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Kinematic Requirements for Machining Cylindrical Surfaces Characterized by Straight Directrix and Circle Generatrix

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Abstract
A workpiece is defined by one or more unit surfaces that are continuous with each other, each unit surface being characterized by a directrix curve and a generatrix curve. Each of these curves can be generated in six ways. Consequently, a unit surface can be obtained in 6×6 = 36 distinct ways using various processing methods (such as cutting, plastic deformation, or unconventional methods). Some of these methods are found in industrial practice, while others are possible but economically inefficient. Depending on the nature of the directrix and generatrix curves, certain theoretical methods for obtaining a unit surface are not yet known and require inventive effort to be revealed. This paper focuses on the methods for obtaining unit cylindrical surfaces where the directrix curve is a straight line described cinematically as the trajectory of a point and the generatrix curve is a circle.

Keywords
surface, directrix curve, generatrix curve, processing scheme, kinematic chain

1. Introduction

Various parts used in machine construction and generally in various technical systems are limited to the external environment by one (extremely rarely) or more continuous surfaces. Real surfaces are technologically generated by various processing methods, presenting asperities and deviations in form and position. From a theoretical point of view, associated ideal surfaces are of particular importance, which are completely smooth and have no deviations. An ideal surface is described by two curves, a directrix curve and a generatrix curve. Both theoretically and technologically, the directrix and generatrix curves can be [1-3]:

- materialized (M, m);
- obtained kinematically:
  - by copying (Co, co);
  - as the trajectory of a point (Ci_{tp}, ci_{tp});
  - as the envelope of a family of curves (Ci_{fc}, ci_{fc});
  - by rolling (R, r);
  - programmed (P, p).

Since there are six ways to obtain the generatrix curve and six ways to obtain the directrix curve, there are 6×6 = 36 possibilities to obtain a surface characterized by a particular directrix curve and a particular generatrix curve, possibilities outlined in Table 1.

Table 1. Possible combinations for obtaining a surface [2]

		Directrix					
		M	Co	Ci_{tp}	Ci_{fc}	R	P
Generatrix	m	M&m	Co&m	Ci_{tp}&m	Ci_{fc}&m	R&m	P&m
	co	M&co	Co&co	Ci_{tp}&co	Ci_{fc}&co	R&co	P&co
	ci_{tp}	M&ci_{tp}	Co&ci_{tp}	Ci_{tp}&ci_{tp}	Ci_{fc}&ci_{tp}	R&ci_{tp}	P&ci_{tp}
	ci_{fc}	M&ci_{fc}	Co&ci_{fc}	Ci_{tp}&ci_{fc}	Ci_{fc}&ci_{fc}	R&ci_{fc}	P&ci_{fc}
	r	M&r	Co&r	Ci_{tp}&r	Ci_{fc}&r	R&r	P&r
	p	M&p	Co&p	Ci_{tp}&p	Ci_{fc}&p	R&p	P&p

The same directrix curve can be obtained using various methods, rarely only one. There are also processing methods that cannot be used as generation modes for obtaining certain directrix curves. This statement is entirely true for obtaining the generatrix curve as well. Both curves that characterize a specific surface must, obviously, be obtained using the same processing method. The chosen method determines both the productivity of the processing, the quality and precision of the resulting real surface. Surface generation is achieved as a result of one or more generating movements relative to the workpiece and the tool used for processing [3]. The directrix curve is obtained as a result of at least one relative generating movement between the tool and the workpiece, with the main movement performed by the machine tool being imperative [2, 4, 5]. If obtained kinematically, the generatrix curve is the result of one or more generating movements. If materialized (by the tool profile), the generatrix curve does not require any movement to be transposed onto the workpiece [2, 4, 5]. Generating surfaces on machine tools also requires some auxiliary movements that are performed outside the piece-tool contact. This paper exemplifies, through processing schemes and kinematic requirements, cases of generating cylindrical surfaces where the directrix curve is a straight line obtained kinematically as the trajectory of a point, with one example for each of the six methods of obtaining the generatrix curve.

2. Kinematic Requirements for Processing Cylindrical Surfaces Characterized by a Directrix Curve Obtained Kinematically As the Trajectory of a Point (Ci_{tp})

As specified in Table 1 above, six methods for obtaining a surface where the directrix curve is described kinematically as the trajectory of a point are identified. These methods, depending on how the generatrix curve is achieved, can be: materialized (m), kinematically obtained by copying (co), kinematically obtained as the trajectory of a point (ci_{tp}), kinematically obtained as the envelope of a family of curves (ci_{fc}), kinematically obtained by rolling (r), programmed (p).

