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Spur Gears with Curved Directrix: Diversity and Methods of Generation. A Patent Study

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Abstract
Gear wheels and mechanisms deploying spur gears have been known since antiquity. Leonardo da Vinci's studies on gears are also exciting. Although there is no explicit information about how gears were cut in the ancient and medieval world, it is unanimously accepted that tooth profiles were not generated kinematically. The modern period is characterized by machines and movement, and gears are practically ubiquitous. Among the cylindrical gears, the most frequently used ones are those with straight directrix and those with helical cylindrical gears (cylindrical gears with inclined teeth), both types being rectifiable. Monographs or specialized treatises are rare that even mention that cylindrical gears can also have curved teeth, i.e. curved directrix to the associated reference rack. In the last five decades, and only sporadically before, explicit interests in and achievements emerged related to cylindrical gears with curved directrices, relevant in this sense being patents and doctoral theses. Most of the works mainly concern cylindrical teeth with involute and circular directrix, as well as methods and machines for their machining. At the limit, cambered teeth can also be considered cylindrical teeth with curved directrix. Cylindrical gears with poly-hypocycloid directrix and corresponding machining methods are a subject initiated by Professor Emil Botez and addressed exclusively by researchers from the *Politehnica* University of Bucharest. Romanian researchers are actively contributing to furthering knowledge about various cylindrical gear with curved directrices and their manufacturing methods.

Keywords
spur gear, curved directrix, methods of generation, specialized machine tools, patent

1. Introduction

In the construction of various machines and equipment, man has been using gears since ancient times. The discovery of the mechanism at Antikythera, figure 1, was a great surprise and generated many studies about the depth of knowledge of the ancients regarding gears and their even sophisticated applications.

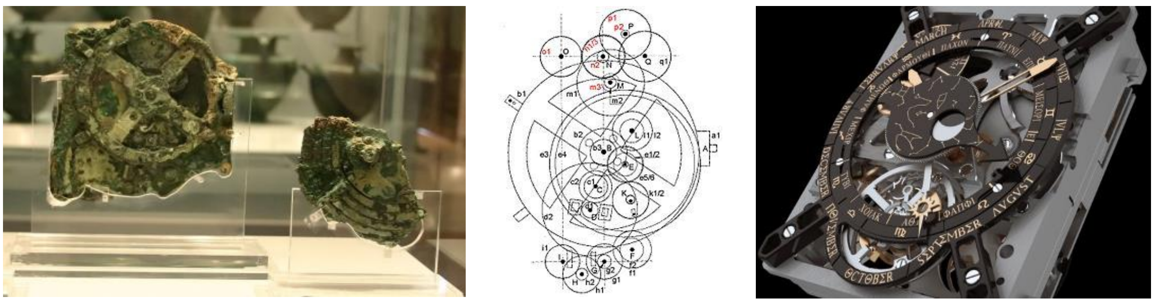

Fig. 1. The Antikythera mechanism (photo [1], diagram [2] and reconstruction [3])

Another exciting historical landmark related to toothed gears is Leonardo da Vinci's studies [4]. It is certain that the qualitative evolution of teeth and knowledge related to them is related to applications in the construction of transmissions used in windmills and water mills, used for milling or for driving various machines, as well as in the construction of clocks as they are installed in the towers of many medieval cathedrals or churches.

Although there is no explicit information about how gears were cut in the ancient and medieval world, it is unanimously accepted that tooth profiles were not generated kinematically.

The modern period is characterized by machines and movement, and gears are practically ubiquitous. Cylindrical and bevel gears are the most common, worm gears are less common.

Like any other surface, the characteristic surface of a tooth is described by a guiding curve (the directrix) and a generating curve (the generatrix) [5, 6]. Various machine-tools are used to cut gear, all of them working on the principle of kinematically generating both directrix ("direction" of the teeth) and generatrix (the "profile").

Among the cylindrical teeth, the most frequently used are those with straight directrix, but also those with helical cylindrical directrix (cylindrical teeth with inclined teeth), both types being rectifiable. Usually bevel gears have straight, inclined or curved teeth, meaning that the direction in the flat reference wheel is a straight line that intersects the axis of the flat wheel, a straight line that does not intersect the axis of the flat wheel or, respectively, a kinematically generated planar curve (circle, Archimedean spiral, epi- or hypocycloid, involute, etc.). Bevel gears with curved teeth are mainly used in power transmissions, due to their higher load capacity and less noisy operation than those with straight or inclined teeth. Bevel gears with straight teeth, inclined teeth and those with circular arc teeth can be rectified.

Modern gear generatrices are mutually wraparound curves. Most often used are gears with involute generatrix that is generated kinematically by rolling.

2. Cylindrical Gears with Curved Directrix

Monographs or specialized treatises are rare that even mention that cylindrical gear can also have curved teeth, i.e. curved directrix to the associated reference rack. In this sense, for example, a vague reference is identified in Schwerd [7], as well as in Lexicon [8]. It is surprising that in his most representative work [9] Botez does not even mention the possibility of cutting cylindrical gear with curved directrix, in particular circular arc, although he was an active participant in devising the Lexicon.

In the last three decades, and only sporadically before, there are explicit interests in and achievements related to cylindrical gear with curved directrix, relevant in this sense being patents and doctoral theses.

2.1. Gears with Involute Directrix

An old reference, from the beginning of the 20th century is identified in [10], where a gear with a pinion and two flat wheels is described, figure 2. The flat wheels A and A' facing one another have front teeth with curved directrix. Implicitly, pinion B in turn has teeth with curved directrix, asymmetrical with respect to the median transverse plane of the pinion.

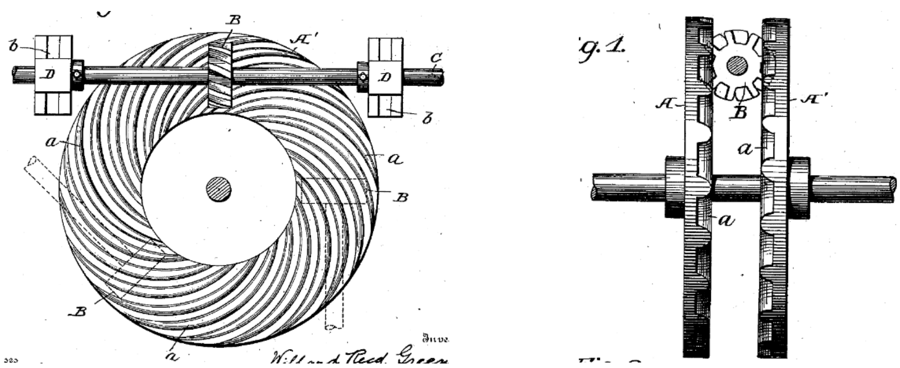

Fig. 2. Gear consisting of a pinion B and two flat wheels A and A' (acc. to [10])

The subject "pinion gear - flat wheel with front teeth" can be found 80 years later, as a research topic at The University of Kentucky Research Foundation, one of the achievements being US 4238970 patent [11]. It is explicitly specified that the teeth of the flat wheel are oriented along a normal involute, figure 3, so that the direction of the toothing is an arc portion of this curve. In order to mesh correctly with the flat wheel teeth, the pinion tooth directrix must be of the same nature.

