alloys with a solidification interval, is justified. In order to obtain correct results there is also the need for specialized software for each solidification type (solid solution, eutectic, hypoeutectic, etc.).

References

1. Ciobanu, I., Dăian, M., Munteanu, S.I., Crişan, A., Bedo. T. (2016): *A Comparative Study on Gravitational and Centrifugal Casting Solidification*. **RECENT**, ISSN 1582-0246, Vol. 17, no. 1(47) p. 11-25
2. Ciobanu, I., Dăian, M., Crişan, A., Bedo, T., Munteanu, S.I. (2016): *Studiu prin simulare pe calculator privind solidificarea unei piese din fontă eutectică turnată centrifugal* (*A Study by Computerized Simulation on Cast Iron Centrifugal Casting Solidification*). Revista de turnătorie, ISSN 1224-2144, nr. 1-2, p. 19-31 (in Romanian)
3. Ciobanu, I., Dăian, M., Munteanu, S.I., Crişan, A., Bedo, T. (2016): *Aspects Regarding the Solidification Kinetics of a Cast Iron Centrifugal Casting*. **RECENT**, ISSN 1582-0246, Vol. 17, no. 2(48), p. 92-101
4. Dăian, M. (2017): *Cercetări privind turnarea centrifugală a aliajelor neferoase* (*Researches Concerning Centrifugal Casting of Neferous Alloys*). Research report, Interdisciplinary Doctoral School, Transilvania University of Braşov, Romania (in Romanian)
5. Dăian, M. (2016): *Cercetări privind turnarea centrifugală a aliajelor neferoase* (*Researches Concerning Centrifugal Casting of Neferous Alloys*). Ph.D. thesis, Transilvania University of Brasov, Romania (in Romanian)
6. Guliaev, B.B., et al. (1972): *Procedee speciale de turnare* (*Special Casting Procedures*). Editura Tehnică, Bucureşti, Romania (in Romanian)
7. Ionescu, Daniela (2014): *Simularea solidificării pieselor turnate din aliaje cu solidificare în interval de temperatură* (*Computer Simulation of the Solidification of Castings from Alloys Solidifying within a Temperature Interval*). Ph.D. thesis, Transilvania University of Braşov, Romania (in Romanian)

