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Mathematical Modelling in Manufacturing Planning and Optimization

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Abstract
The company's production planning involves determining the quantity of products that could be produced in a certain period under specified technical and organizational conditions. In the conditions of demand fluctuations, the problem of establishing an optimal production schedule arises. Usually, optimization involves evaluating the different possible variants using an efficiency criterion and choosing the variant that corresponds to the convenient value in terms of the type of criterion (maximum or minimum). The evaluation can be done in two ways: 1) creating production scheduling variants, checking them in terms of their consistency with the available capacity, evaluating them in terms of the costs involved or 2) mathematical modelling - a faster and more efficient way to solve the planning. The paper presents aspects regarding the hierarchical nature of the planning process, the content of the annual production plan, as well as a case study detailing the method of establishing an optimal manufacturing schedule using mathematical modelling through linear programming.

Keywords
manufacturing planning, optimization, linear programming, efficiency

1. Introduction

Every management act has as its starting point the creation of a plan. Planning is the managerial sequence on which all others are based: an unplanned thing cannot be organized or controlled.

Planning aims to orient the activity, establish the objectives to be achieved, the means and resources necessary to achieve them. Plans can be extremely simple or very complex, the main elements that differentiate them being the planning horizon and the degree of detail. Considering these aspects, at the organizational level there are multiple forms of planning: strategy, business plan, current plan, operational-calendar plans [1, 2, 3].

The process of planning productive activities, like the decision-making process, has a hierarchical character [2] unfolding in stages and materializing in different approaches in terms of the tools and working methods used. These stages are:

- strategic planning – issues of management strategy are considered (assimilation in manufacturing of products, directions and size of investments, etc.), it targets time intervals longer than one year, uses a high level of information aggregation and works with synthetic indicators of the organization's activity;
- tactical (current) planning – aims at implementing strategic decisions, the planning interval is usually one year, or divided into quarters, months. The tool used in planning the activity at this level is the production plan (annual/quarterly/monthly) in which information is centralized by product types is used;
- operational planning (production scheduling) – aims at implementing decisions from the previous planning level, detailing up to the job level information regarding types and quantities to be manufactured, delivery dates, purchases to be made, equipment loading, accepted level of scrap, stocks, etc. The planning horizon is the week, day, hour and the tool used is the manufacturing order.

The annual production plan is a tactical planning tool, can be developed for departments, industrial segments, production workshops, specifying the types of products to be manufactured, the quantities requested by customers [3, 4]. This information serves the coordinated development of the other plans of the enterprise's activity: supply, personnel, equipment maintenance, investments, etc. The diagram in Figure 1 illustrates the main elements of a plan and the correlations between them [5].

The central place in the organization's plan is occupied by production (Figure 1). The plan specifies the type/quantity of products to be manufactured and the value of production. These levels are based on firm orders or forecasts. On the other hand, the planned quantity is established and adjusted considering the existing production capacity, the level of which measures the maximum productive potential of the organization. If the forecasted demand and the planned production exceed the production capacity, then investments must be planned. On the other hand, the need for material resources, labor, and specific consumption are separately highlighted, which are subsequently found in the production costs. Costs, along with income and profit, are the main economic and financial indicators characterizing the organization's activity [1, 2, 6].