It should be emphasized that the US 4238970 patent also gives a solution for processing the pinion teeth, proposing a tool (a type of mill with frontal teeth) that uses rack shapers to mortise cylindrical teeth.

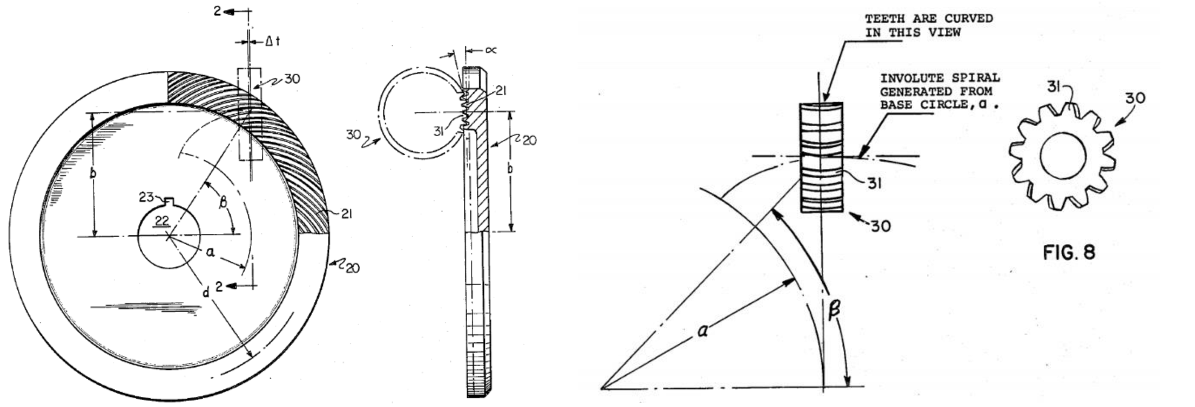

Fig. 3. Flat wheel gear (20) with front teeth – pinion (30) with curved teeth (acc. to [11])

Lately, research carried out at GM Global Technology Operations (Detroit, MI, US) and embodied in at least two patents [12, 13] (of very good quality) also addresses cylindrical pinion gears - flat wheel with front teeth (pinion – face gear) in which the direction is an involute (normal or elongated), but in which the teeth have an asymmetric profile. The generatrices of the two flanks of a tooth have different involutes, figure 4, thus resulting in a gear with a smaller number of teeth benefitting from increased bending strength. In the gears described, the numbers of teeth of pinions and face gear can vary largely, figure 5, and as a result, very different transmission ratios can be obtained in terms of their values.

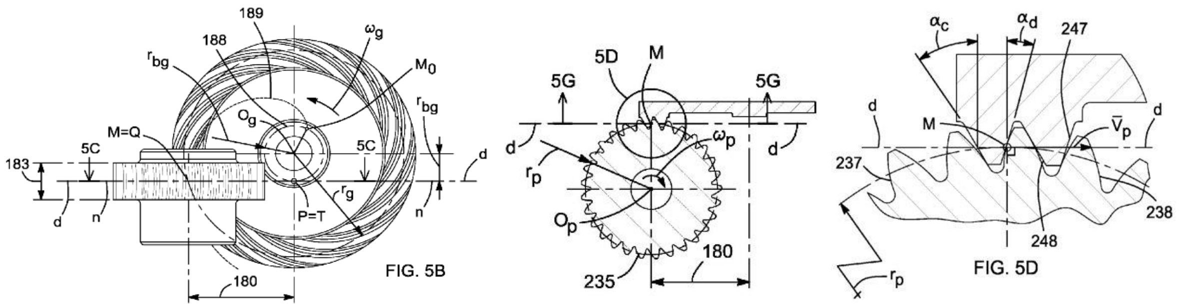

Fig. 4. Pinion - face gear (acc. to [12]) with asymmetric teeth

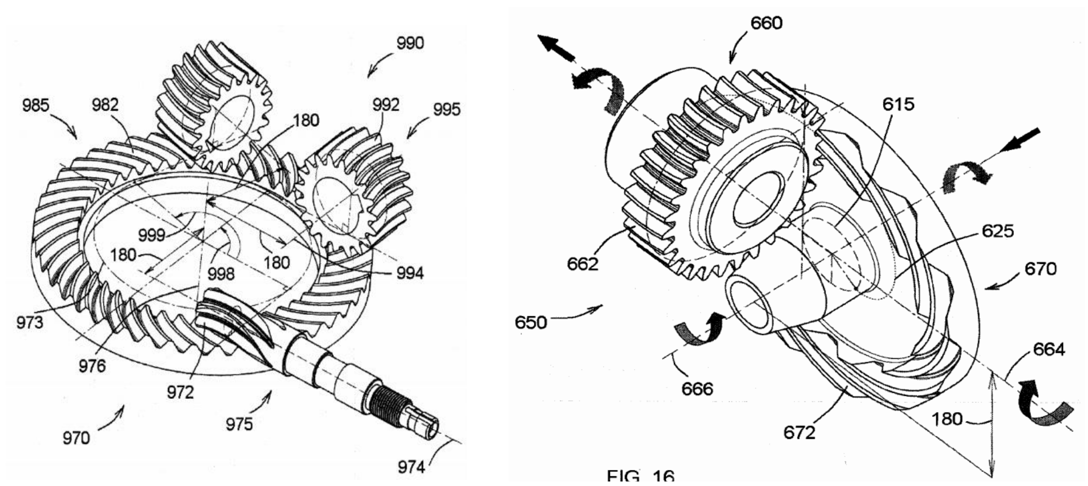

Fig. 5. Pinion - face gears (acc. to [12]) with various numbers of teeth at the pinion and the face gear

In [12] it is also emphasized that in the described gears, the driving wheel and implicitly the driven one can have any sense of rotation, see figure 5 again, so that there is no preferential or even inadequate direction. This observation refers exclusively to pinion gears - gears with symmetrical teeth. In gears with asymmetric teeth [14], a certain sense of rotation of the driving wheel is strongly recommended.

Methods of generating the teeth that form pinion – face gears with asymmetric teeth are addressed and relevant solutions are given for tools and machine tools that can be used for this. Recommended for processing cylindrical teeth with curved directrix in an involute arc (normal or elongated) is milling with specialized tools, face hob and conical hob on a milling machine of the type used to process spur gears with hob, figure 6.

