RECENT J. (2019), 58:064-069
https://doi.org/10.31926/RECENT.2019.58.064

Mathematical Model Using Regression Functions for Determining the Expression of Surface Roughness at Plain Grinding with Diamond Wheels of Aluminum Oxide Ceramic (Al_2O_3)

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Abstract
The aim of the work is to determine, through mathematical modeling using regression functions, the dependency expression between the surface roughness R_a at grinding with diamond wheels of aluminum oxide ceramic (Al_2O_3) for plain grinding with longitudinal advance, and the values of the grinding processing mode as $R_a = f(S_l, t, V_d)$.

Keywords
aluminum oxide ceramic, roughness, mathematical modeling, regression functions

1. Introduction

Mathematical modelling of the data gathered through experiments is used to determine the expression of roughness $R_a = f(S_l, t, V_d)$, at plain grinding with longitudinal advance, where:

S_l [m/min] – longitudinal advance;
t [mm] – depth of cut;
V_d [m/s] – peripheral speed of diamond wheel,

tested on the parameters of grinding, as function of multiple linear regression.

2. Mathematical Model

For determination of roughness expression at plain grinding, tested on the parameters of grinding, as function of multiple linear regression [1]:

$$Y_i = \beta_0 + \beta_1 \cdot X1_i + \beta_2 \cdot X2_i + \ldots + \beta_p \cdot Xp_i + \varepsilon_i \quad (1)$$

where:
p – represent the number of independent variables in multiple linear regression;
$X1_i, X2_i, \ldots, Xp_i$ – independent variables;
0, 1, ..., p – regression coefficients who must be determined.

For this on made the variable matrix A [2]:

$$\mathbf{A} = \begin{vmatrix} X11 & X12\ldots & X1p \\ X21 & X22\ldots & X2p \\ \ldots & \ldots & \ldots \\ Xn1 & Xn2\ldots & Xnp \end{vmatrix} \quad (2)$$

where n is the number of determinations for an variable.

Considering the experimental values Y_{exp} like an equation:

$$Y_{exp} = b_0 + b_1 \cdot X1 + b_2 \cdot X2 + \ldots + b_p \cdot Xp \quad (3)$$

where: $b_0, b_1, b_2 \ldots b_p$ – regression coefficients, X1, X2, ..., Xp – the rows of which correspond to variables in the matrix of residual errors given by the equation:

$$e_i = yi - b_0 + b_1 \cdot X1_i + b_2 \cdot X2_i + \ldots + b_p \cdot Xp_i \quad (4)$$

for which the mean square error is the following expression:

$$SSE = \sum [yi - (b_0 + b_1 \cdot X1_i + b_2 \cdot X2_i + \ldots + b_p \cdot Xp_i)]^2 \quad (5)$$

The calculations could devein very laborious if it would keep the number of variables $p - 1 > 3$, so given the parameters of the cutting regime in the correction plane is considered $p - 1 = 3$, and which result in the normal equation of multiple regression:

$$n \cdot \mathrm{b0} + b1 \cdot \sum X1 + b2 \sum X2 + b3 \sum X3 = \sum Y \tag{6}$$

$$\mathrm{b0} \cdot \sum X1 + b1 \cdot \sum X1^2 + b2 \sum (X1 \cdot X2) + b3 \sum (X1 \cdot X3) = \sum (X1 \cdot Y) \tag{7}$$

$$\mathrm{b0} \cdot \sum X2 + b1 \cdot \sum (X2 \cdot X1) + b2 \sum X2^2 + b3 \sum (X1 \cdot X3) = \sum (X2 \cdot Y) \tag{8}$$

$$\mathrm{b0} \cdot \sum X3 + b1 \cdot \sum (X3 \cdot X1) + b2 \sum (X3 \cdot X2) + b3 \sum X3^2 = \sum (X3 \cdot Y) \tag{9}$$

a normal equation solving involves assigning known values to regression coefficients $b_0=1$, $b_1=1$, $b_2=1$, $b_3=1$, which values are obtained for multiple linear regression coefficients b_0, b_1, b_2, b_3:

$$Y = b_0 + b_1 \cdot X1 + b_2 \cdot X2 + b_3 \cdot X3 \tag{10}$$

3. Experimental Results

The material used is a ceramic material with a high content of aluminum oxide, whose properties are shown in Table 1 [3]:

Table 1. Aluminum oxide properties

A_2O_3	%	99.5
Density	kg/m^3	3450
Porosity	%	1
Breaking resistance	MPa	250
Compressing resistance	MPa	1180
Flexion resistance	MPa	175
Resilience	N/mm^2	2.9
Hardness	Vikers	2000
Elasticity module	MPa	210
Linear thermal expansion	$10^{-6}K^{-1}$	8.3 – 12

Processing was carried out on a grinding machine RPO-200, and measurements was performed on a roughness score FORM-Talysurf-120. Given the possibility of kinematic machine tool, the parameters of the system have changed in next order [3]: processing depth t [mm], longitudinal advance S_l [m/min], the longitudinal peripheral speed of the disc V_d [m/s], the values shown in the Table 2 [3].

Table 2. Cinematic parameters

Diamond disc: 1 A1-150-10-4-D125-R-C75		
Parameters	Values	Unity
Depth of cut, t	0.02; 0.03; 0.04; 0.05; 0.06; 0.07	mm
Longitudinal advance, S_l	1; 2; 2.5; 3	m/min
Peripheral speed of diamond wheel, V_d	15; 23; 28; 31	m/s

The parameter of surface roughness R_a was measured in three different areas of the workpiece surface processed, and the values taken into account is the average of the three measured values. The schematic diagram used to determine the roughness is shown in Figure 1 [3].

With each longitudinal advances S_l, and depth of machining t, have grinding to the four peripheral speed V_d of diamond disc, every time the measured parameter R_a at condition presented in the previous chapter, the results are presented in Table 3 (roughness values representing the average of the measurements at the three points of measurement) [3].

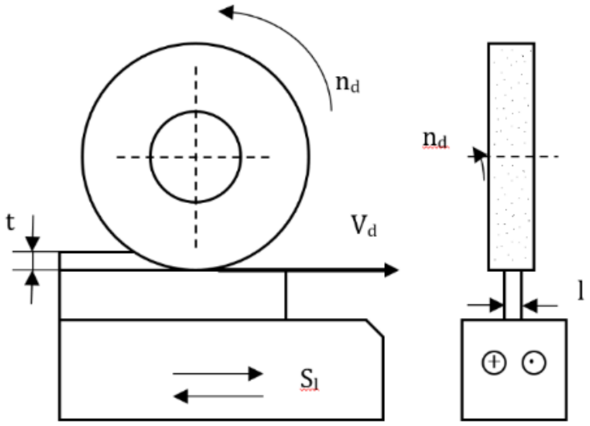

Fig. 1. Plan grinding

Table 3. Roughness R_a

t [mm]	S_l [m/min]	R_a [µm]			
		V_d = 15 m/s	V_d = 23 m/s	V_d = 28 m/s	V_d = 31 m/s
0.02	1	0.8431	0.7485	0.6524	0.6475
0.03	1	0.8564	0.7623	0.6595	0.6491
0.04	1	0.8827	0.7761	0.6687	0.6511
0.05	1	0.8912	0.7922	0.6732	0.6572
0.06	1	0.8953	0.7981	0.6996	0.6832
0.07	1	0.9058	0.8024	0.7036	0.6934
0.02	2	0.8614	0.7628	0.6727	0.6398
0.03	2	0.8725	0.7745	0.6812	0.6413
0.04	2	0.8963	0.7896	0.6983	0.6487
0.05	2	0.8975	0.7951	0.7075	0.6528
0.06	2	0.9012	0.8076	0.7181	0.6669
0.07	2	0.9427	0.8154	0.7234	0.7104
0.02	2.5	0.8763	0.7717	0.6793	0.6654
0.03	2.5	0.8824	0.7897	0.7021	0.6797
0.04	2.5	0.8979	0.7984	0.7153	0.6821
0.05	2.5	0.9233	0.8057	0.7334	0.7169
0.06	2.5	0.9357	0.8238	0.7645	0.7483
0.07	2.5	0.9421	0.8345	0.7892	0.7651
0.02	3	0.8862	0.7851	0.6859	0.6712
0.03	3	0.8998	0.7978	0.6908	0.6748
0.04	3	0.9371	0.8026	0.6993	0.6875
0.05	3	0.9547	0.8264	0.7367	0.7247
0.06	3	0.9772	0.8415	0.7782	0.7323
0.07	3	0.9964	0.8597	0.7956	0.7767