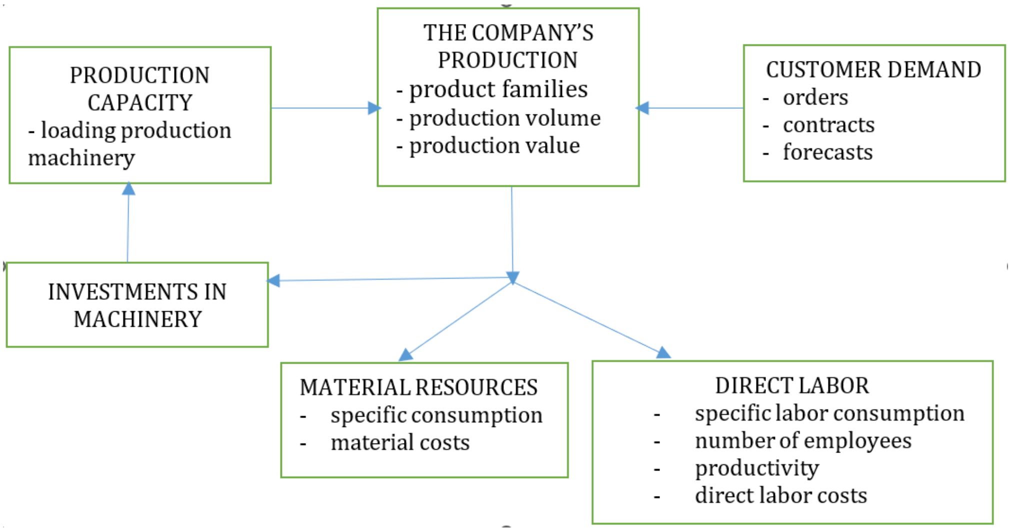

Fig. 1. Production's organization within the company

2. Case study on manufacturing planning and optimization using the linear programming method

2.1. The general framework

Considering the above, the company's production plan will include [2]:

- specific product types, which are part of families: products that have common characteristics and are similar in terms of manufacturing technology.
- firm orders received from various partners, regarding the manufacture of products or works based on collaboration contracts, etc.

As a result, the targets for current planning will be the delivery terms requested by the beneficiaries and specified in the contracts concluded. However, there may be situations in which, due to fluctuations in demand and the impossibility of forecasting these fluctuations, the production capacity is insufficient. For this reason, one of the solutions would be that in periods of excess capacity, more is manufactured than is required to be delivered, to create stocks from which to supplement the quantities that will be delivered in periods of capacity deficit. The stocks created in these situations will be limited to the strict minimum in order not to generate additional costs, therefore inefficiency.

Under these conditions, the problem of establishing an optimal manufacturing program arises. Optimization involves evaluating the different possible variants using an efficient criterion and choosing the variant that corresponds to the appropriate value in terms of the type of criterion (maximum or minimum). The evaluation can be done empirically or using mathematical modeling through the linear programming method. The second option is a faster and more efficient way to solve the problem and guarantees the achievement of the optimum [1, 6].

If that lasts for n months, the company must manufacture a certain quantity of products, q_i, $1 \le i \le n$, of a certain type, requested by customers.

The production capacity of the company allows for the achievement of a production volume, X_i, $1 \le i \le n$, pcs/month, under the conditions of a normal established working regime. To avoid the situation of a deficit in productive capacity (which could be generated by delays in the delivery of materials, machine failures, exceeding the allowed percentage of scrap, etc.), an additional production, Y_i, $1 \le i \le n$, pcs/month, can be provided. The additional production could be manufactured during overtime hours (e.g. Saturday and Sunday).

2.2. Basic information for the case study

Manufacturing planning involves determining the quantity of products that could be produced under normal working conditions, the quantity of products that could be produced during overtime and the quantity that should be stored from the quantity produced, provided that a chosen efficiency criterion is met [3, 7].

The situation can be modeled using mathematical programming – linear programming. A linear programming model aims to maximize or minimize an expression – objective function, whose variables satisfy certain restrictions (conditions). In the linear programming model, both the objective function and the restrictions are expressed mathematically in the form of linear functions. The objective function and the restrictions are constructed considering the concrete characteristics of the real situation being modeled [1, 6, 8].

To create the linear programming model, the following were considered:

- c_i, the unit cost of production, under the established normal working regime.
- c_{si}, the unit cost of production, during overtime.
- h_i, $1 \leq i \leq n$, the unit costs of storage, monthly.
- x_i, the quantity of product P_i to be manufactured under normal working regime, in month i;
- y_i, the quantity of product P_i to be manufactured under overtime regime, in month i;
- S_i, the quantity of product P_i to be stored in month i.

To establish an optimal manufacturing schedule, the criterion of minimizing the costs involved (manufacturing and storage) (Eq. 1) was used.

Considering the above elements, the mathematical model of linear programming contains: the objective function (Eq. 1) and the constraints (Eq. 2, 3, 4, 5).

$$\min f = \min CT = \sum_{i=1}^{n}(c_i * x_i + c_{si} * y_i + h_i * S_i) + c_1 * x_1 + c_{s1} * y_1 \tag{1}$$

$$0 \leq x_i \leq X_i\,; 1 \leq i \leq n \tag{2}$$

$$0 \leq y \leq Y_i\,; 1 \leq i \leq n \tag{3}$$

$$S_i \geq 0\,; 1 \leq i \leq n \tag{4}$$

$$\sum_{k=1}^{i}[(x_k + y_k) - q_k] = S_{i+1}\,; 1 \leq i \leq n-1 \tag{5}$$

in which:

$x_k + y_k$, the quantity of products produced monthly in normal and additional working mode;

q_k, the customer demand, monthly.