2.1. Case Ci_{tp}&m

A systematic research is currently underway to identify the possibilities of generating common surfaces through various processing methods (cutting, plastic deformation, unconventional methods), including those where the directrix curve is straight and the generatrix curve is a circle. The set of processing methods identified at this stage as possible for the Ci_{tp}&m case in question is given by relation (1), namely:

$$M_{dr(Citp)\&circle(m)} = \\ = M^{Aşc}_{dr(Citp)\&circle(m)} \bigcup M^{Def}_{dr(Citp)\&circle(m)} \bigcup M^{PrN}_{dr(Citp)\&circle(m)} = \\ = \begin{Bmatrix} planing;\ shaping; broaching; punching/perforating\ with\ dies; \\ electrical\ discharge\ machining\ with\ a\ solid\ electrode \end{Bmatrix} \quad (1)$$

The methods of broaching and punching with dies can be considered representative for the Ci_{tp}&m case. Figure 1 exemplifies the generation by broaching of an internal cylindrical surface characterized by a directrix curve that is a straight line obtained kinematically as the trajectory of a point and a generatrix curve that is a materialized circle.

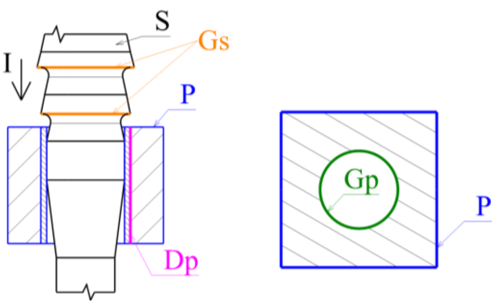


Fig. 1. Processing scheme corresponding to the case Ci_{tp}&m

The process requires a single generating movement, the primary translational movement I performed by tool S. As a result of this movement, the directrix curve Dp of the workpiece P is obtained. The generatrix curve Gs of the tool is transposed as the generatrix curve Gp of the workpiece. Since the generatrix curve Gp of the workpiece is materialized, it does not require any generating movement to be obtained.

Most often, broaching machines are hydraulically operated. A hydraulic diagram that minimally satisfies the kinematic requirements indicated in the processing scheme from Figure 1 is shown in Figure 2. The scheme identifies the following components: motor M1 (with rotational speed n_{M1}), safety coupling C_{S1}, hydraulic pump PH1, three-position distributor, linear hydraulic motor MHL1, hydraulic oil tank T1, maximum valve with inlet control.

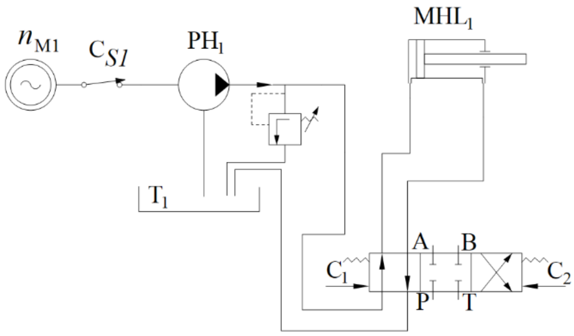


Fig. 2. Hydraulic actuation scheme corresponding to the case Ci_{tp}&m

The primary translational movement is performed by the rod of the linear hydraulic motor MHL_1, and the direction of this movement is changed via the hydraulic distributor.

2.2. Case Ci_{tp}&co

The set of processing methods identified at this stage as possible for the Ci_{tp}&co case in question is given by relation (2), namely:

$$M_{dr(Citp)\&circle(co)} = \\ = M^{Aşc}_{dr(Citp)\&circle(co)} \bigcup M^{Def}_{dr(Citp)\&circle(co)} \bigcup M^{PrN}_{dr(Citp)\&circle(co)} = \\ = \left\{ \begin{matrix} turning; planing; \ shaping; broaching; nibbling; \\ electrical\ discharge\ machining\ with\ a\ solid\ electrode \end{matrix} \right\} \quad (2)$$

The Ci_{tp}&co case can be exemplified by the scheme in Figure 3, which depicts a nibbling process for sheet metal processing.