4. Determination the Expression of Roughness as Plain Grinding Regression

To determine the expression of roughness R_a depending on the parameters of the cutting regime as a linear function of multiple regression as: $R_a = b_0 + b_1 \cdot S_l + b_2 \cdot t + b_3 \cdot V_d$, where: b_0, b_1, b_2, b_3, b_s, S_l, t, V_d have the meaning mentioned, it solves the normal equations (6) ... (9), are constituting the matrix of **B** variables at the following form [4]:

$$\mathbf{B} = \begin{vmatrix} Sl11 & t11 & Vd11 & Ra11 \\ Sl21 & t21 & Vd21 & Ra21 \\ \dots & \dots & \dots & \dots \\ Sln1 & tn1 & Vdn1 & Ran1 \end{vmatrix}, \quad (11)$$

where:

X1:=B<0> - column with the values of longitudinal advance S_l in the Table 3;

X2:=B<1>- column with the values of depth of cut t in the Table 3;

X3:=B<2>- column with the values of peripheral speed of diamond wheel V_d in the Table 3;

Y:=B<3>- column with the values of roughness R_a in the Table 3.

Determination of R_a = f(S_l, t, V_d) have been made for next peripheral speed of the diamond disc V_d = 15 m/s, V_d = 23 m/s, V_d = 28 m/s and V_d = 31m /s are presented in the following subsections.

The term roughness R_a = f(S_l, t, V_d) to the plane of the correction V_d = 15 m/s.

For the determination of multiple linear regression coefficients, with the data from Table 3 are made matrix of the variables **B** and by solving the normal equations (6) ... (9) were obtain the following values for multiple linear regression coefficients [4]:

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1.083 \\ 0.029 \\ 1.605 \\ -0.275 \end{bmatrix} \quad (12)$$

and the regression expression would have the form:

$$R_a = 1.083 + 0.029 \cdot S_l + 1.605 \cdot t - 0.275 \quad (13)$$

The term roughness R_a = f(S_l, t, V_d) to the plane of the correction V_d = 23 m/s.

For the determination of multiple linear regression coefficients, with the data from Table 3, are made matrix of the variables **B** and by solving the normal equations (6) ... (9) were obtain the following values for multiple linear regression coefficients [4]:

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1.394 \\ 0.021 \\ 1.531 \\ -0.71 \end{bmatrix} \quad (14)$$

and the regression expression would have the form:

$$R_a = 1.394 + 0.021 \cdot S_l + 1.531 \cdot t - 0.71 \quad (15)$$

The term roughness R_a = f(S_l, t, V_d) to the plane of the correction V_d = 28 m/s.

For the determination of multiple linear regression coefficients, with the data from Table 3 are made matrix of the variables **B** and by solving the normal equation (6) ... (9) were obtain the following values for multiple linear regression coefficients [4]:

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1.454 \\ 0.030 \\ 1.684 \\ -0.899 \end{bmatrix} \quad (16)$$

and the regression expression would have the form:

$$R_a = 1.454 + 0.030 \cdot S_l + 1.684 \cdot t - 0.899 \quad (17)$$

The term roughness R_a = f(S_l, t, V_d) to the plane of the correction V_d = 31 m/s.