8. Ionescu, I.C. (2015): *Cercetări privind simularea solidificării pieselor turnate cu simetrie de rotație* (*Researches Regarding the Solidification Simulation of Casting with Rotational Simetry*). Ph.D. thesis, Transilvania University of Braşov, Romania (in Romanian)
10. Ionescu, I., Dăian, M., Ciobanu, I., Varga, B., Bedo, T., Stoicănescu, M. (2016): *Experimental Verification of a Software for Simulation of Centrifugal Casting Solidification*. **RECENT**, ISSN 1582-0246, Vol. 17, no. 1(47), p. 26-32
11. Ionescu, I., Ionescu, D., Ciobanu, I., Jiman, V. (2012): *Mathematical Modelling of eutectic alloy cylindrical castings solidification*. Metalurgia, ISSN 0461-9579, Vol. 64, no. 8, p. 10-20
12. Ionescu, I., Ciobanu, I., Munteanu, S.I., Bedo, T. (2014): *Studiu prin Simulare privind Solidificarea Pieselor Cilindrice Tubulare din Aliaj Al–Si și din Fontă* (*A study by Simulation Concerning Cylindrical Tubular Casting of Al-Si and Cast Iron Alloys Solidification*). Cercetări metalurgice și noi materiale, ISSN 1221-5503, Vol. XXI, nr. 3, p. 37-48 (in Romanian)
13. Ionescu, I., Ciobanu, I., Munteanu, S.I., Bedo, T. (2014): *Studiu privind solidificarea pieselor tubulare cilindrice* (*A study concerning cylindrical tubular casting solidification*). Revista de turnatorie, ISSN 1224-2144, no. 7-8, p. 2-7
14. Iudin, S.B., et al. (1976): *Turnarea centrifugă* (*Centrifugal casting*). Editura Tehnică, Bucureşti, Romania (in Romanian)
15. Jiman, V., Ciobanu, I., Munteanu, S.I (2009): *Particularities of the Mathematical Modelling of Centrifugally Cast Part Solidification*. Metalurgia Internaţional, ISSN 1582-2214, Special Issue no. 10, p. 84-88
16. Jiman, V., Ciobanu, I., Carcea, I., Munteanu, S.I., Bedo. T., Monescu. V. (2011): *The verification by Experiment of Sim 3D Software invented for Castings Solidification Simulation*. Metalurgia Internaţional, ISSN 1582-2214, no. 5, p. 67-70
17. Madhusudhan, S., Narendranath, S., Kumar, G.C.M, Mukunda, P.G. (2010): *Experimental study on rate of solidification of centrifugal casting*. International Journal of Mechanical and Materials Engineering (IJMME), ISSN 1823-0334, Vol. 5, no. 1, p. 101-105
18. Maşniță, M. (2007): *Cercetări privind influența unor factori tehnologici și constructivi asupra solidificării pieselor turnate* (*A Researches Regarding the Influence of Technical and Design Factors on Casting Solidification*). Ph.D. thesis, Transilvania University of Brasov, Romania (in Romanian)
19. Missenard, A. (1975): *Conductivité thermique des solides, liquides, gas et de leur mélanges*. Éditions Eyrolles, Paris, France (in French)
20. Monescu, V. (2010): *Realizarea unui program 3D pentru simularea solidificarii pieselor turnate* (*3D Software for casing solidification simulation*). Ph.D. thesis, Transilvania University of Brasov, Romania (in Romanian)
21. Sofroni, L., et al. (1980): *Bazele teoretice ale turnării* (*Theoretical bases of solidifications*). Editura Didactică și Pedagogică, Bucureşti, Romania (in Romanian)
22. Soporan, V., Constantinescu, V. (1995): *Modelarea la nivel macrostructural a solidificării aliajelor* (*Macro-solidification Modelling of alloys solidification*). Editura Dacia, Cluj-Napoca, Romania (in Romanian)
23. Soporan, V., Constantinescu, V., Crișan M. (1995): *Solidificarea aliajelor, preliminarii teoretice* (*Solidification of Alloys, Theoretical Preliminaries*). Editura Dacia, ISBN 973-97041-1-5, Cluj-Napoca, Romania (in Romanian)
24. Soporan, V., et al. (2010): Modelarea matematică a proceselor care au loc la turnarea pieselor metalice (Mathematical Modelling of the Process from Metalic Castining). Editura Casa Cărţii de Ştiinţă, ISBN 978-973-133-059-4, Cluj-Napoca, Romania (in Romanian)
25. Soporan, V., Vamoş, C., Pavai, C. (2003): *Modelarea numerică a solidificării* (*Numerical Modelling of Solidification*). Editura Dacia, ISBN973-35-1645-7, Cluj-Napoca, Romania (in Romanian)
26. Soporan, V. (1994): Stadiul actual al modelării prin metode numerice a procesului de solidificare a pieselor turnate (*The Actual Level of the Casting Solidification Simulation by Numerical Methods*). Metalurgia, ISSN 0461-9579, Vol. 46, no. 9, p. 20-23
27. Ștefănescu, D. (2001): *Science and Engineering of Casting Solidification*. Department of Material Engineering, University of Alabama, Tuscalosa, Kluwer Academic/Plenum Publishers, ISBN 0-306-46750-X
28. Wei, S., Lampman, S. (2008): *Centrifugal Castin*g. ASM Handbook, Vol. 15: Casting, p. 667-673, DOI: 10.1361/asmhba0005257
29. Suzuki, K., Yao, M. (2004): *Simulation of Mold Filling and Solidification during Centrifugal Precision Casting of Ti-6Al-4V Alloy*. Metals and Materials International, ISSN 1598-9623, Vol. 10, iss. 1, p. 33-38, doi:10.1007/BF03027361
30. Treho, E. (2011): *Centrifugal Casting of an Aluminium Alloy*. Ph.D. thesis, University of Birmingham, UK
31. Bedo, T., Dăian, M., Varga, B., Stoicănescu, M., Crișan, A., Ciobanu, I. (2016): *A Simulation and Experiment Concerning the Solidification of an Al-Zn Alloy Casting*. **RECENT**, ISSN 1582-0246Vol. 17, no. 2(48), p. 83-91

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