S_{i+1}, the quantity of products that will be stored in the following month, in the event of a capacity deficit.

Equation (5) shows that the difference between what was manufactured monthly ($x_k + y_k$) and what should have been manufactured as a customer requirement, q_{k-1}, is stored in the following month, S_{i+1}.

The objective of this case study is to establish the optimal monthly manufacturing schedule (for the next six months of 2026) for a company in the automotive industry. To achieve the objective, the linear programming method was used.

To build the mathematical model, the data presented in Table 1 were considered:

- demand for a certain reference – representative serial product for the company, demand expressed through the real contracts.
- available production capacity for normal working hours (1/8-hour work shift, 5 days/week). The production capacity was calculated considering the productivity of the assembly process on the studied line = 180 pcs/hour.

- possibility of working overtime (1/8-hour work shift, Saturday and/or Sunday), to cover a possible production capacity deficit.

Daily production capacity = 7.5 hours/shift × 180 pcs/hour = 1350 pcs/day.

Depending on the number of working days, respectively the number of Saturdays and Sundays in each month (for the first months of 2026), the production capacity values were determined, in normal working mode and overtime. The values are shown in Table 1.

- production costs (lei/pcs.) for normal working mode and overtime
- storage costs (lei/month). For the calculation of storage costs, a coefficient of 12% was considered applied to the unit cost in normal mode compared to the number of months for which the planning is done (six months).

Table 1. Collected and calculated data

Month	January	February	March	April	May	June
Demand (pcs/month)	30878	28991	31009	29540	28257	29657
Production capacity -normal working conditions (pcs/month)	22950	27000	29700	27000	27000	28350
Production capacity-overtime (pcs/month)	8100	9450	10800	6750	9450	10800
Unit production cost - normal conditions (lei/pcs)	36	38	36	35	39	36
Unit production cost -overtime (lei/pcs)	39	37	38	36	37	39
Unit holding cost (lei/ lei. Month)	0.72	0.76	0.72	0.7	0.78	0.72

The evaluation criterion used is that of minimizing the costs involved in the adopted manufacturing and storage program. The cost minimization criterion means a level of sales prices that allows maximizing the benefits obtained, as a premise for achieving the desired profitability.

For the mathematical model, 17 integer variables non-negative were used, and 17 restrictions related to the adopted working regime and the conditions relating to stocks were defined. The linear programming model is presented in Eq. (6) ... (18).

$$\min CT = 36x_1 + 39y_1 + 38x_2 + 37y_2 + 0.76S_2 + 36x_3 + 38y_3 + 0.72S_3 + +35x_4 + 36y_4 + 0.7S_4 + 39x_5 + 37y_5 + 0.78S_5 + 36x_6 + 39y_6 + 0.72S_6 \tag{6}$$

$$x_1 \leq 22950; \quad y_1 \leq 8100 \tag{7}$$

$$x_2 \leq 27000; \quad y_2 \leq 9450 \tag{8}$$

$$x_3 \leq 29700\ ; \quad y_3 \leq 10800 \tag{9}$$

$$x_4 \leq 27000; \quad y_4 \leq 6750 \tag{10}$$

$$x_5 \leq 27000; \ y_5 \leq 9450 \tag{11}$$

$$x_6 \leq 28350; y_6 \leq 10800 \tag{12}$$

$$x_1 + y_1 - S_1 = 30878 \tag{13}$$

$$x_1 + y_1 + x_2 + y_2 - S_3 = 59869 \tag{14}$$

$$x_1 + y_1 + x_2 + y_2 + x_3 + y_3 - S_4 = 90878 \tag{15}$$

$$x_1 + y_1 + x_2 + y_2 + x_3 + y_3 + x_4 + y_4 - S_5 = 120418 \tag{16}$$

$$x_1 + y_1 + x_2 + y_2 + x_3 + y_3 + x_4 + y_4 + x_5 + y_5 - S_6 = 148675 \tag{17}$$

$$x_i \geq 0;\ y_i \geq 0\ ;S_i \geq 0 \qquad (18)$$

3. Results

The solution of the system of inequalities was performed using the WinQSB (Windows Quantitative Systems for Business) software. Figure 2 shows the data entry method and Figure 3 shows the results obtained after running the program.