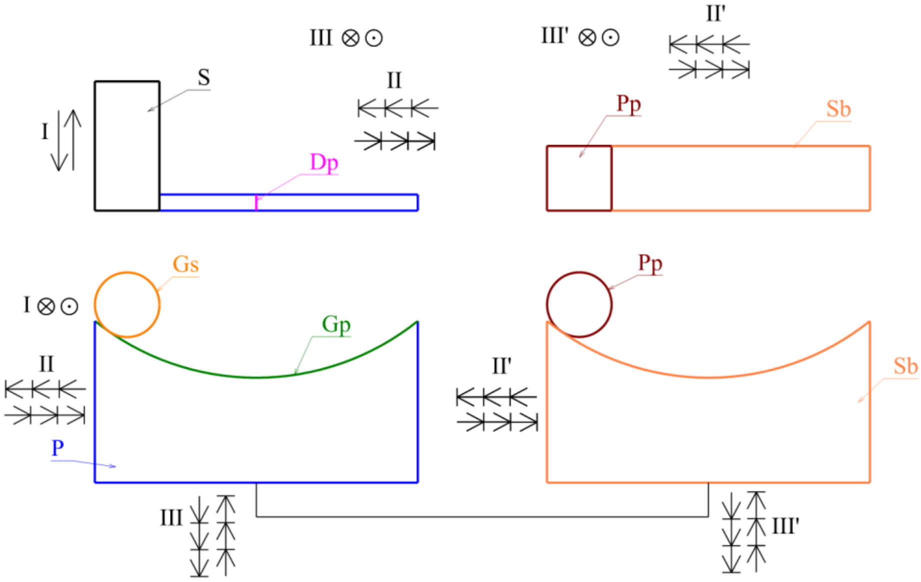


Fig. 3. Processing scheme for punching corresponding to the case Ci_{tp}&co

The directrix curve Dp of the workpiece is obtained as a result of the main movement, which is the rectilinear-alternating translation of tool S, a cylindrical punch with generating curve Gs. The workpiece P performs an intermittent translational movement II, with a small and constant step size. A template Sb is attached to the workpiece P and performs an identical movement II' to that of the workpiece.

The template Sb is always in contact with a fixed feeler Pp, which has an identical profile to tool S. Consequently, the template Sb implicitly performs another intermittent movement III', perpendicular to movement II', with the same frequency as movement II' but variable in magnitude. Movement III' is transmitted in a 1:1 ratio to the workpiece P as movement III. As a result of movements II and III, the generatrix curve Gp of the workpiece P, which copies the profile of the template Sb, is obtained. The fact that the generatrix curve Gp results as the envelope of the successive positions of the workpiece P relative to tool S is of little relevance in this case.

Outside the processing operation, movements II and III also serve as auxiliary movements for relative positioning between the workpiece and the tool.

A principal kinematic scheme, with links between the kinematic chains corresponding to the described Ci_{tp}&co case, is presented in Figure 4.

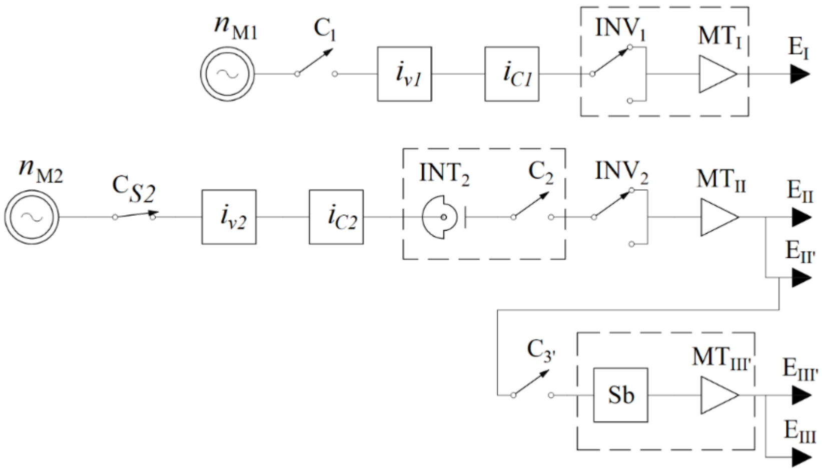


Fig. 4. Principal kinematic scheme with links corresponding to the case Ci_{tp}&co

In the structure of the kinematic chain of movement II, the presence of an intermittent mechanism INT_2 is imperative, ensuring its discrete nature. Translational movement II cannot be infinite in the same direction, thus necessitating the presence of a reversing mechanism INV_2. The transformation mechanism MT_2 ensures the conversion of rotational movement (transmitted from the driving electric motor M2) into the translational movement of the effector element E_{II}, a sled that drives the workpiece P.