For the determination of multiple linear regression coefficients, with the data from Table 3 are made matrix of the variables **B** and by solving the normal equation (6) ... (9) were obtain the following values for multiple linear regression coefficients [4]:

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0.850 \\ 0.270 \\ 1.606 \\ -0.78 \end{bmatrix} \tag{18}$$

and the regression expression would have the form:

$$R_a = 0.85 + 0.27 \cdot S_l + 1.606 \cdot t - 0.78 \tag{19}$$

5. Evaluation of the Roughness

The diagrams of Figures 2 ... 9 was raised to the basis of experimental dates, being necessary to consider the influence of the parameters of the processing regime of the roughness.

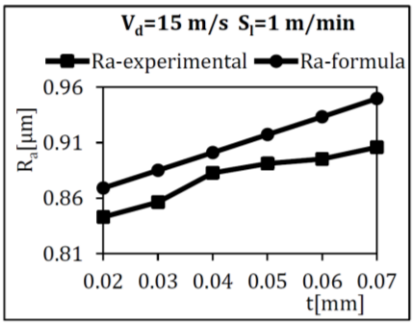

Fig. 2. The dependence of the roughness R_a at grinding with V_d = 15 m/s, and S_l = 1 m/min

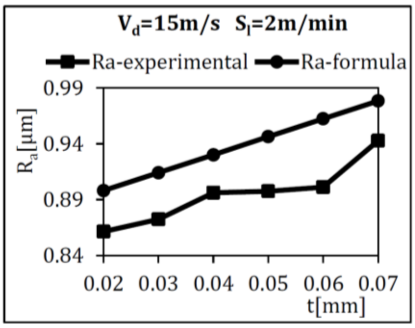

Fig. 3. The dependence of the roughness R_a at grinding with V_d = 15 m/s, and S_l = 2 m/min

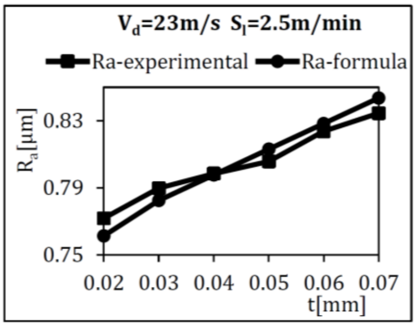

Fig. 4. The dependence of the roughness R_a at grinding with V_d = 23 m/s, and S_l = 2.5 m/min

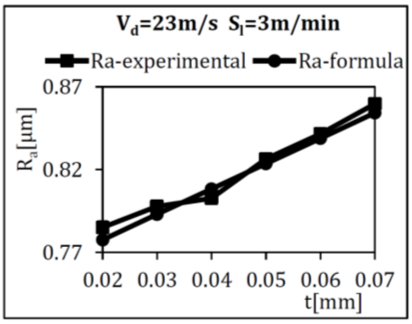

Fig. 5. The dependence of the roughness R_a at grinding with V_d = 23 m/s, and S_l = 3 m/min

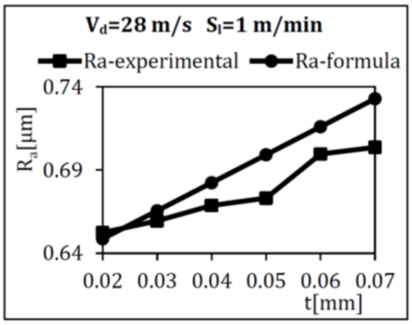

Fig. 6. The dependence of the roughness R_a at grinding with V_d = 28 m/s, and S_l = 1 m/min

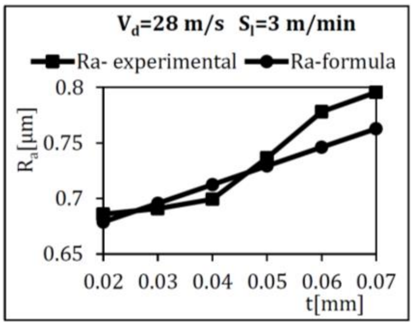

Fig. 7. The dependence of the roughness R_a at grinding with V_d = 28 m/s, and S_l = 3 m/min