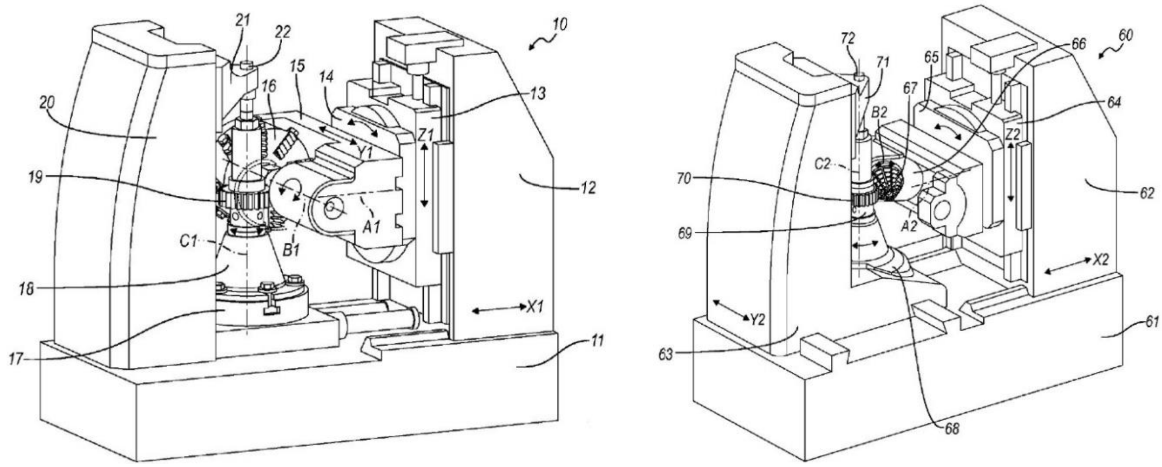

Fig. 6. Tooth milling machines equipped with face hob or conical hob (acc. to [13]), for processing cylindrical teeth with curved directrices in involute arc (normal or elongated)

Milling machines of the type shown in figure 6 are also used to process the front teeth of the flat wheels. Milling with a conical hob, figure 7, planing (!!) with a rack cutter, figure 8, the workpiece performing rotation and translation simultaneously, and turning (!!) using a mortising gear shaper as a tool, figure 9, whose geometry is similar to the macrogeometry of a pinion that can engage with the face gear being machined.

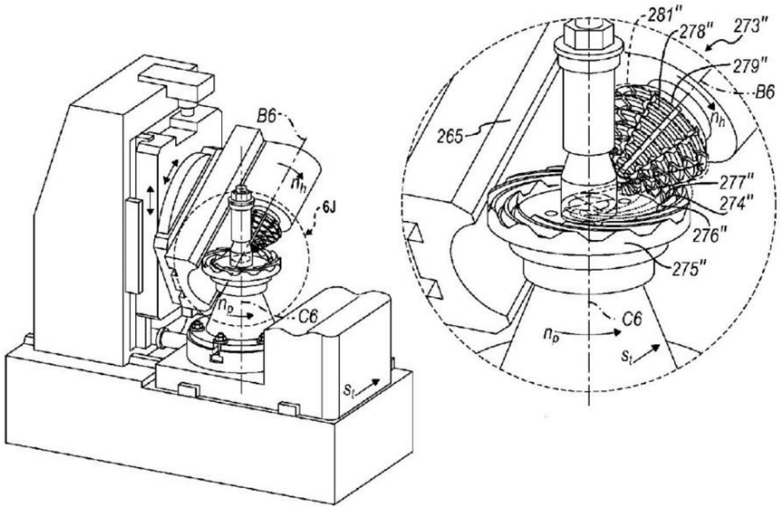

Fig. 7. Tooth milling machine equipped with a conical hob (acc. to [13]), intended for machining teeth on a face gear with directrix in involute arc (normal or elongated)

2.2. Gears with Arc Circle Directrix

Nearly 100 years ago (the patent application was filed in 1914!) a patent was issued [15] that addresses teeth with circular arc directrix, as well as a method of making them. The teeth can be symmetrical with respect to the median transverse plane of the wheel or not, figure 10. The involute generatrix (profile) is obtained kinematically by rolling, as the machined wheel rolls over the frontal plane of the tool used, a milling head with frontal teeth. The rolling motion is achieved by a rack-and-pinion mechanism.

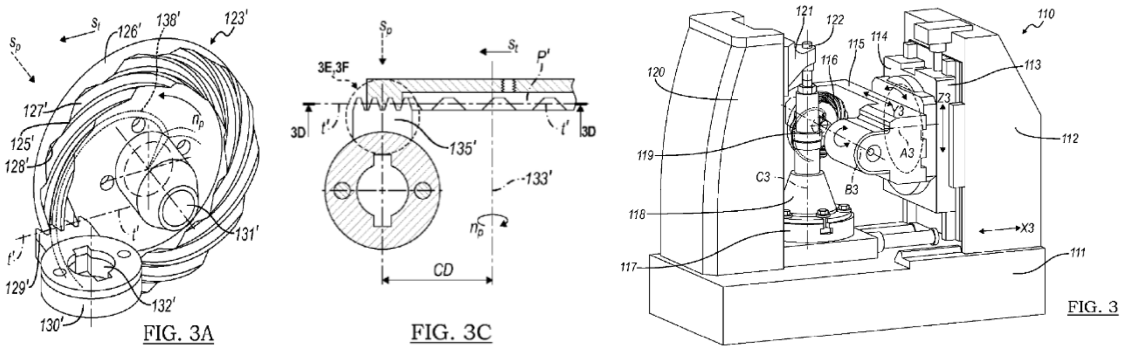

Fig. 8. Tooth milling machine equipped with a rack cutter (acc. to [13]), intended for machining teeth on a face gear with directrix in involute arc (normal or elongated)

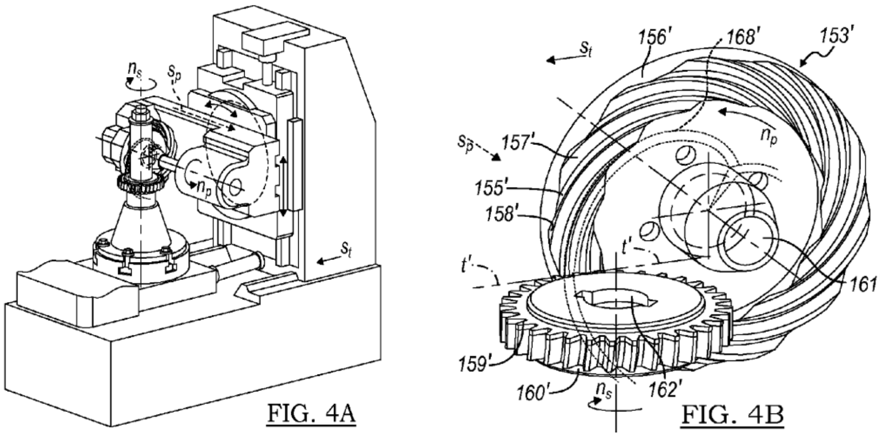

Fig. 9. Tooth milling machine equipped with a gear shaper (acc. to [13]), intended for machining teeth on a face gear with directrix in involute arc (normal or elongated)