The template-feeler mechanism, which copies the profile of the template, implicitly contains the mechanism for transforming (in terms of direction and magnitude) movement II' into movement III'.

2.3. Case Ci_{tp}&ci_{tp}

The set of processing methods identified at this stage as possible for this case is expressed by relation (3), namely:

$$\begin{aligned} M_{dr(Citp)\&circle(citp)} &= \\ &= M^{Aşc}_{dr(Citp)\&circle(citp)} \bigcup M^{Def}_{dr(Citp)\&circle(citp)} \bigcup M^{PrN}_{dr(Citp)\&circle(citp)} = \\ &= \begin{Bmatrix} turning;\ sawing;\ \ laser\ cutting; \\ plasma\ cutting; abrasive\ water\ jet\ cutting \end{Bmatrix} \end{aligned} \quad (3)$$

This is a more particular case, as there are not many methods available that provide both a straight directrix curve and a circular generatrix curve, both obtained kinematically as the trajectory of a point. The author has devised an original processing scheme with a chain saw, depicted in Figure 5, which is currently unidentified in the state of knowledge and practice of surface generation.

Primary Movement I is the guided translational movement of the chain teeth, obtained by transforming the rotational movement of the driving wheel of the wheel-chain mechanism. In this way, the straight directrix curve Dp is obtained kinematically as the trajectory of a point.

Movement II is a continuous circular feed movement in both directions, determining the generatrix curve Gp, also kinematically obtained as the trajectory of a point.

Auxiliary movements III, IV, and V are for the relative positioning of the tool and workpiece, including adjusting the radius of the workpiece's generatrix curve Gp and thus the position of the rotational axis of movement II relative to the workpiece. Movement III also serves as a penetration feed movement, necessary for removing a new layer of material.

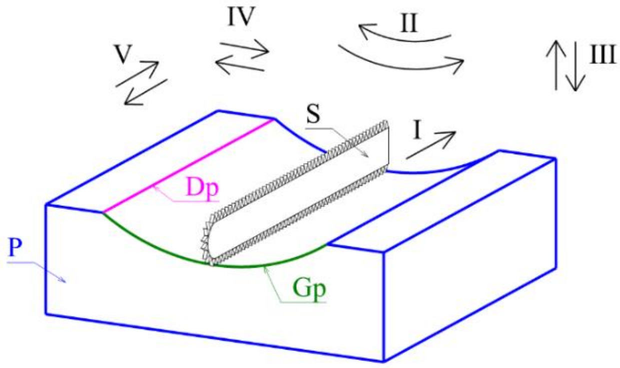


Fig. 5. Processing scheme for cutting with a chain saw, corresponding to the case $Ci_{tp}\&ci_{tp}$

Using the kinematic chains-type theory, the principal kinematic scheme without links, depicted in Figure 6, can be easily constructed. This scheme is associated with the processing scheme in Figure 5, allocating an appropriate kinematic chain-type to each movement.

This method ensures that each movement in the process has a corresponding kinematic chain, providing a clear and organized representation of the entire processing operation.

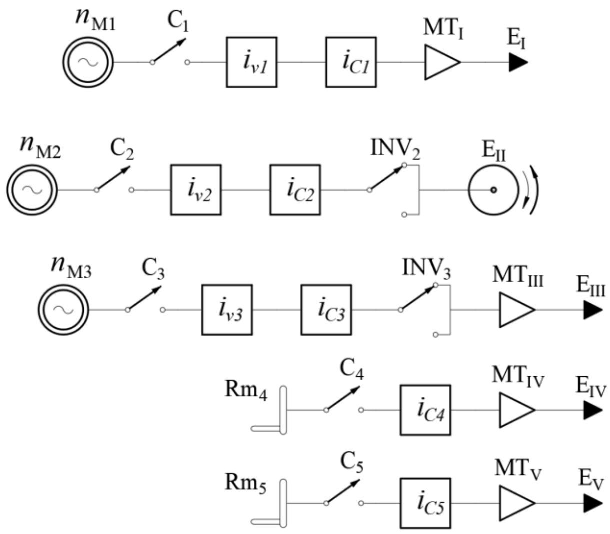


Fig. 6. Principal kinematic scheme without links corresponding to the case $Ci_{tp}\&ci_{tp}$

A detailed, minimalist kinematic scheme that aligns with the principal kinematic scheme shown in Figure 6 is presented in Figure 7 as a 3D representation. This detailed scheme does not include the auxiliary kinematic chains corresponding to the movements IV and V for driving the workpiece.