Linear and Integer Programming - [LP Sample Problem]

File Edit Format Solve and Analyze Results Utilities Window WinQSB Help

Minimize : X13 0.76

Variable -->	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	Direction	R. H. S.
Minimize	36	39	38	37	36	38	35	36	39	37	36	39	0.76	0.72	0.7	0.78	0.72		
C1	1	1											-1					=	30878
C2	1	1	1	1										-1				=	59869
C3	1	1	1	1	1	1									-1			=	90878
C4	1	1	1	1	1	1	1	1								-1		=	120418
C5	1	1	1	1	1	1	1	1	1	1							-1	=	148675
C6	1																	=	22950
C7		1																<=	8100
C8			1															=	27000
C9				1														<=	9450
C10					1													=	29700
C11						1												<=	10800
C12							1											=	27000
C13								1										<=	6750
C14									1									=	27000
C15										1								<=	9450
C16											1							=	28350
C17												1						<=	10800
LowerBound	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
UpperBound	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M		
VariableType	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer	Integer		

Fig. 2. Coefficients of the model variables

11-28-2025 16:42:18	Decision Variable	Solution Value	Unit Cost or Profit C(j)	Total Contribution	Reduced Cost	Basis Status
1	X1	22,950.0000	36.0000	826,200.0000	0	basic
2	X2	7,928.0000	39.0000	309,192.0000	0	basic
3	X3	27,000.0000	38.0000	1,026,000.0000	0	basic
4	X4	3,300.0000	37.0000	122,100.0000	0	basic
5	X5	29,700.0000	36.0000	1,069,200.0000	0	basic
6	X6	0	38.0000	0	0.2800	at bound
7	X7	27,000.0000	35.0000	945,000.0000	0	basic
8	X8	3,797.0000	36.0000	136,692.0000	0	basic
9	X9	27,000.0000	39.0000	1,053,000.0000	0	basic
10	X10	0	37.0000	0	0.2200	at bound
11	X11	28,350.0000	36.0000	1,020,600.0000	0	basic
12	X12	0	39.0000	0	39.0000	at bound
13	X13	0	0.7600	0	2.7600	at bound
14	X14	1,309.0000	0.7200	942.4800	0	basic
15	X15	0	0.7000	0	2.4200	at bound
16	X16	1,257.0000	0.7800	980.4600	0	basic
17	X17	0	0.7200	0	37.5000	at bound
	Objective	Function	(Min.) =	6,509,907.0000		

Fig. 3 Results obtained

Table 2. Optimal manufacturing schedule

	x_i (pcs)	y_i (pcs)	x_i+y_i (pcs)	Demand (pcs/month)	Stock (S_i) (pcs/month)
January	22950	7928	30878	30878	-
February	27000	3300	30300	28991	+1309 (S_3)
March	29700	0	29700	31009	-1309
April	27000	3797	30797	29540	+1257 (S_5)
May	27000	0	27000	28257	-1257
June	28350	0	28350	29657	-1307

Using a linear programming model the optimal planning variant was determined, which would meet the requirements imposed by the concrete working conditions.

It is observed that in March there will be no additional work (y_i = 0) because the stock of products (S_3 = 1309 pieces) in February covers the demand of 31009 pieces. Also, in May there is no additional work, the difference of 1257 pieces will be covered by the production in April. In June we observe a deficit in quantity (-1307 pieces). In this case, overtime could be worked to cover this deficit. The optimal manufacturing schedule is obtained with a minimum total cost of 6509907 lei/6 months.

4. Conclusions

The planning process is dynamic and complex. Due to the disruptive factors that can affect the production system (machine malfunctions, cancellation of some ordered quantities, the appearance of additional orders), the planning must be resumed, revised to be in accordance with the new conditions. Revising the plan means repeatedly solving the mathematical model, at a certain period, updating the parameters that have changed during that period.

The use of mathematical modeling for such situations has advantages:

- It is easy to use, many situations in productive activity can be modeled using linear programming models;
- A wide variety of factors that may intervene in the planning process can be considered. In this way, the premises for achieving well-founded scheduling are created;
- The decision-maker - the person who makes the plan will solve routine situations faster and will be able to dedicate more time to exceptional situations.

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