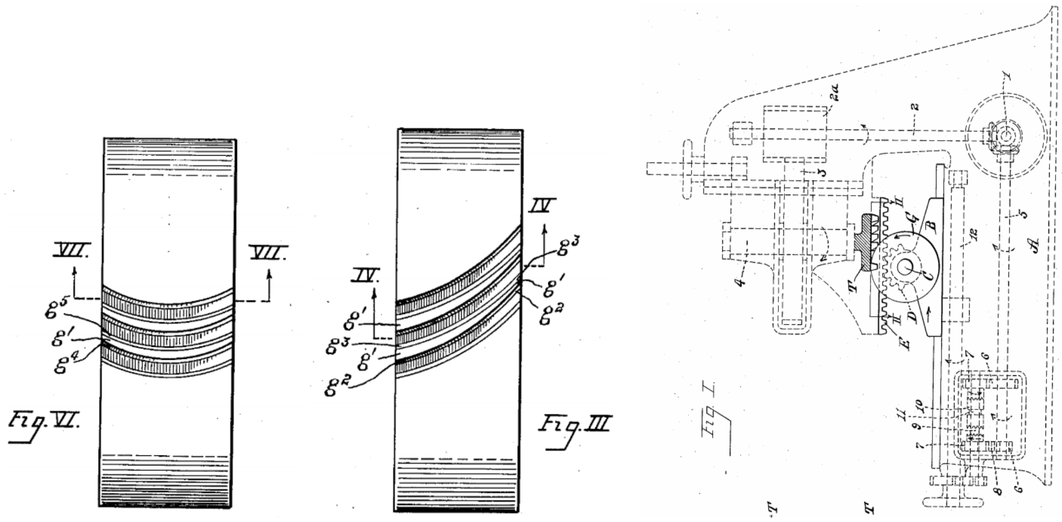

Fig. 10. Cylindrical gears with directrix in a circular arc and manufacturing method, acc. to [15]

In order to generate cylindrical gears with directrix in a circular arc by rolling, Boor [16] proposes the adaptation of a universal milling machine with a horizontal main shaft, figure 11. Rolling is achieved [15] by a pinion-rack mechanism, with a fixed rack attached to the machine frame. The longitudinal motion ensures the translation of the workpiece, and implicitly the rotation necessary for rolling. Indexing is discrete, manually executable.

Remarkably the possibility is emphasized that cylindrical gears with teeth in a circular arc that form a gear do not necessarily have parallel axes.

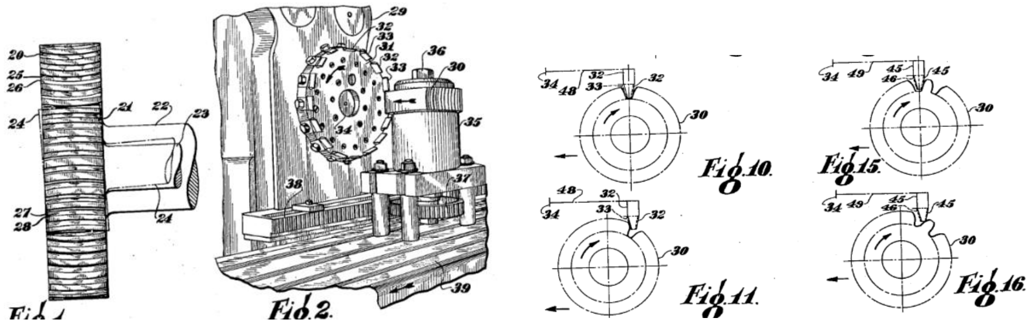

Fig. 11. Cylindrical gears with circular arc directrix and processing method, acc. to [16]

From the same period as [15], around 1920, is the US 1469290 patent [17], which refers to both spur gear and bevel gear, figure 12. Milling heads with front teeth (rotary cutter) described by the same inventor and in a previous patent [18] are used to process the teeth, the generatrix of the cutting edges of the active parts of the tool being in a circular arc. The gear tooth generatrices that are the subject of US 1469290 patent are also circular arcs. The teeth have spherically-curved active surfaces some of which are convex and the others concave; one active surface of each tooth is convex, and the other active surface of the same tooth is concave. As a result, the generatrices are not generated by rolling, but are materialized, i.e. they copy the active profile of the tools of the front milling head used for processing. Discrete indexing is required for processing all the teeth of the wheel.

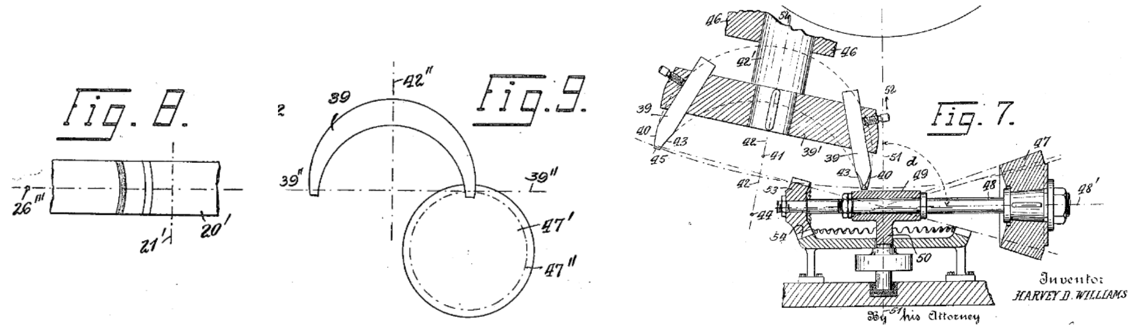

Fig. 12. Teeth with circular arc directrix, acc. to [17], generated with milling heads with front teeth

An interesting solution for milling heads with front teeth intended for machining teeth with circular arc directrix for cylindrical gears and racks is presented in US 3915060 patent [19], figure 13. Relative to the frontal plane of the tool, its front teeth are located inclined by a relatively small angle θ. The front teeth materialize either a tooth of the reference rack, in which case the width of the tooth gap is constant along the directrix in a section normal to it, or a gap of the reference rack, in which case the width of the tooth gap is constant along the directrix.