2.4. Case $Ci_{tp}\&ci_{fc}$

The set of possible processing methods for this case is expressed by relation (4), namely:

$$M_{dr(Citp)\&circle(cifc)} = \\ = M^{Aşc}_{dr(Citp)\&circle(cifc)} \bigcup M^{Def}_{dr(Citp)\&circle(cifc)} \bigcup M^{PrN}_{dr(Citp)\&circle(cifc)} = \\ = \left\{ \begin{matrix} turning;\ planing;\ shaping;\ broaching, sawing; \\ nibbling;\ electrical\ discharge\ machining\ with\ a \\ solid\ electrode \end{matrix} \right\} \tag{4}$$

The case is illustrated by a shaping processing scheme, shown in Figure 8, which is sometimes encountered in industrial practice.

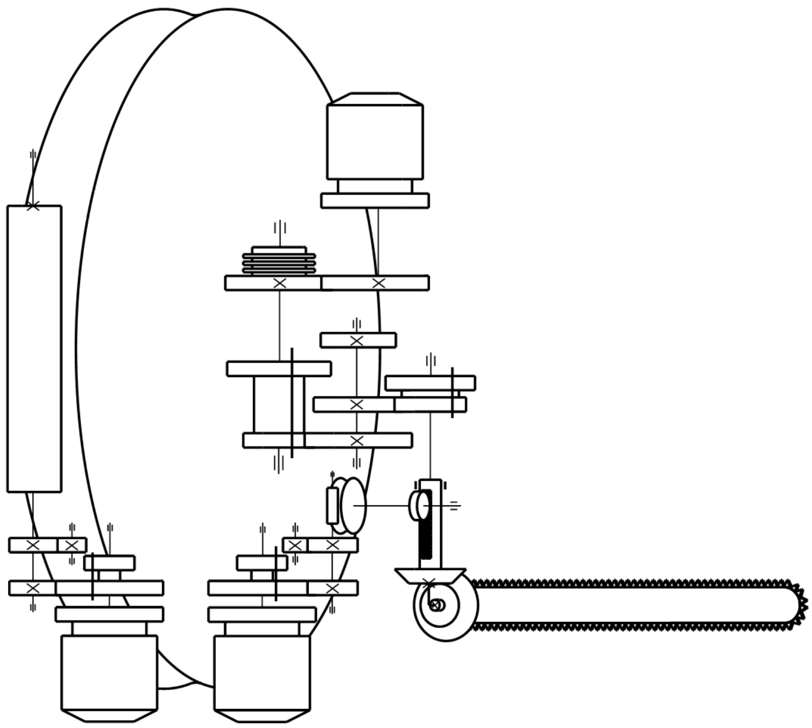


Fig. 7. Detailed kinematic scheme corresponding to the case $Ci_{tp}\&ci_{tp}$ – 3D view

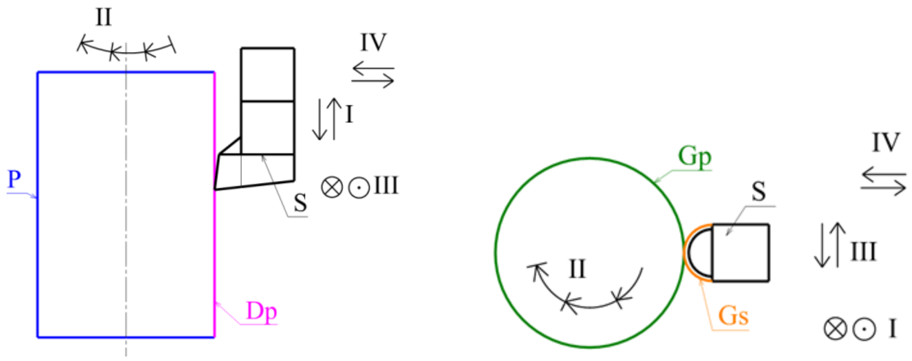


Fig. 8. Shaping processing scheme corresponding to the case $Ci_{tp}\&ci_{fc}$

Movement I: The rectilinear-alternating translation executed by the cutting tool is necessary to obtain the straight directrix curve Dp of the workpiece, generated as the trajectory of a point. Generatrix curve Gp is obtained as the envelope of the successive positions of the traces left on the workpiece by the generatrix curve Gs of the tool. These traces are created due to the successive angular repositioning of the workpiece P relative to the tool S, as a result of the discrete circular feed movement II of the workpiece P. Auxiliary movements III and IV are for positioning the workpiece P relative to the tool S.