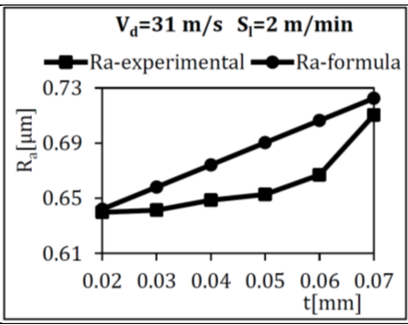	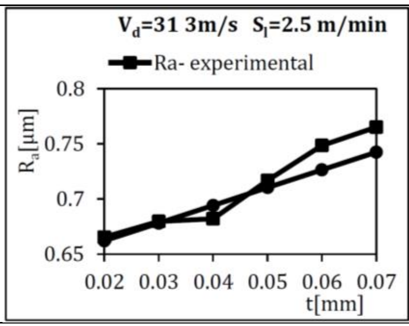
Fig. 8. The dependence of the roughness R_a at grinding with V_d = 31 m/s, and S_l = 2 m/min	Fig. 9. The dependence of the roughness R_a at grinding with V_d = 31 m/s, and S_l = 2.5 m/min

6. Conclusions

The analysis of the diagrams presented in Figures 2 … 9 can lead to the following conclusions:

1. The variation curves of the roughness R_a analytically deducted, approximate better the experimental results for the peripheral speed of the diamond wheel of V_d = 31 m/s than for the speeds of V_d = 28 m/s, V_d =23 m/s or V_d =15 m/s;
2. The variation curves of the roughness R_a analytically deducted, approximate better the experimental results for the longitudinal advance S_l = 1 m/min;
3. The variation curves of the roughness R_a analytically deducted, approximate better the experimental results for the depth of cut t = 0.2 mm;
4. The variation of the experimental roughness is between 0.84-0.99 µm for the peripheral speed of the diamond wheel of V_d = 15 m/s is between 0.74-0.85 µm for the peripheral speed of the diamond wheel of V_d = 23 m/s, between 0.65-0.74 µm for the peripheral speed of the diamond wheel of V_d = 28 m/s and between 0.64-0.77 µm for the peripheral speed of the diamond wheel of V_d = 31 m/s;
5. The variation of the roughness for the analytical linear curves is between 0.84-0.93 µm for the peripheral speed of the diamond wheel of V_d = 15 m/s is between 0.73-0.84 µm for the peripheral speed of the diamond wheel of V_d = 23 m/s, between 0.65-0.79 µm for the peripheral speed of the diamond wheel of V_d = 28 m/s and between 0.63-0.73 µm for the peripheral speed of the diamond wheel of V_d = 31 m/s;
6. The variation of the experimental roughness increases steadily with increasing of depth of cut of processing;
7. The variation of the experimental roughness increases steadily with increasing of longitudinal advance S_l of processing;
8. In all cases the variation curves of the roughness R_a based on the peripheral speed of the diamond wheel are strictly decreasing;
8. It is recommended to be used the following expressions to determine the roughness at the plain grinding of the aluminum oxide:

$R_a = 1.083 + 0.029 \cdot S_l + 1.605 \cdot t - 0.275$ µm to the plane of the correction V_d = 15 m/s;
$R_a = 1.394 + 0.021 \cdot S_l + 1.531 \cdot t - 0.71$ µm to the plane of the correction V_d = 23 m/s;
$R_a = 1.454 + 0.030 \cdot S_l + 1.684 \cdot t - 0.899$ µm to the plane of the correction V_d = 28 m/s;
$R_a = 0.85 + 0.27 \cdot S_l + 1.606 \cdot t - 0.78$ µm to the plane of the correction V_d = 31 m/s.

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