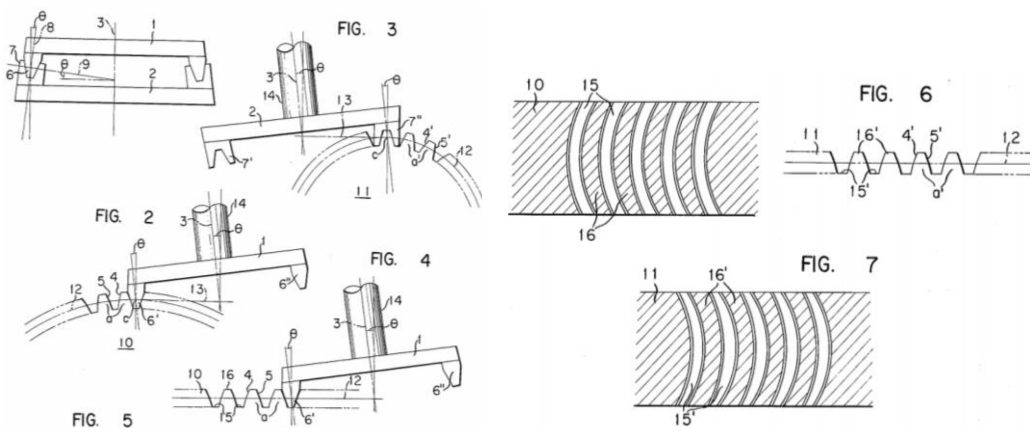

Fig. 13. Milling heads with front teeth for machining gear wheels with directrix in circular arc, acc. to [19]

"Rack with curved teeth (curved directrix)", figure 14, is the subject of some patents obtained in the last decade [20, 21]. The study of cylindrical gears with circular arc teeth, variable in height, is the subject of a doctoral thesis [22].

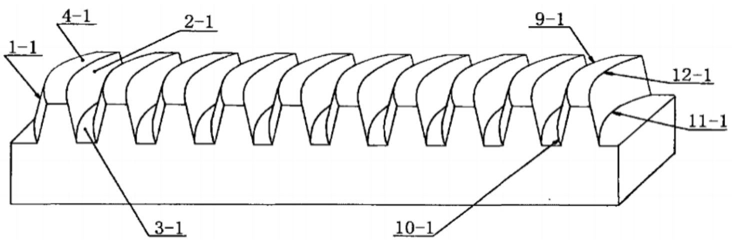

Fig. 14. Rack with curved teeth, acc. to [20, 21]

It is possible that the teeth of the cylindrical gears obtained according to [17] and [19] are not of constant height. Certainly, the cylindrical gear with curved teeth proposed in [23] are of variable height, figure 15, any active cutting edge of the tool being tilted towards the tool axis at an angle β. In this sense, a device [24] is also designed, usable for universal vertical milling machines, figure 16. A mechanism with metal bands is used to obtain the involute generatrix of gear by rolling, the ends of the bands being fixed to the machine frame.

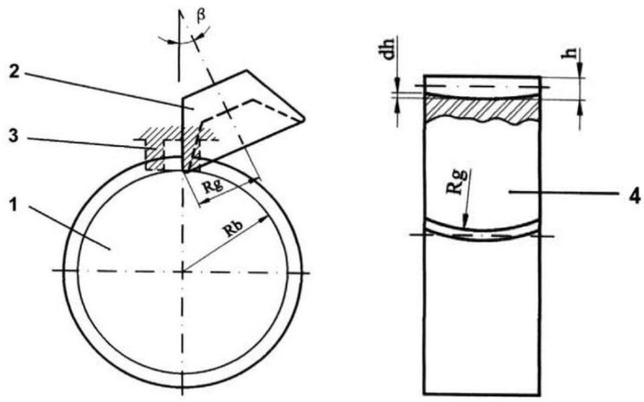

Fig. 15. Gear wheel with directrix in circular arc, acc. to [23], with variable height along the directrix

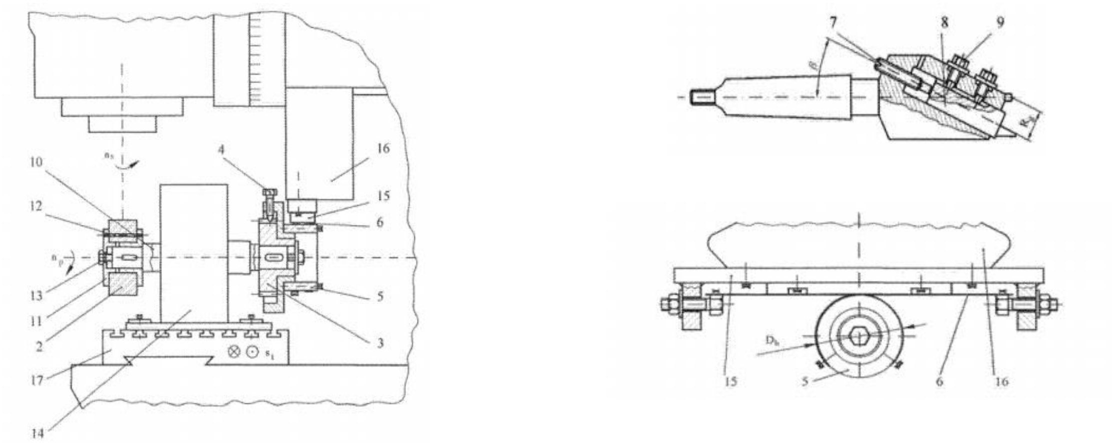

Fig. 16. Device for processing cylindrical gear with a circular arc directrix,
with variable teeth height along the directrix, acc. to [24]

A mechanism with bands for rolling teeth with a circular arc directrix for hydraulic pumps is also identified in the device proposed in patent JPH 11118023 [25], but the position of the metal bands is indicated erroneously, figure 17.

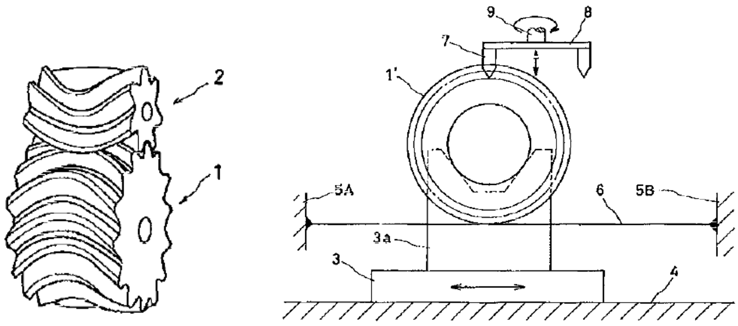

Fig. 17. Cylindrical gears with circular arc directrix and method of tooth cutting, acc. to [25]

There are, of course, also specialized machine tools for processing cylindrical teeth with circular arc directrix. At least three relevant examples are available, from the 1940s, 1970s and 2010s, all of which are milling machines and all of using a milling head with front teeth arranged along a circle whose radius is equal to the radius of the directrix of the processed teeth.