A principle kinematic diagram corresponding to this case is presented in Figure 9.

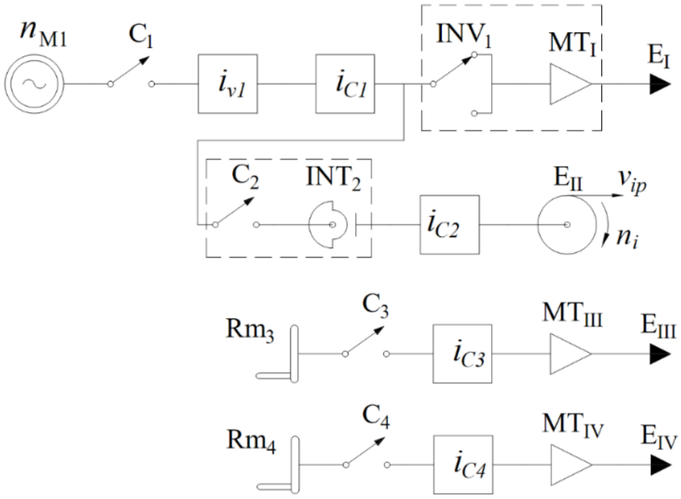


Fig. 9. Principle kinematic diagram with links corresponding to the case $Ci_{tp}\&ci_{fc}$

To achieve the primary movement I, which is a rectilinear-alternating translation with auto-return, the main kinematic chain includes the transformation mechanism MT_1. This mechanism implicitly contains a reversing mechanism INV_1. The frequency of movement I is adjusted via the mechanism i_{v1}.

Kinematic chain of movement II: The discrete circular feed movement of the workpiece is connected to the main kinematic chain of movement I, ensuring that movements I and II have the same frequency.

Auxiliary movements III and IV are manually driven using handwheels R_{m3} and R_{m4}, ensuring precise positioning of the workpiece relative to the tool.

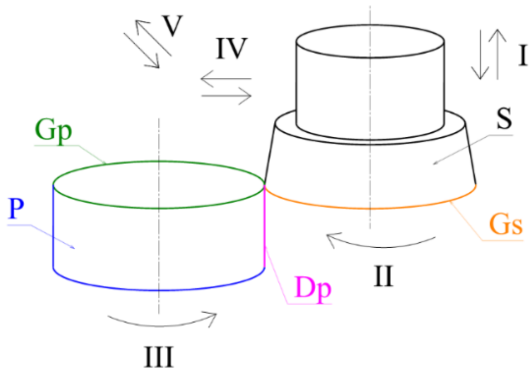
Although shown as a circular arc in Figure 8 for direct and explicit comparison with the similar generatrix curve obtained through nibbling in Figure 3, the tool's generatrix curve Gs does not have to be a circular arc. For cost efficiency, it is preferable for the generatrix curve Gs to be straight, leading to the profile Gp of the workpiece being generated by tangents. Generally, Gs can be any convex curve.

2.5. Case $Ci_{tp}\&r$

Possible processing methods for this case are:

$$\begin{aligned} &M_{dr(Citp)\&circle(r)} = \\ &= M^{Aşc}_{dr(Citp)\&circle(r)} \bigcup M^{Def}_{dr(Citp)\&circle(r)} \bigcup M^{PrN}_{dr(Citp)\&circle(r)} = \\ &= \left\{ \begin{matrix} turning; planing;\ shaping; broaching; \\ electrical\ discharge\ machining\ with\ a\ solid\ electrode \end{matrix} \right\} \end{aligned} \tag{5}$$

Figure 10 presents a shaping processing scheme suitable for the $Ci_{tp}\&r$ case. This is supported by an example found in industrial practice, where shaping with an appropriately profiled knife-wheel is used for machining chain wheels.