In the machine proposed by Jury through US 2426774 patent [26], figure 18, both the main shaft 21 (the tool holder shaft) and the part holder shaft 4 have horizontal axes. The speed of the main shaft is not adjustable. The electric motor 20 drives both the main movement (rotation of the tool), as well as the rotation movements of part 5 and translation of a vertical slide 7 carrying the main shaft (as well as the driving motor). The aforementioned rotation of the part and translation of the tool ensure the generation by rolling of the involute profile of the workpiece teeth. The speed of the part carrier shaft is adjusted by the change gears 30, and the transfer ratio of the rolling linkage is adjusted by the change gears 39. Worms 34 and 36 rotate together with machine shaft 29, causing the rotation of worm wheels 6 and 37, respectively. Worm 36 is translatable along shaft 29 together with the tool holder shaft, thus allowing adjustment of the distance between the axis of the workpiece and the frontal plane of the tool, a distance dependent on the geometric characteristics of the workpiece teeth.

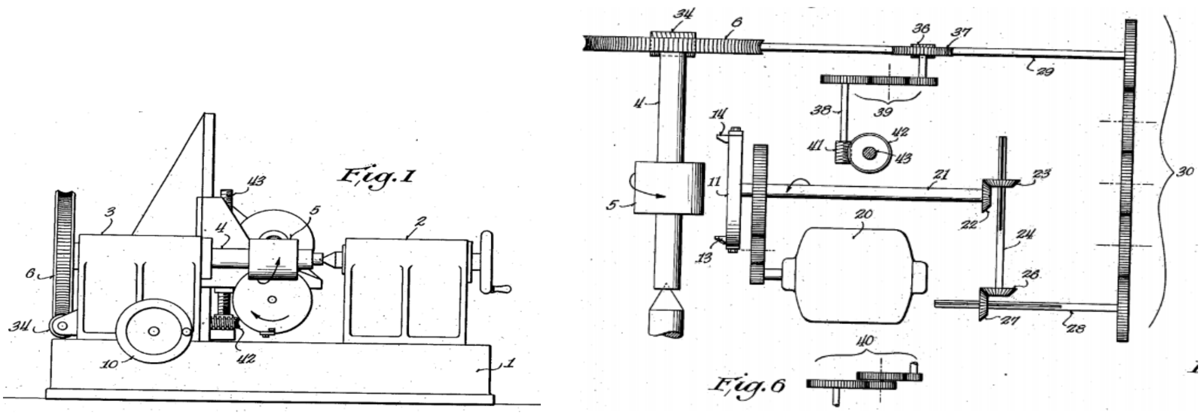

Fig. 18. Specialized machine for processing cylindrical teeth with circular arc directrix.
Front view and kinematic diagram, acc. to [26]

In good agreement with the general appearance and kinematics of the curved bevel gear milling machines made by the Swiss company Öerlikon is the gear cutting machine proposed by Kotthaus through US 4211511 patent [27], figure 19. The kinematic diagram that is part of the patent documentation does not specify how movement E is achieved, namely the vertical translation of the tool-carrying slide 13, a movement necessary to generate the teeth profile of part 16 by rolling. A particularity explicitly mentioned in the patent description should be, however, noted: the teeth of part 16 can be obtained either by rolling without a feed movement, or only as a result of a feed movement (movement D of slide 13 of the tool holder shaft) unaccompanied by any generation movement. If gear cutting is not done by rolling, then the height of the teeth cannot be constant, and their profile strictly copies the profile of the cutting edges of tools 17 located in milling head 14.

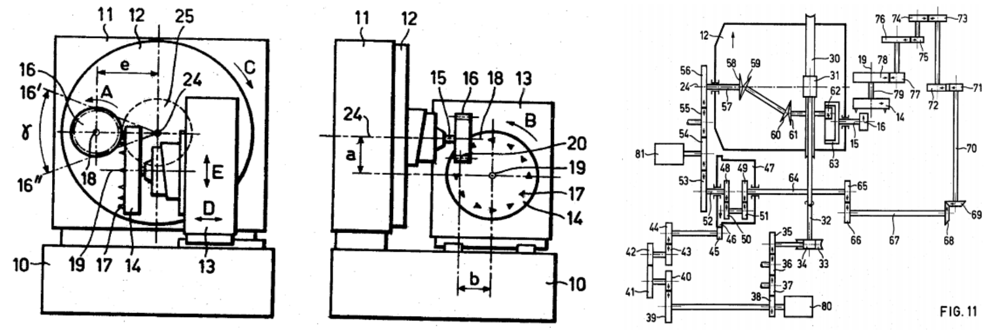

Fig. 19. Specialized machine for processing cylindrical teeth with circular arc directrix.
Views and kinematic diagram, acc. to [27]

The interest in the processing of cylindrical teeth with circular arc directrix of the author of the present paper and some collaborators led to invention patent RO 125758 [28]. The part-carrying shaft 47 is vertical and together with slide 52 performs the longitudinal feed movement, figure 20. A screw-nut mechanism is used for the drive, nut 67 carrying out translation together with slide 52.

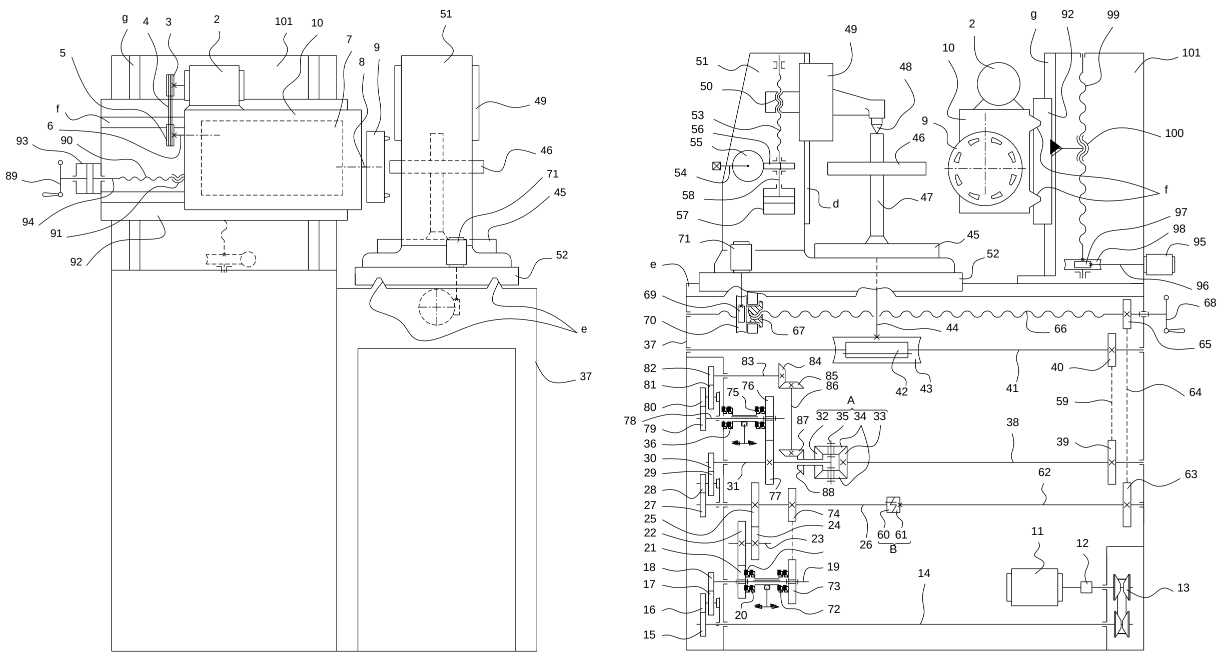

Fig. 20. Specialized machine for processing cylindrical teeth with circular arc directrix.
View and kinematic diagram, acc. to [28]

The profile of the processed teeth is obtained by rolling. During the kinematic generation of a gap between the teeth the rolling linkage ensures the connection between its rotation and translation. The rolling linkage includes the part of the linkage responsible for the circular feed of the part - located between part 46 and shaft 26 - and the part of the linkage responsible for the longitudinal feed - located between shaft 26 and longitudinal slide 52. This linkage is adjusted by change gears 27, 28, 29 and 30.

Indexing is discrete (intermittent) and is done automatically. Indexing takes place during the withdrawal phase of slide 52 to its initial position, by the additional rotation of the machined part 46 by one or multiple teeth.

2.3. Cylindrical gears with crowning

One cause of gear failure is known to be the uneven distribution of load on the tooth flank. Load distribution on the flank of the tooth is determined by factors independent of the load itself - deviations and manufacturing errors – as well as by factors dependent on the applied load - elastic deformations of the component elements [29]. A solution to compensate the uneven distribution of load on the flank of the tooth due to deformations is to modify to tooth profile, for example by crowning or endrelief, figure 21.

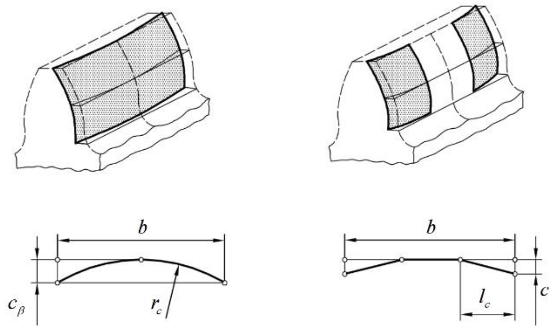

Fig. 21. Changing the directrix by crowning and endrelief [29]

Spur gears with crowning can be obtained as a result of a later operation, or even at generation. Finishing by shaving can also be accompanied by the crowning of the teeth. Changes in the direction of the teeth (crowning or endrelief) in the case of grinding with a worm-gear hob stone are obtained by combining axial, radial and angular feeds.

The kinematic generation of crowning actually requires that the gear directrix be curved, even if its radius of curvature is very large. It is possible to obtain gear crowning through milling with worm-gear hob cutters, disk cutters and even end-mills if the feed along the axis of the part is adequately combined with a small-amplitude translation in the radial direction of the part. The crowning is obtained as the implicit result of a small variation in the height of the teeth.

It is usually considered that crowning can only be positive. Various modifications of the gear profile, including types with negative crowning, figure 22, are proposed by US 9091338 patent [30].

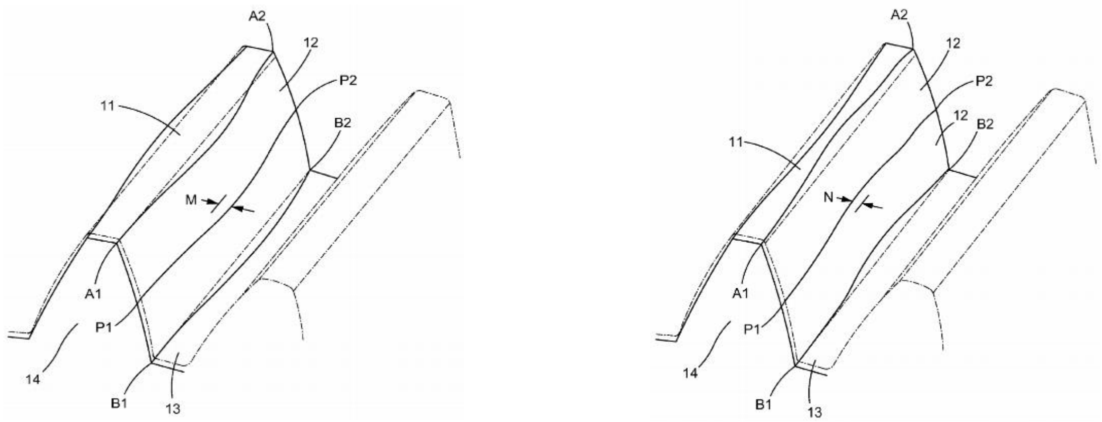

Fig. 22. Convex-crowned tooth profile and concave-crowned tooth profile, acc. to [30]

Professor Emil Botez studied the possibility of obtaining spur gears with crowning by direct kinematics and looked for a directrix curve that would meet the requirements. He identified certain hypocycloids that meet the requirements, like those with three or four cusps (Steiner's hypocycloid (tricuspid hypocycloid) and the astroid, respectively). The research generated a patent [31], but also evolved towards the generation of cylindrical gears with curved directrix (various hypocycloids) in the true sense of the word.

2.4. Cylindrical hypocycloidal gears

Starting from the initial aim of generating cylindrical gear with convex flanks [31], in order to favorably position the actual contact surface research evolved and the possibility was demonstrated, including practically, of generating cylindrical teeth with curved directrix in a hypocycloid arc on specialized milling machines [32]. The shape of the directrix of poly-hypocycloidal teeth, designed in the plane of the reference rack, and the kinematic diagram of principle of a gear milling machine equipped for the generation of cylindrical poly-hypocycloidal teeth are shown in figure 23.