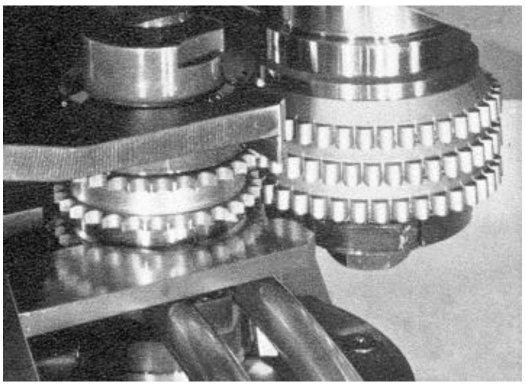


Fig. 10. Processing scheme for the $Ci_{tp}\&r$ case and practical example

The scheme has all the characteristics corresponding to a shaping machine with a cylindrical-tooths knife-wheel, such as the MD 250 machine. Primary movement I: rectilinear-alternating translation with auto-return, which describes the directrix curve Dp of the workpiece. Circular feed movement II is made by the tool. Circular feed III of the workpiece in a rigid kinematic connection imposed by the rolling condition, both necessary to achieve the generatrix curve Gp through rolling. Auxiliary movements IV and V: for relative positioning between the tool and the workpiece. A technological movement for discrete and small-amplitude translation to approach and retract between the tool and the workpiece to avoid friction during idle time is not represented.

The scheme in Figure 10 does not capture the details regarding the tool's profile and the profile of the processed workpiece. The kinematic requirements associated with the scheme in Figure 10 are specified in the principle kinematic diagram with links represented in Figure 11. Strict proportionality is only imposed between rotational movements II and III, ensured by a rolling-type kinematic chain [1, 2].

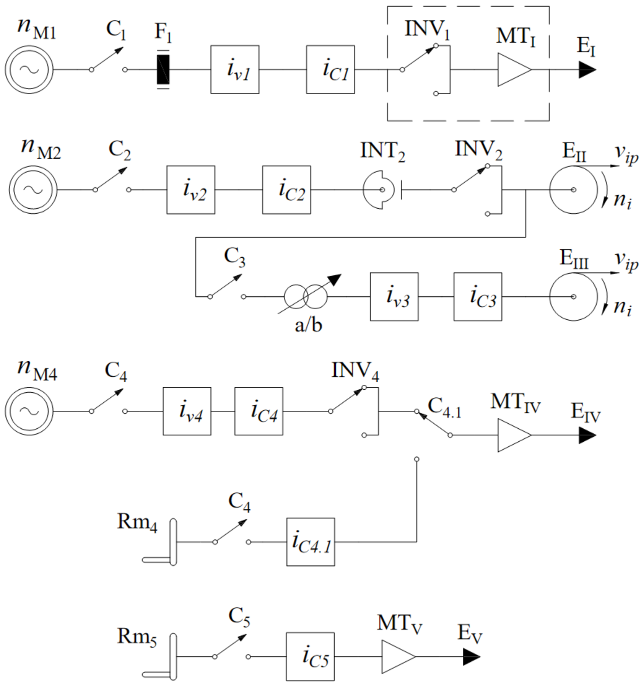


Fig. 11. Principle kinematic scheme with links corresponding to the case Ci_{tp}&r

Frequency adjustment of movement II is managed through the regulation mechanism i_{v2}. Ratio between movements III and II: controlled by the change gear mechanism a/b, expressly indicated. Reversing mechanism INV_3 allows the workpiece to be processed either on its exterior or interior. Auxiliary movement IV is achieved with a mixed translational motion kinematic chain, allowing the selection of either mechanical or manual drive. Auxiliary movement V is achieved with a simple translational motion kinematic chain, driven manually.

2.6. Case Ci_{tp}&p

It should be emphasized that any processing scheme where the generatrix curve is obtained kinematically (co, ci_{tp}, ci_{fc}, or r) can be easily converted into a processing scheme where the generatrix curve is programmed. Consequently, the set of processing methods is richer. For the case in question, this set is expressed by relation (6).

The current case is exemplified by the processing scheme shown in Figure 12. This scheme is derived from the one corresponding to the Ci_{tp}&co case, depicted in Figure 3, where the generation of the curve by copying is replaced with a programmed generator.