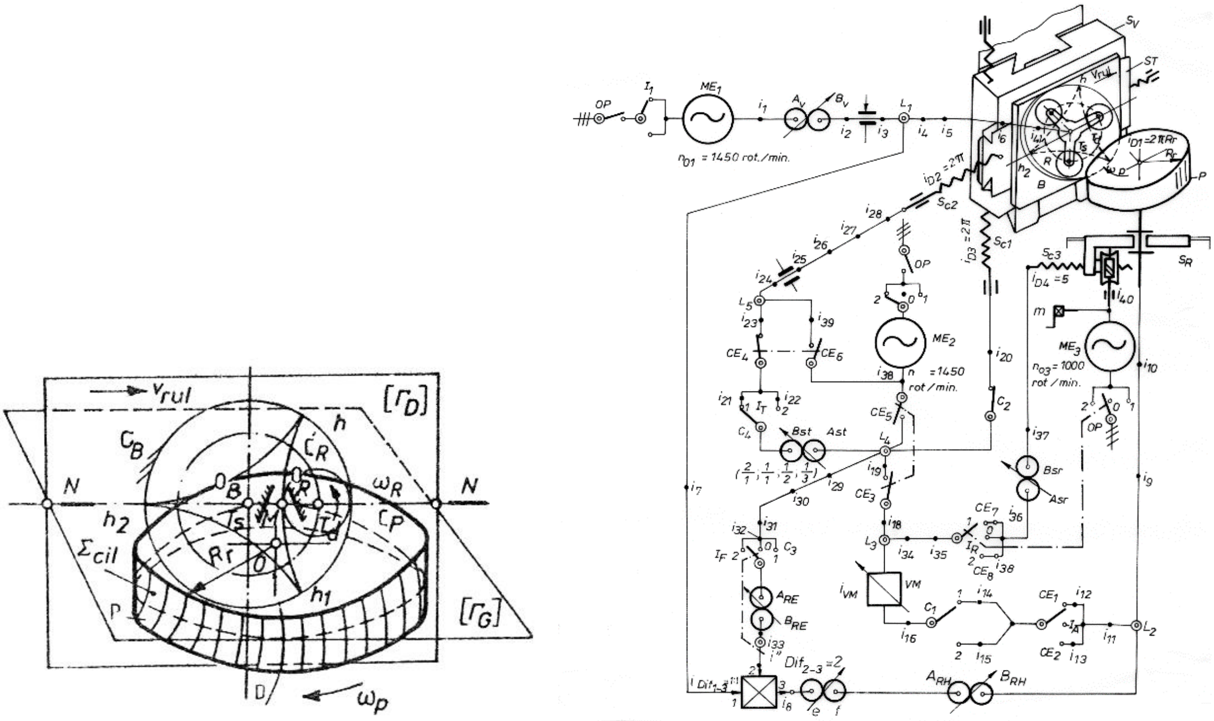

Fig. 23. The shape of the poly-hypocycloidal gear directrix and kinematic scheme of a milling machine intended for processing such gear [32]

Various hypocycloids - normal, elongated or shortened, as appropriate - can be used as directrices of the curved cylindrical teeth. Patent RO 108942 refers to a milling head of poly-hypocycloidal teeth where the directrix is an astroid arc, figure 24.

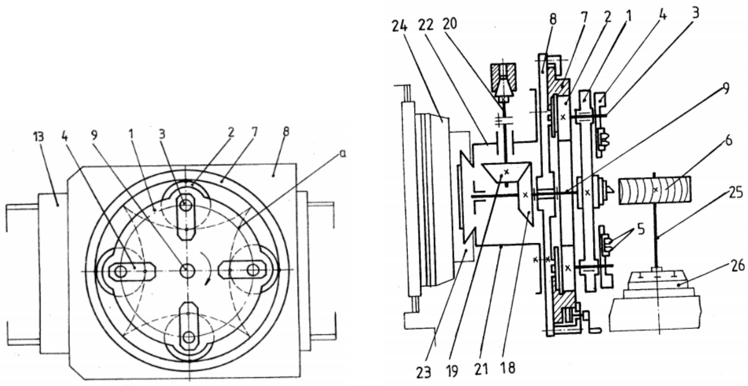

Fig. 24. Milling head for the processing of curved cylindrical teeth, with directrix in an asteroid arc [33]

It should be noted that the concept of using a hypocycloid as a directrix of a cylindrical or conical gear is quite old. In 1925 a patent [34] application was filed for a machine for cutting double helical or double spiral gears similar to herringbone gears, figure 25. The disadvantages of V teeth are eliminated and the advantages are preserved, including during manufacturing. A shortened hypocycloid arc is used as the directrix (figure 26 shows as an example a hypocycloid with five cusps) which approximates as closely as possible the considered herringbone configuration.

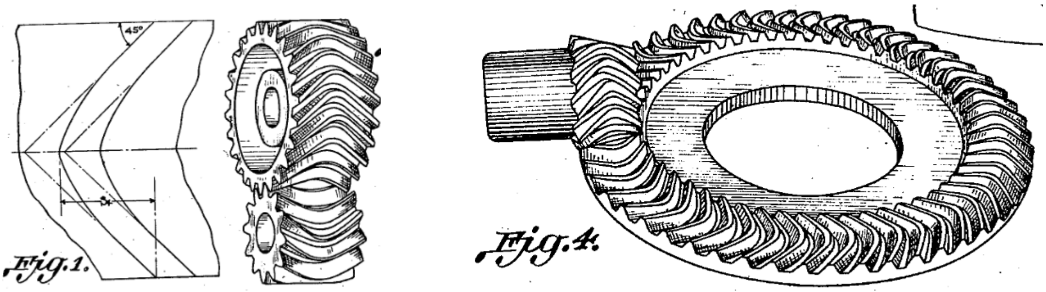

Fig. 25. Cylindrical and bevel gear with teeth similar to the herringbone configuration

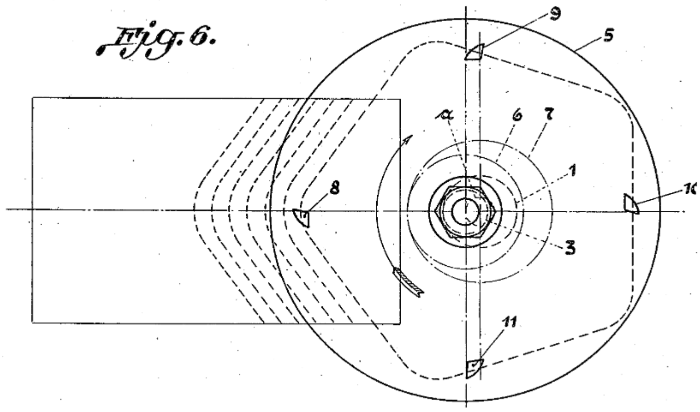

Fig. 26. Shortened hypocycloid generatrix, also represented in the reference rack plan

3. Conclusions

Cylindrical gears are machine parts used since antiquity, but the kinematic generation of teeth has been known and applied only since the second half of the 19th century.

Usually spur gears have a straight directrix or a cylindrical helix. However, there are also cylindrical gears with a curved directrix, of various shapes. The most common curved directrix is a circle arc, the finishing of the teeth being possible by grinding; however, curved teeth with involute arc or hypocycloid arc directrix are also identified.

Milling is the most frequently used machining method, using milling heads with front teeth, but it is also possible to deploy planing with a face hob or rack cutter.

For the processing of cylindrical teeth with curved directrix, universal milling machines equipped with suitable specialized devices and appropriate tools (milling heads with front teeth) are used, or specialized gear milling machines are designed, often similar to gear milling machines with worm-gear hob cutters .

From time to time various researchers pleasantly surprise the scientific community with new proposals regarding cylindrical teeth with curved directrix, including their machining technology.

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