Main movement I, which is a rectilinear-alternating movement with auto-return, is executed by the tool S, a cylindrical punch. The directrix curve Dp is described kinematically as the trajectory of a point. The generatrix curve Gp is obtained as a result of the correlated action of movements II and III, both numerically controlled, both discrete, and executed during the idle stroke of movement I while the tool S is no longer in contact with the workpiece P.

$$M_{dr(Citp)\&circle(p)} = \\ = M^{Aşc}_{dr(Citp)\&circle(p)} \bigcup M^{Def}_{dr(Citp)\&circle(p)} \bigcup M^{PrN}_{dr(Citp)\&circle(p)} = \\ = \left\{ \begin{array}{c} turning; planing;\ shaping; broaching;\ sawing; \\ stamping; \\ laser\ cutting; plasma\ cutting; abrasive\ water\ jet\ cutting; \\ electrical\ discharge\ machining\ with\ a\ solid\ electrode \end{array} \right\} \quad (6)$$

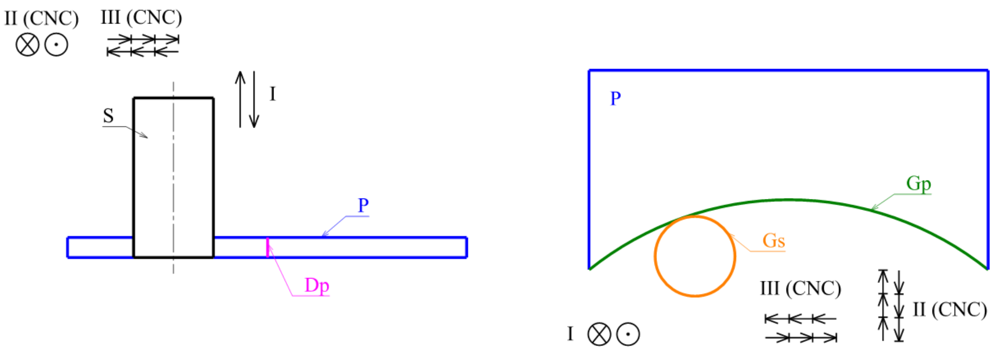


Fig. 12. Processing scheme corresponding to the case Ci_{tp}&p

A suitable principle kinematic scheme is illustrated in Figure 13. Movement I is independent and does not require numerical control. Movements II and III also serve as auxiliary movements for relative positioning in the horizontal plane of the workpiece relative to the tool outside the processing. Movements II and III are performed by distinct numerically controlled kinematic chains but are correlated through the action of the ENC (Numerical Control Equipment).

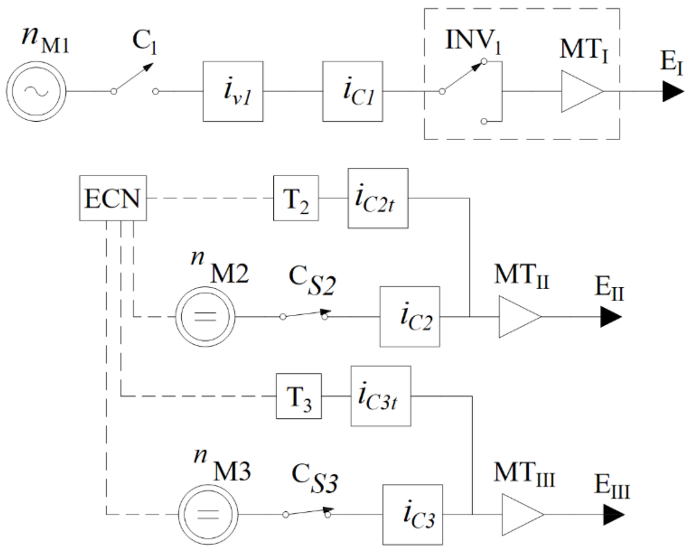


Fig. 13. Principal kinematic scheme corresponding to the case Ci_{tp}&p

3. Conclusions

The same surface can be obtained using various processing methods and, at least theoretically, using any combination of generatrix and directrix curve realizations. The work addresses cases characterized by a straight directrix curve obtained kinematically as the trajectory of a point (Ci_{tp} cases) and a circular generatrix curve for all six modes of realization (m, co, ci_{tp}, ci_{fc}, r, and p). Each case is exemplified by a

processing scheme and an appropriate principle kinematic scheme, which relies on the theory of type kinematic chains. Most of the examples presented are found in the current state of the art and correspond to either cutting processes or plastic deformation processes. There are few specialized works that approach plastic deformation processes from the perspective of surface generation theory.
One of the examples given, the $Ci_{tp}\&ci_{tp}$ case, is original and results from an invention process. This reflects the fact that it is rare to identify cases where both the guiding curve and the generating curve are obtained kinematically as the trajectory of a point, but also that through creative and innovative effort, new processing schemes can be conceived.

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